APPLIED MATHEMATICS

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Mathematics

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Preface

The following notes have been arranged in an effort to meet the needs for a text involving practical application of elementary mathematics to the common everyday problems in business matters, industrial activities, manufacturing, and trade work. They relate primarily to fractions, decimals, money, percentage, linear measure, board measure, square measure, cubic measure, determining quantities, making estimates, and a wide variety of calculations involving the use of shop formulas.

Its practical nature with its informational value marks it as being different from most texts of this kind, making it suitable for Junior and Senior High Schools, Vocational Schools, Trade Schools, and Apprenticeship Courses. It is also very well adapted for evening-school use and for home study.

The instructional material and the 1100 and more problems meet full requirements for elementary mathematics in the building-trades branches, including carpentry, cabinetmaking, lathing, plastering, painting, paperhanging, sheet-metal work, brick, mason, and concrete work; the printing trades; the machinist and toolmaking trades; the auto-mechanics trades; and the general factory type of work. The draftsmen and the technical branches of industry will also find it of much help to them in the solution of the many problems that confront them daily.

The text is divided into suitable units of instruction followed by the answers that relate to the problems in each unit. As a check upon the student's ability the instructor is provided with selected review problems.

The several hundred illustrations, sketches, and drawings throughout the text are so arranged that the student may receive a gradual training in the interpretation and understanding of shop sketches and working drawings.

Selected references and shop formulas are compiled at the end of the book for use in the solution of the problems involving shop formulas.

TO THE TEACHER

Attention of the teacher is directed to the selection of instructional material, the large number of illustrated examples and the completeness of the explanations; to the arrangement of this material upon a unit basis, together with the answers to the problems that relate to each unit; to the selected review problems for checking and examination; to the gradual development of an understanding and use of shop sketches and working drawings; to the many reference tables and shop formulas and to the general guidances, suggestions that are ever present in the trade information and the problems.

The teacher should encourage the student to use the reference tables in solving the problems relating to shop formulas, much as he would use a handbook, and to make simple shop sketches to use in connection with his calculations. These practices are of great value to the student. It would be well also to encourage him to preserve the complete solution of his problems in a suitable notebook for future reference, as there is every probability that he will use some of these later on when he goes to work.

In the preparation and arrangement of this text the author has been fortunate in having valuable assistance and advice from many sources including business, industry, skilled tradesmen, and educators. He fully realizes that without such help it would be difficult to provide this complete text.

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Contents

FRACTIONS

Addition; subtraction; multiplication; division; cancellation; applications to daily problems in the home, in business, and in industry.

Fractions

A clear understanding of what a fraction is, may be obtained from a careful examination of the inch measurement, as found on the ordinary 12-inch rule.

12-INCH RULE

Fig. 1

Fig. 2

It will be seen that each inch is divided into 16 equal parts, or divisions, as shown in enlargement in Figure 2. Each one of these divisions equals one-sixteenth part of the whole unit, the inch. This amount, one sixteenth, expressed by a number, is written as $\frac{1}{16}$. This kind of a number is called a *fraction*.

It represents, in this case, one of the sixteen equal parts into which this unit, one inch, is divided; namely, $\frac{1}{16}$ of an inch. Three of these divisions would, accordingly be expressed in fractional form as $\frac{3}{16}$ of an inch. In the same way five divisions would be expressed as $\frac{5}{16}$ of an inch.

From this brief explanation it may be seen that a fraction is *one* or *more of the equal parts* into which a unit is divided.

By examining each of these fractions, $\frac{1}{16}$, $\frac{3}{16}$, and $\frac{5}{16}$, it may be noted that each is made up of two parts, or terms. One term tells into how many equal parts the unit is divided. This is called the *denominator.* In the above fractions, 16 is the denominator, as it tells into how many equal parts the unit, one inch, is divided.

The other term of the fraction is called the *numerator.* This tells how many of the equal parts are taken to form, or to make up, the fraction. In the fraction $\frac{1}{16}$ the figure 1 is the numerator, as it tells that 1 of the 16 equal parts of the inch is taken to form the fraction $\frac{1}{16}$. In the fraction $\frac{3}{16}$, the figure 3 is the numerator, and in the fraction $\frac{5}{16}$, the figure 5 is the numerator, also for this same reason.

In each of these fractions, the figures 1, 3, and 5 tell *how many equal parts* are taken to form the given fractions.

It will be noted that the terms of each of these fractions are separated by a short straight line. The number *below* this line is the *denominator*, the number *above* the line is the *numerator.*

The number which separates the numerator from the denominator is sometimes referred to as a *division sign*, indicating that the *numerator* is *divided* by the *denominator.* This fact is used later on in dealing with certain processes involving fractions.

PROPER FRACTIONS

Fractions like $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{8}$, $\frac{8}{9}$, and such, where the numerator is *smaller* than the denominator, are called *proper fractions.*

Proper fractions always indicate an amount which is *less than* 1.

IMPROPER FRACTIONS

Fractions like $\frac{3}{2}$, $\frac{17}{8}$, $\frac{11}{6}$, $\frac{9}{8}$, and such, where the numerator is *larger* than the denominator, are called *improper fractions.* As may be seen these fractions are greater than 1.

Fractions like $\frac{2}{2}$ (read "two halves"), $\frac{3}{3}$ (read "three thirds"), and $\frac{4}{4}$ (read "four fourths"), in which the numerator equals the denominator, are also *improper fractions.*

Since the line between the numerator and the denominator indicates that the numerator may be divided by the denominator then the above improper fraction $\frac{2}{2}$ becomes $2 \div 2$, which in turn equals 1. In the same manner the fraction $\frac{3}{3}$ becomes $3 \div 3$, which also equals 1. Likewise $\frac{4}{4}$ equals 1.

This shows that a fraction whose numerator and denominator are the same is equal to 1.

From the above explanation it may also be seen that 1 may be changed to a fractional form by writing a fraction whose numerator and denominator are equal. For example, 1 may be changed to such fractional forms as $\frac{7}{7}$, $\frac{5}{5}$, $\frac{17}{17}$, $\frac{23}{23}$, $\frac{533}{533}$, and the like.

Accordingly, it may be stated that *improper fractions* may be changed to either a *whole number*, or to a combination of a *whole number and a fraction, as the case may be, by dividing the numerator by the denominator.*

To illustrate:

$$\frac{9}{9} = 9 \div 9 = 1$$
$$\frac{6}{3} = 6 \div 3 = 2$$
$\frac{3}{2} = 3 \div 2 = 1\frac{1}{2}$ (read "one and one half")
$\frac{17}{8} = 17 \div 8 = 2\frac{1}{8}$ (read "two and one eighth")
$\frac{9}{5} = 9 \div 5 = 1\frac{4}{5}$ (read "one and four fifths")

MIXED NUMBERS

When a whole number and a fraction are combined in one, as for example, $2\frac{1}{8}$ or $1\frac{4}{5}$, the combination is called a *mixed number*.

Such mixed numbers may be readily changed to *improper fractions* by multiplying the *whole* number by the *denominator* of the fraction, and then *adding* to this amount the *numerator* of the fractional part of the mixed number. This gives the number that is the new numerator of the improper fraction. This *new* numerator is then placed over the *original* denominator to form the *equivalent* improper fraction.

For example, in order to change the mixed number $2\frac{1}{8}$ to an improper fraction, the whole number 2 is multiplied by the denominator 8, giving as a result 16. To this is added the

numerator 1, giving a total of 17. Placing 17 over the denominator 8, there results the improper fraction $\frac{17}{8}$.

In this improper fraction the numerator 17 also indicates that in the mixed number $2\frac{1}{8}$ there are 17 eighths.

Applications of this rule are seen in the following examples.

Example 1:

Change $11\frac{13}{16}$ to an improper fraction.

Solution and Explanation:

Using the above rule, the first step is to determine the numerator of the desired improper fraction. This is done by multiplying 11 by 16 and adding to that the number 13.

11×16 equals 176. $\quad$ $176 + 13$ equals 189.

Placing this new numerator 189 over the original denominator 16, there results $\frac{189}{16}$ as the equivalent improper fraction.

$11\frac{13}{16}$ changed to an improper fraction, therefore, equals $\frac{189}{16}$.

Example 2:

How many $\frac{1}{8}$ inches are there in the measurement $5\frac{3}{8}$ inches?

Solution and Explanation:

The number of $\frac{1}{8}$ inches in $5\frac{3}{8}$ inches is determined by changing $5\frac{3}{8}$ to an improper fraction. As previously explained the numerator will indicate the number of $\frac{1}{8}$ inches in the original number.

According to the above rule, the numerator is 5×8 or 40, plus 3, which equals 43. The improper fraction then becomes $\frac{43}{8}$.

The numerator of this improper fraction indicates that in the measurement $5\frac{3}{8}$ inches there are 43 one eighth inches.

Problems Involving Expressing Fractions

1. Determine which of the following are mixed numbers, proper fractions, or improper fractions: $\frac{1}{4}$, $3\frac{3}{4}$, $\frac{5}{17}$, $\frac{13}{15}$, $\frac{19}{7}$, $1\frac{1}{2}$, $\frac{21}{5}$, $\frac{8}{4}$, $6\frac{7}{8}$.

2. Change to mixed numbers the following fractions: $\frac{5}{2}$, $\frac{9}{2}$, $\frac{11}{5}$, $\frac{19}{8}$, $\frac{113}{12}$, $\frac{16}{3}$, $\frac{13}{10}$, $\frac{41}{13}$, $\frac{18}{7}$, $\frac{23}{12}$.

3. How many eighths in $1\frac{1}{8}$ in., $2\frac{7}{8}$ in., $5\frac{1}{8}$ in., $11\frac{3}{8}$ in., $9\frac{5}{8}$ in.?

NOTE: The word *inch* or *inches* is indicated by the abbreviation in., or by the symbol (″) as in the measurement expressed as 5 in. or 5″.

4. How many pieces $\frac{1}{2}$ in. long can be cut from a strip of aluminum that measures $11\frac{1}{2}$ in. long?

5. It is desired to divide a line $4\frac{1}{4}$ in. long into spaces that measure $\frac{1}{4}$ in. long. How many such spaces in this line?

6. Change the following to improper fractions:

$$5,\ 71\tfrac{1}{16},\ 12\tfrac{3}{16},\ 8,\ 10\tfrac{9}{13},\ 9\tfrac{1}{8},\ 108\tfrac{15}{64}.$$

7. What fractional part of 9 is 5? What part of 65 is 48?

8. Which is greater, $2\frac{1}{8}$ or $\frac{19}{8}$; $5\frac{8}{9}$ or $\frac{17}{3}$; $\frac{13}{6}$ or $1\frac{5}{6}$?

9. In measuring the width of a strip of wood, a boy counts the divisions on his ruler and finds that the piece measures $\frac{15}{8}$ in. wide. What measurement is this equal to when expressed as a mixed number?

10. The thickness of a brass bar measures $\frac{23}{16}$ of an inch. What is this measurement equal to when expressed as a mixed number?

11. How many 32nds, in $3\frac{5}{32}$ in.; in $1\frac{7}{32}$ in.; in $19\frac{11}{32}$ in.?

12. If there are 36 in. in one yard what fractional part of a yard is 11 in.; 7 in.; 25 in.?*

REDUCTION OF FRACTIONS

Proper fractions like $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{8}$, and $\frac{9}{16}$ are said to be in their *lowest terms*. That is, both the numerator and the denominator cannot be further reduced by dividing each of them by the same number.

When in this condition they are referred to as being in their *simplest form*.

Such fractions as $\frac{2}{4}$, $\frac{12}{16}$, $\frac{9}{24}$, and $\frac{18}{64}$, are *not* in their lowest terms, nor their simplest form, because each may be further reduced by dividing both terms by a number which will be exactly contained in them. This exact divisor is called a *factor*,

*Answers to these problems will be found on page 14.

and because it is common to both terms it is known as a *common factor* of these terms.

Both the numerator and the denominator of $\frac{2}{4}$ may be divided by 2. This number 2, therefore is a common factor of these terms. The division *reduces* both terms of the fraction by changing the numerator to 1 and the denominator to 2, making the fraction read $\frac{1}{2}$ instead of $\frac{2}{4}$. This process is known as *factoring.*

Although the form of this fraction has been changed by this reduction its *value* has not changed. To illustrate this further: $\frac{2}{4}$ of an inch is equal to $\frac{1}{2}$ of an inch; $\frac{2}{4}$ of a yard equals $\frac{1}{2}$ of a yard.

In like manner $\frac{12}{16}$ in., $\frac{6}{21}$ in., and $\frac{18}{64}$ in. when reduced to *lowest* terms become $\frac{3}{4}$ in., $\frac{2}{7}$ in., and $\frac{9}{32}$ in. respectively. Here again the *form* of the fraction has changed but the *value* has not changed.

A fraction should always be expressed in its *simplest form.* That is, the numerator and the denominator should be reduced as low as possible by dividing both terms by common factors as above explained.

This is illustrated in the following practical problem.

Example:

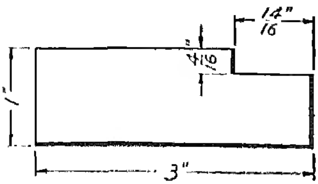
Redraw the sketch at top of page 9 placing all measurements in their lowest terms.

Explanation:

NOTE: Measurements are indicated as shown in the drawing by what are known as *dimensions*. For this purpose, *dimension lines* are used. These dimension lines have an *arrowhead* at each end and are drawn parallel to the side, or the edge, measured. While the measurement figure is usually located in a suitable space in the dimension line as shown in the drawing, it may be located elsewhere if advisable or more convenient.

By examining the measurements in this drawing it may be seen that both $\frac{14}{16}$ in. and $\frac{4}{16}$ in. are not in their lowest terms.

Both the numerator and the denominator in the fraction $\frac{14}{16}$ may be divided by the common factor 2. This reduces it to $\frac{7}{8}$.

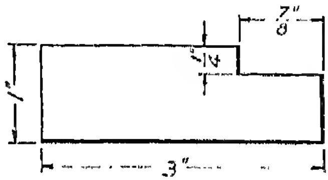


The result $\frac{7}{8}$, cannot be further reduced. Hence, $\frac{14}{16}$ in. expressed in simplest form, or lowest terms, becomes $\frac{7}{8}$ in.

In the fraction $\frac{4}{16}$, both the numerator 4 and the denominator 16 may be reduced to lower terms by dividing each by 2, reducing it to $\frac{2}{8}$. Again, 2 is a factor of both terms 2 and 8, reducing the fraction $\frac{2}{8}$ to $\frac{1}{4}$. The terms of this fraction $\frac{1}{4}$, cannot be reduced further, and the measurement $\frac{4}{16}$ in. in simplest form becomes $\frac{1}{4}$ in.

Hence the measurement $\frac{14}{16}$ in. expressed in its lowest terms is $\frac{7}{8}$ in., while the measurement $\frac{4}{16}$ in. in its lowest terms equals $\frac{1}{4}$ in.

These measurements should be used in the redrawn sketch as shown to the right.



NOTE: The reduction of the fraction $\frac{4}{16}$ might have been accomplished in one operation by dividing both terms by 4, the highest common factor. When the highest factor cannot be determined readily it is satisfactory to use such other factors as are correct.

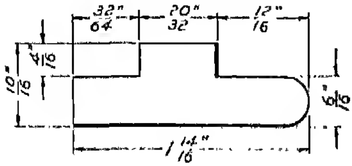
Problems Involving Factoring

1. Express in simplest form:
$\frac{10}{16}$ of an inch; $\frac{2}{4}$ of a yard; $\frac{12}{16}$ in.; $\frac{6}{12}$ of a foot; $\frac{24}{32}$ in.; $\frac{18}{64}$ in.

2. In measuring an iron bar it is found to be $1\frac{10}{16}$ in. thick. What should this measurement read when properly expressed?

3. Reduce to lowest terms:

$\frac{125}{325}$; $\frac{64}{120}$; $\frac{12}{84}$; $8\frac{44}{121}$; $3\frac{60}{144}$; $36\frac{84}{108}$; $19\frac{30}{48}$; $21\frac{48}{64}$ in.



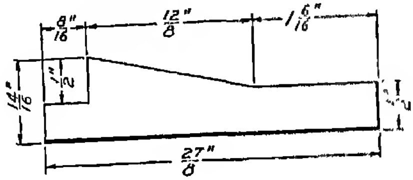
4. Redraw the sketch on the left, giving dimensions in lowest terms.

5. Which of the following

are *not* in their lowest terms? Why? Express them in their lowest terms.

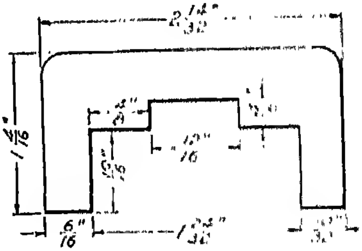
$\frac{3}{18}$; $\frac{14}{8}$; $\frac{28}{37}$; $\frac{18}{32}$ in.; $2\frac{30}{64}$ in.; $11\frac{13}{16}$ in.

6. Redraw the following template giving dimensions in simplest form.



7. Is $\frac{24}{32}$ greater than $\frac{3}{4}$? Is $\frac{150}{24}$ greater than $6\frac{1}{4}$? Is $3\frac{7}{8}$ greater than $\frac{60}{16}$? Prove each answer.

8. Redraw the template as illustrated below putting all dimensions in lowest terms.



9. Reduce to simplest form:
$\frac{24}{56}$; $\frac{90}{18}$; $\frac{144}{60}$; $\frac{48}{64}$; $\frac{120}{32}$; $\frac{36}{48}$; $\frac{64}{96}$; $\frac{72}{132}$.

10. Which of the following is the larger?
$\frac{40}{32}$ in. or $1\frac{1}{4}$ in.; $\frac{63}{8}$ or $7\frac{5}{8}$; $8\frac{9}{16}$ or $\frac{130}{16}$; $\frac{19}{4}$ in. or $4\frac{3}{4}$ in.; $\frac{42}{8}$ or $5\frac{1}{4}$.*

Changing Fractions to Equivalent Fractions Having a Given Denominator

In the foregoing problems it was seen that when both terms of the fraction were *divided* by the same number, the *value of the fraction* did *not* change. In the following example it will be seen that when both terms are *multiplied* by the same number, the *value* of the fraction does *not* change either, although the *form* of the fraction *does change*. To illustrate:

*Answers to these problems will be found on page 14.

By multiplying both terms of the fraction $\frac{2}{5}$ by 12, for example, it becomes changed to

$$\frac{2 \times 12}{5 \times 12} \text{ or } \frac{24}{60}$$

But $\frac{24}{60}$ does not look much like $\frac{2}{5}$. However, it has the same value as $\frac{2}{5}$, and may be actually changed to $\frac{2}{5}$. This change is accomplished by first dividing both terms of $\frac{24}{60}$ by 2, thus reducing it to $\frac{12}{30}$. In turn, by dividing both terms of $\frac{12}{30}$ by 3, that fraction becomes changed to $\frac{4}{10}$. Again, by dividing both terms of $\frac{4}{10}$ by 2, that becomes reduced to the original fraction $\frac{2}{5}$.

That is, $\frac{2}{5} = \frac{4}{10} = \frac{12}{30} = \frac{24}{60} = \frac{2 \times 12}{5 \times 12}$.

This proves that when both terms of a fraction are multiplied by the same number, the *value* of the fraction does not change, although the *form* does change.

This fact is made use of in changing fractions so as to give them *denominators* like other fractions. This kind of a change is sometimes referred to as *raising*, or *increasing*, a given fraction to a *required denomination*, or changing a fraction to an *equivalent* fraction of *another denomination*. How this is applied is shown in the following problem:

Example 1:

Change the fraction $\frac{3}{8}$ to an equivalent fraction having the denominator 32.

Solution and Explanation:

To find the number to be used in multiplying both terms of the fraction $\frac{3}{8}$ in order to change the denominator to 32's (thirty-seconds), *divide* the desired *denominator*, 32, by the *original* denominator, 8. This gives 4 as a quotient. Then *multiply both terms* of the fraction $\frac{3}{8}$ by the number 4. This will give a *new denominator*, also a *new numerator*. When placed in fractional form this becomes

$$\frac{3 \times 4}{8 \times 4} \text{ or } \frac{12}{32}$$

As explained above, this multiplication does *not* change the value of the fraction.

That is, $\frac{3}{8}$ changed to 32's becomes $\frac{12}{32}$.

This same method is also followed in changing *whole numbers* to given fractional denominations, as illustrated below.

Example 2:

Determine how many 64ths of an inch there are in 2 inches.

Solution and Explanation:

As previously explained, a whole number may be expressed in fractional form by using it as the numerator of a fraction whose denominator is 1. In this case, 2 changed to fractional form, becomes $\frac{2}{1}$.

After this is so expressed, both terms are multiplied by 64 in order to change the fraction to one with the *required* denominator.

This becomes $\dfrac{2 \times 64}{1 \times 64} = \dfrac{128}{64}$.

That is, in 2 inches there are $\frac{128}{64}$ inches, or 128 sixty-fourths of an inch.

The correctness of this solution may be proven by dividing 128 by 64, showing that $\frac{128}{64}$ equals 2.

Proof: $128 \div 64 = 2$

To change a *mixed number* to a fraction of a *given* denomination the same process as above is also followed, the mixed number being first changed to an *improper fraction*.

Example 3:

How many 32nds of an inch in $3\frac{3}{8}$ in.?

Solution and Explanation:

$3\frac{3}{8}$ changed to an improper fraction equals $\frac{27}{8}$.

To find the number which is to be used in multiplying both terms of this fraction in order to change it to 32nds, divide 32 by 8. This gives 4, which is to be used as the multiplier.

Proceeding as in the previous example:

$$\frac{27 \times 4}{8 \times 4} = \frac{108}{32}. \quad \text{That is, } 3\tfrac{3}{8} = \tfrac{108}{32}.$$

This shows that in $3\frac{3}{8}$ in. there are 108 thirty-seconds of an inch, the numerator of the fraction indicating the number of 32nds.

Knowledge of this process is of great help in both addition and subtraction of fractions.

Problems Involving Changing Fractions to Equivalent Fractions of Another Denomination

1. Change:

$\frac{3}{16}$ in. to 64ths; $1\frac{3}{8}$ in. to 32nds; $21\frac{3}{4}$ in. to 64ths; $3\frac{1}{4}$ in. to 16ths.

2. Change:

$1\frac{1}{4}$ to 20ths; $\frac{4}{7}$ to 63rds; $\frac{11}{15}$ to 120ths; $4\frac{3}{7}$ to 42nds.

3. Change:

$\frac{3}{4}$; $\frac{9}{16}$; $\frac{1}{2}$; $\frac{7}{8}$ to 48ths.

4. What is the equivalent number of 16ths of an inch in $1\frac{3}{8}$ in.; in $\frac{3}{4}$ in.; in $\frac{1}{2}$ in.; in $1\frac{7}{8}$ in.; in $1\frac{9}{16}$ in.; in 3 in.; in $3\frac{1}{2}$ in.?

5. How many 64ths in $\frac{5}{8}$ in.; $2\frac{3}{4}$ in.; $1\frac{11}{16}$ in.; $7\frac{1}{2}$ in.; 4 in.; $1\frac{3}{8}$ in., $\frac{9}{32}$ in.?

6. Change each of the following to 30ths:

$\frac{7}{5}$; $1\frac{1}{3}$; $\frac{1}{2}$; $9\frac{1}{6}$; $\frac{11}{15}$.

7. A strip of sheet steel measures $2\frac{3}{8}$ in. wide. Of how many 32nds is this the equivalent?

8. A young man spent two and one half dollars for tools to be used in his workshop. This is equal to how many quarters?

9. Change 2 in.; $4\frac{1}{4}$ in.; and $8\frac{11}{16}$ in. to 64ths of an inch.

10. How many 32nds of an inch in $1\frac{5}{8}$ in.; $9\frac{11}{16}$ in.; $\frac{5}{8}$ in.; $6\frac{3}{8}$ in.; $12\frac{1}{2}$ in.?

11. How many pieces of copper $\frac{1}{2}$ in. long may be cut from a strip of thin copper $17\frac{1}{2}$ in. long?

12. Reduce $7\frac{1}{2}$ to 10ths; $3\frac{1}{3}$ to 15ths; $2\frac{1}{7}$ to 35ths.*

*Answers to these problems will be found on page 14.

ANSWERS TO PROBLEMS

Pages 6 and 7.

1. $3\frac{3}{4}$; $1\frac{1}{2}$; $6\frac{7}{8}$ mixed numbers. $\frac{1}{4}$; $\frac{7}{17}$ proper fractions. $\frac{13}{13}$; $\frac{10}{7}$; $\frac{21}{5}$; $\frac{8}{4}$ improper fractions.
2. $2\frac{1}{2}$; $4\frac{1}{2}$; $2\frac{1}{6}$; $1\frac{5}{8}$; $9\frac{5}{12}$; $5\frac{1}{3}$; $1\frac{3}{10}$; $3\frac{9}{13}$; $2\frac{4}{7}$; $1\frac{11}{12}$.
3. 9; 23; 41; 91; 77.
4. 23 half-inch pieces.
5. 17 quarter inches.
6. $\frac{10}{2}$; $\frac{1137}{16}$; $\frac{105}{16}$; $\frac{10}{2}$; $\frac{130}{13}$; $\frac{73}{8}$; $\frac{6927}{64}$.
7. $\frac{5}{9}$; $\frac{48}{65}$.
8. $\frac{19}{8}$ is greater than $2\frac{1}{8}$; $5\frac{2}{3}$ equals $\frac{17}{3}$; $\frac{15}{8}$ is greater than $1\frac{5}{8}$.
9. $1\frac{7}{8}$ in.
10. $1\frac{7}{16}$ in.
11. 101; 39; 619.
12. $\frac{11}{36}$ yd.; $\frac{7}{36}$ yd.; $\frac{25}{36}$ yd.

Pages 9 and 10.

1. $\frac{5}{8}$ in.; $\frac{1}{2}$ yd.; $\frac{3}{4}$ in.; $\frac{3}{4}$ in.; $\frac{9}{32}$ in.
2. $1\frac{5}{8}$ in.
3. $\frac{5}{13}$; $\frac{8}{15}$; $\frac{1}{7}$; $8\frac{4}{11}$; $3\frac{2}{3}$; $36\frac{1}{2}$; $19\frac{3}{4}$; $21\frac{3}{4}$ in.
4. $\frac{1}{2}$ in.; $\frac{5}{8}$ in.; $\frac{3}{4}$ in.; $\frac{3}{8}$ in.; $1\frac{7}{8}$ in.; $\frac{5}{8}$ in.; $\frac{1}{4}$ in.
5. $\frac{1}{6}$; $1\frac{3}{4}$; $\frac{9}{16}$ in.; $2\frac{9}{16}$ in.
6. $\frac{1}{2}$ in.; $1\frac{1}{2}$ in.; $1\frac{3}{8}$ in.; $3\frac{3}{8}$ in.; $\frac{7}{8}$ in.
7. $\frac{3}{4}$ equals $\frac{24}{64}$; $6\frac{3}{4}$ is greater than $\frac{132}{21}$; $3\frac{7}{8}$ is greater than $\frac{72}{18}$.
8. $2\frac{7}{16}$ in.; $1\frac{1}{4}$ in.; $\frac{3}{8}$ in.; $1\frac{3}{4}$ in.; $\frac{5}{16}$ in.; $\frac{3}{8}$ in.; $\frac{1}{2}$ in.; $\frac{1}{4}$ in.; $\frac{1}{4}$ in.
9. $\frac{8}{7}$; $5\frac{1}{3}$; $2\frac{2}{5}$; $\frac{3}{4}$; $3\frac{3}{4}$; $\frac{9}{4}$; $\frac{2}{3}$; $\frac{9}{11}$.
10. $\frac{40}{32}$ in. equals $1\frac{1}{4}$ in.; $\frac{63}{8}$ is greater than $7\frac{5}{7}$; $8\frac{7}{16}$ is less than $\frac{130}{16}$; $\frac{19}{4}$ equals $4\frac{3}{4}$; $\frac{42}{8}$ equals $5\frac{1}{4}$.

Page 13.

1. $\frac{12}{64}$ in.; $\frac{44}{32}$ in.; $\frac{1302}{64}$ in.; $\frac{52}{16}$ in.
2. $\frac{25}{20}$; $\frac{45}{63}$; $\frac{112}{120}$; $\frac{180}{42}$.
3. $\frac{36}{48}$; $\frac{27}{48}$; $\frac{24}{48}$; $\frac{168}{48}$.
4. $\frac{22}{16}$ in.; $\frac{12}{16}$ in.; $\frac{8}{16}$ in.; $\frac{30}{16}$ in.; $\frac{25}{16}$; $\frac{48}{16}$ in.; $\frac{56}{16}$ in.
5. $\frac{40}{64}$ in.; $\frac{170}{64}$ in.; $\frac{108}{64}$ in.; $\frac{480}{64}$ in.; $\frac{250}{64}$ in.; $\frac{88}{64}$ in.; $\frac{18}{64}$ in.
6. $\frac{42}{30}$; $\frac{40}{30}$; $\frac{15}{30}$; $\frac{275}{30}$; $\frac{22}{30}$.
7. $\frac{76}{32}$.
8. 10 quarters.
9. $\frac{128}{64}$; $\frac{272}{64}$; $\frac{556}{64}$
10. $\frac{52}{32}$ in.; $\frac{310}{32}$ in.; $\frac{20}{32}$ in.; $\frac{204}{32}$ in.; $\frac{400}{32}$ in.
11. 35 pieces.
12. $\frac{75}{10}$; $\frac{50}{15}$; $\frac{75}{35}$.

Review Problems Involving Expression and Reduction of Fractions

1. Change $3\frac{1}{7}$ to 63rds; $5\frac{1}{3}$ to 48ths; $\frac{2}{3}$ to 57ths; $4\frac{1}{4}$ in. to 8ths of an inch.

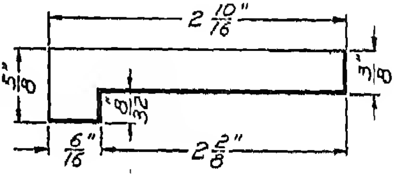
2. How many 16ths of an inch in $5\frac{1}{4}$ in.? How many $\frac{1}{32}$ in. in $18\frac{1}{2}$ in.?

3. Which of the following are in their simplest form:

$$\frac{10}{4};\ 6\frac{18}{32}\text{ in.};\ \frac{63}{72};\ 3\frac{9}{17};\ \frac{48}{15}?$$

4. Is $\frac{24}{32}$ larger than $\frac{13}{16}$? Is $1\frac{1}{2}$ larger than $\frac{21}{14}$? Is $2\frac{15}{40}$ larger than $2\frac{18}{24}$?

5. Some of the dimensions on the opposite drawing are not correctly expressed because they are not in their simplest form. Make a new drawing giving all dimensions in their simplest form.

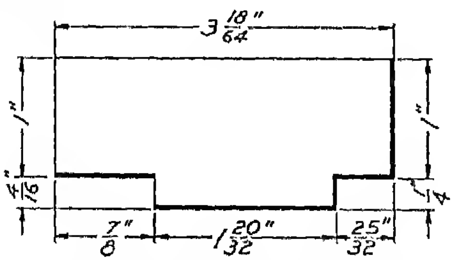


6. In the following list of numbers which are improper fractions and which are mixed numbers:

$$3\frac{1}{4};\ \frac{14}{6};\ \frac{11}{10};\ 2\frac{9}{14};\ \frac{12}{8};\ 1\frac{8}{15}.$$

7. A strip of thin brass $7\frac{3}{4}$ in. wide is to be cut into pieces that measure $\frac{1}{4}$ in. wide. Into how many pieces can this strip be cut?

8. Redraw the following sketch giving all dimensions in their lowest terms.



9. There are 12 inches in one foot. What fractional part of one foot is 6 in.; 9 in.; 4 in.; 3 in.; 10 in.; 7 in.; 8 in.?

ADDITION OF WHOLE NUMBERS AND A FRACTION

When one or more whole numbers are to be *added* to a fraction, the sum of the whole numbers is simply placed in front of the fraction. The result is a *mixed number*, which, as previously explained, consists of a whole number and a fraction combined.

For example, to find the sum of 7, 3, 5, and $\frac{11}{16}$, the first step is to add the whole numbers. This equals $7 + 3 + 5$, or 15. Placing this sum in front of the fraction $\frac{11}{16}$, the resulting sum of the whole numbers, and the fraction becomes $15\frac{11}{16}$.

ADDITION OF SIMILAR FRACTIONS

To add two or more fractions it is necessary that they have the *same name*, or the *same denominator*. Such fractions as $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{7}{8}$ are of this kind because they have the same name or same denominator, that is, they are all eighths. Because they have the *same* denominator they are called *similar fractions*.

Such similar fractions as those above can be added just as they are by adding the numerators together and placing this sum over the common denominator.

According to this, the sum of the similar fractions $\frac{1}{8}$, $\frac{3}{8}$, and $\frac{7}{8}$ is determined by adding the numerators 1, 3, and 7, and placing this sum over the common denominator 8. This gives as a result the *new* fraction $\frac{11}{8}$. By changing this to a mixed number, $\frac{11}{8}$ becomes $1\frac{3}{8}$.

ADDITION OF MIXED NUMBERS HAVING THE SAME COMMON DENOMINATOR

To add *mixed* numbers which have fractional parts that are of the same *common denominator*, first add the fractions as above explained, and then add the whole numbers. By placing the *sum* of the whole numbers in front of the *sum* of the fractions the combined result gives the correct sum.

This is illustrated in the following example.

Example:

What is the sum of $2\frac{1}{8}$, $9\frac{3}{8}$, and $1\frac{5}{8}$?

Solution and Explanation:

Following the procedure as above explained,

$\frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$, as the sum of the fractional parts.

$2 + 9 + 1 = 12$, as the sum of the whole numbers.

The sum of the three mixed numbers, $2\frac{1}{8}$, $9\frac{3}{8}$, and $1\frac{3}{8}$, therefore equals $12\frac{7}{8}$.

The addition of such mixed numbers is usually expressed as follows, the fractional parts being added in a *vertical* row.

$$\begin{array}{r} 2\frac{1}{8} \\ 9\frac{3}{8} \\ 1\frac{3}{8} \\ \hline \text{Sum} = 12\frac{7}{8} \end{array}$$

COMBINATION OF WHOLE NUMBERS, MIXED NUMBERS, AND SIMILAR FRACTIONS

A combination of whole numbers, mixed numbers, and fractions having the same common denominator, are added in the same manner as above noted. They are placed in the usual vertical row and the fractional parts are added as explained. Their sum is then reduced to lowest terms.

This sum is then combined with the sum of the whole numbers for the final result.

How such a combination is added is shown in the following example.

Example:

Determine the sum of the following measurements: $1\frac{19}{64}$ in., $\frac{37}{64}$ in., 2 in., $6\frac{41}{64}$ in., $\frac{9}{64}$ in.

Solution and Explanation:

Following the arrangement as above explained this works out as:

$$\begin{array}{r} 1\frac{19}{64} \\ \frac{37}{64} \\ 2\phantom{\frac{00}{00}} \\ 6\frac{41}{64} \\ \frac{9}{64} \\ \hline \text{Sum} = 9\frac{106}{64} \end{array}$$

Reducing the fractional part $\frac{106}{64}$ to simplest form it becomes $1\frac{42}{64}$, which in turn reduces to $1\frac{21}{32}$.

The sum $9\frac{106}{64}$ then becomes changed to $9 + 1\frac{21}{32}$, or $10\frac{21}{32}$.

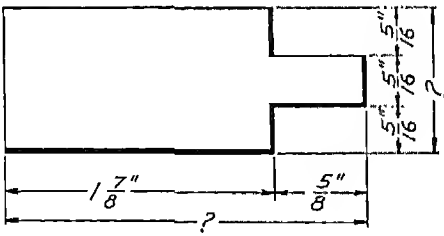
That is, the sum of the measurements $1\frac{19}{64}$ in., $\frac{37}{64}$ in., 2 in., $6\frac{41}{64}$ in., and $\frac{9}{64}$ in., equals $10\frac{21}{32}$ in.

Problems Involving the Addition of Similar Fractions

1. What is the sum of each of the following?
   a) $\frac{17}{64}$, $\frac{3}{64}$, 2, $6\frac{9}{64}$, $11\frac{13}{64}$, 21.
   b) 8, $\frac{9}{25}$, $\frac{8}{25}$, $4\frac{1}{25}$, $9\frac{18}{25}$.
   c) $\frac{1}{8}$ in., $\frac{3}{8}$ in., $7\frac{7}{8}$ in., 5 in., $2\frac{5}{8}$ in.
   d) 6, 5, 19, $\frac{1}{4}$, $4\frac{3}{4}$, $2\frac{3}{4}$.

2. From a bar of iron a boy cuts off four pieces measuring $3\frac{3}{16}$ in., $1\frac{5}{16}$ in., $8\frac{7}{16}$ in., and $10\frac{13}{16}$ in. respectively. How many inches of stock did he cut off in all, disregarding the thickness of the saw blade?

3. What is the total length of the piece illustrated below? What is the total width? Redraw the piece placing on the drawing all dimensions including the total length and the total width as indicated by the question marks.



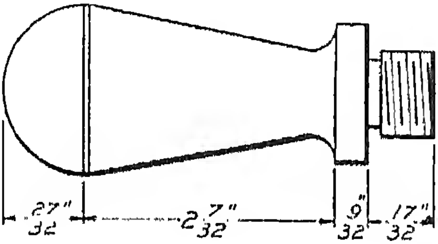
4. Four strips of brass measure each, $\frac{3}{32}$ in., $\frac{5}{32}$ in., $\frac{9}{32}$ in., and $\frac{11}{32}$ in. thick. What is their combined thickness?

5. In repairing a damaged "oil line" on an automobile, the mechanic uses three pieces of copper tubing. One of these pieces measures $\frac{3}{4}$ ft., another 1 ft., and the other measures $1\frac{1}{4}$ ft. How many feet of this tubing were used for this particular repair job?

6. A workman assigned to the job of repairing a leaking roof finds that he needs four strips of sheet metal. One piece of this sheet metal measures 3 ft. long, another measures $4\frac{1}{4}$ ft. long, another $3\frac{3}{4}$ ft. long, while the fourth piece measures $2\frac{3}{4}$ ft. long. How many feet of sheet metal in all were needed for this job?

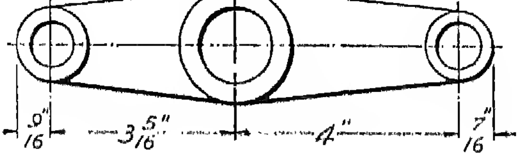
7. Cracks and knots in a white pine board necessitate that a piece $1\frac{3}{4}$ ft. long be cut off one end of the board, while a piece that measures $\frac{3}{4}$ ft. long be cut off the other end. What is the total length in feet, of this waste material?

8. Calculate the total length of the handle as illustrated below.

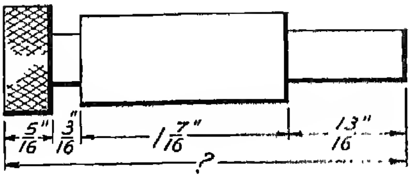


9. After completing a job of wiring, an electrician finds that he has left four pieces of wire. These measure $9\frac{1}{4}$ ft., 8 ft., $10\frac{3}{4}$ ft., and $12\frac{1}{4}$ ft. respectively. What is the total length of these four pieces?

10. Determine the total length of the special link that is illustrated in the following drawing.



11. What is the total length of the pin in the following sketch?



12. In checking over the repairs needed in his cellar workshop a boy finds that he needs the following pieces of lumber: one piece $5\frac{3}{4}$ ft. long, one piece 5 ft. long, one piece $4\frac{1}{4}$ ft. long, and another piece $3\frac{3}{4}$ ft. long. What is the total length of the pieces needed?*

LEAST COMMON DENOMINATOR

To add fractions having different denominators, it is necessary first to change them to *equivalent fractions* which have the *same common denominator.* This common denominator should be the *least* number which can be exactly divided by *each* of the different denominators. Such a denominator is called the *least common denominator.*

The fractions are then added the same as the *similar* fractions referred to on page 16.

Sometimes this common denominator may be readily determined by a careful examination of the denominators of the several fractions to be added.

For example:

In the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$, an examination shows that 8 is the least number that can be exactly divided by each of the three denominators. This number 8 therefore is the least common denominator of the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$.

To add these fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$, each must first be changed to an equivalent fraction that has 8 as the common denominator. They are then added as similar fractions in the manner already explained.

*Answers to these problems will be found on page 47.

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Changing $\frac{1}{2}$ and $\frac{1}{4}$ to equivalent fractions which have 8 as the denominator there results the similar fractions $\frac{4}{8}$ and $\frac{2}{8}$. These added with the fraction $\frac{3}{8}$ give $\frac{9}{8}$, or $1\frac{1}{8}$, as the sum of the three fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$.

When the least common denominator cannot be so readily determined it may be calculated as explained in the following problem:

Example 1:

What is the sum of $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{1}{3}$?

Solution and Explanation:

The denominators 8, 6, 4, and 3 must all be changed to the least *common denominator*. As explained, this is the smallest denominator that is common to *all* these fractions.

The first step in this process is to arrange 8 – 6 – 4 – 3 as illustrated below, separating them by short dashes. They are then divided by the smallest divisor, or factor, that will go into as many of the numbers as possible.

For example, 2 will exactly divide 8, 6, and 4, but will not exactly divide 3. This number, 3, is *brought down* to the next row as shown, in line with the quotients 4, 3, and 2.

The second row 4 – 3 – 2 – 3 is then divided by 2. Then the third row, and so on, until no number remains that can be further divided.

How this division is usually arranged is shown as follows:

$$\begin{array}{r|l} 2) & 8 - 6 - 4 - 3 \\ 2) & 4 - 3 - 2 - 3 \\ 2) & 2 - 3 - 1 - 3 \\ 3) & 1 - 3 - 1 - 3 \\ & 1 - 1 - 1 - 1 \end{array}$$

After *all* divisions have been completed the divisors, or factors, are multiplied together as follows:

$2 \times 2 \times 2 \times 3$, which equals 24.

The resulting product 24, is the *least* number which will exactly contain each of the denominators 8, 6, 4, and 3. Accordingly, 24 is the *least common denominator* of the fractions $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{1}{3}$.

After the least common denominator has been obtained the next step is to change these fractions to equivalent fractions which have 24 as the *new* denominator. This change is made in the same manner as that relating to similar fractions. As a result, $\frac{7}{8}$ becomes $\frac{21}{24}$, $\frac{5}{6}$ becomes $\frac{20}{24}$, $\frac{3}{4}$ becomes $\frac{18}{24}$, and $\frac{1}{3}$ becomes $\frac{8}{24}$. These are similar fractions because they have the *same* denominator and therefore may be added as similar fractions.

In this particular case the sum of the numerators equals 21 + 20 + 18 + 8, or 67. The sum of the fractions accordingly is $\frac{67}{24}$.

This result, $\frac{67}{24}$ reduced to a mixed number equals $2\frac{19}{24}$.

As more commonly arranged this addition is as follows:

$$\begin{array}{rcl} \frac{7}{8} & = & \frac{21}{24} \\ \frac{5}{6} & = & \frac{20}{24} \\ \frac{3}{4} & = & \frac{18}{24} \\ \frac{1}{3} & = & \frac{8}{24} \\ \hline \text{Sum} & = & \frac{67}{24} \text{ or } 2\frac{19}{24} \end{array}$$

That is, the sum of $\frac{7}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{1}{3}$ is $2\frac{19}{24}$.

To find the sum of *mixed numbers* whose fractional parts are *not* of the same denomination, it is necessary also to change the fractional parts to *equivalent* fractions having a common denominator.

After this is done they are added in the same manner as mixed numbers which have similar fractional parts, as explained on pages 16 and 17.

Example 2:

Find the sum of $9\frac{1}{2}$ in., $1\frac{3}{16}$ in., and $7\frac{15}{32}$ in.

Solution and Explanation:

As explained, it is first necessary to change the fractional parts $\frac{1}{2}$, $\frac{3}{16}$, and $\frac{15}{32}$ to equivalent fractions having the same common denominator.

To do this, it is necessary to find the least common denominator as explained on page 20. This works out as follows:

$$\begin{array}{r|l} 2) & 2 - 16 - 32 \\ 2) & 1 - 8 - 16 \\ 2) & 1 - 4 - 8 \\ 2) & 1 - 2 - 4 \\ 2) & 1 - 1 - 2 \\ & 1 - 1 - 1 \end{array}$$

Multiplying the divisors there results:

$2 \times 2 \times 2 \times 2 \times 2 = 32$, which is the least common denominator.

NOTE: After sufficient practice it is possible to quickly determine the least common denominator of such fractions as those above. Where this cannot be readily done it is advisable to work out the calculations in detail.

The numbers to be added are then placed in a vertical row as shown and each is changed to its equivalent mixed number which has 32 as the denominator. The details of the addition are the same as that already explained.

$$\begin{array}{lrcl} & 9\frac{1}{2} & = & 9\frac{16}{32} \\ & 1\frac{3}{16} & = & 1\frac{6}{32} \\ & 7\frac{15}{32} & = & 7\frac{15}{32} \\ \hline \text{Sum} & & = & 17\frac{37}{32}, \text{ or } 18\frac{5}{32} \end{array}$$

In the above addition, the sum of the fractional parts is $\frac{37}{32}$. This changed to a mixed number equals $1\frac{5}{32}$. The sum of the whole numbers is 17. By adding these two together the final result becomes $18\frac{5}{32}$.

That is, the sum of $9\frac{1}{2}$ in., $1\frac{3}{16}$ in., and $7\frac{15}{32}$ in. is $18\frac{5}{32}$ in.

ADDITION OF FRACTIONS FOR SPECIAL CONDITIONS

The addition of fractions has a wide application in practical shop problems, especially in those calculations relating to finding the length of pieces having fractional measurements.

The process of addition in such cases is the same as that already explained. The student will be helped in the solution

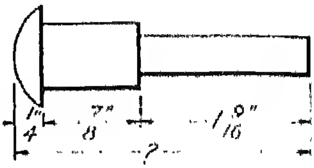
of these problems if he will make a sketch, or a rough drawing, of the part wherever possible.

The following is typical of such problems.

Example 1:

Determine the length of the pin in the following drawing.

Solution and Explanation:



The length of this pin is equal to the sum of the three separate dimensions $\frac{1}{4}$ in., $\frac{7}{8}$ in., and $1\frac{9}{16}$ in. This length is usually indicated by another dimension placed just below the separate dimensions as shown. This dimension, giving the length of the pin, is referred to as the *total length*, sometimes as the *over-all length*.

Before the sum of the several fractions can be found they must be reduced to equivalent similar fractions. In this process the first step is to find the least common denominator.

By careful inspection it may be seen that 16 is the least common denominator of the fractions $\frac{1}{4}$, $\frac{7}{8}$, and $1\frac{9}{16}$.

Changing these fractions to equivalent similar fractions and adding them as previously explained there results:

$$\begin{array}{lcl} \frac{1}{4} & = & \frac{4}{16} \\ \frac{7}{8} & = & \frac{14}{16} \\ 1\frac{9}{16} & = & 1\frac{9}{16} \\ \hline \text{Sum} & = & 1\frac{27}{16}, \text{ or } 2\frac{11}{16} \end{array}$$

That is, the total length of the above pin equals $2\frac{11}{16}$ in.

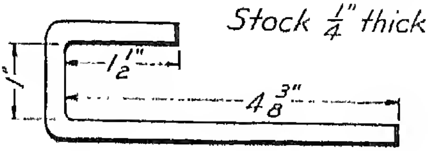
A further application of the addition of fractions may be seen in the method of determining the length of bent metal.

In calculating the total length of such pieces the *sum* of the *inside* dimensions of the finished piece is first determined. To this is added an allowance for each square corner. This varies for different kinds of material, but for the problems in this book, add half the thickness of the metal for each square corner.

How this is done is shown in the following problem.

Example 2:

Calculate the length of the iron strip that is to be bent according to the shape in the following drawing.



Solution and Explanation:

The inside dimensions of this piece are $1\frac{1}{2}$ in., 1 in., $4\frac{3}{8}$ in.

Adding these dimensions in the usual manner the sum becomes:

$$\begin{array}{lrcl} & 1\frac{1}{2} & = & 1\frac{4}{8} \\ & 1 & = & 1 \\ & 4\frac{3}{8} & = & 4\frac{3}{8} \\ \hline \text{Sum} & & = & 6\frac{7}{8} \end{array}$$

The sum of the inside dimensions therefore is $6\frac{7}{8}$ in.

Since the thickness is $\frac{1}{4}$ in. then one half of this measures $\frac{1}{8}$ in.

This amount, $\frac{1}{8}$ in., is to be added for each square corner bend. There being two such bends the total amount to be added equals $\frac{1}{8} + \frac{1}{8}$, or $\frac{1}{4}$ in.

Adding this amount, $\frac{1}{4}$ in., to the sum of the inside measurements, $6\frac{7}{8}$ in., the result becomes:

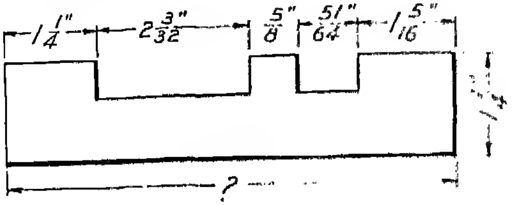
$$\begin{array}{lrcl} & 6\frac{7}{8} & = & 6\frac{7}{8} \\ & \frac{1}{4} & = & \frac{2}{8} \\ \hline \text{Sum} & & = & 6\frac{9}{8}, \text{ or } 7\frac{1}{8} \end{array}$$

That is, the total length of the above piece before bending to the shape noted should be $7\frac{1}{8}$ in.

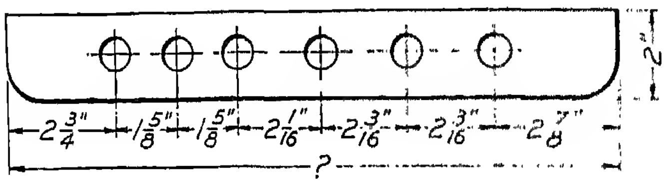
Problems Involving Addition of Fractions Having Different Denominators

1. Find the sum of $3\frac{1}{16}$ in., $1\frac{3}{4}$ in., $6\frac{7}{8}$ in.
   Find the sum of $\frac{11}{12}$, $\frac{3}{8}$, $5\frac{9}{16}$, $12\frac{19}{24}$.
   Find the sum of $8\frac{2}{15}$, $6\frac{1}{2}$, $1\frac{3}{8}$, $4\frac{5}{6}$, $5\frac{1}{6}$.

2. Find the total length of the material used in making the following template.

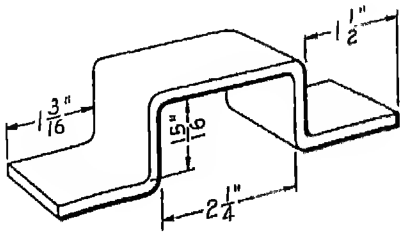


3. A boy is given the following drawing of a tool rack and is asked to build one for his workbench. What is the length of the material needed?

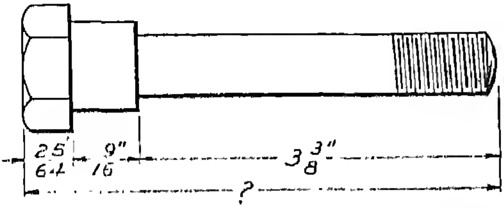


4. Upon looking over materials on hand, a carpenter finds he has 5 hemlock boards of the same width and thickness, but whose lengths measure 3 ft., $7\frac{3}{4}$ ft., $9\frac{3}{4}$ ft., 10 ft., and $9\frac{1}{2}$ ft. What is the sum of these lengths?

5. Determine the length of $\frac{1}{8}$-in. band iron needed for a strap as per the following dimensions.

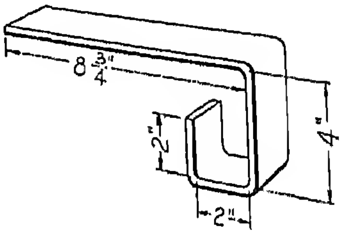


6. What is the total length of the bolt illustrated below?



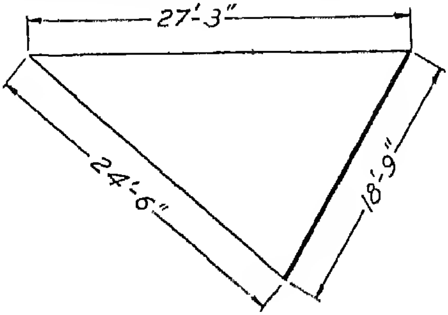
7. To complete a job in the printshop required the work of three printers. It took one printer $3\frac{3}{4}$ hr., another $1\frac{3}{4}$ hr., and the third one $2\frac{1}{2}$ hr. What was the total time in hours required to complete the job?

8. What is the length of $\frac{1}{4}$-in. iron stock required to bend the piece illustrated in the sketch to the right?

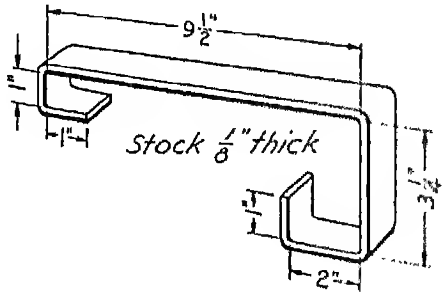


9. In order to do a certain job, a mechanic needs three pieces of steel measuring $3\frac{11}{32}$ in., $7\frac{1}{4}$ in., and $5\frac{1}{2}$ in. respectively. These are cut from a piece of material that measures 5 ft. long. Allowing $\frac{1}{16}$ in. for each saw cut how much material in all was cut from the 5-ft. piece?

10. What is the total length of wire fence needed to enclose the three-sided plot of ground as illustrated?



11. How much material is needed to construct the bracket arm in the following drawing?



12. To lay out a certain job for the machine shop required $2\frac{1}{2}$ hr. The piece was then sent to the drill press where $\frac{1}{4}$ hr. work was done upon it. From there it was sent to the milling machine where $6\frac{3}{4}$ hr. work was done upon it. After this $2\frac{1}{4}$ hr. were required on the grinding machine. What was the total time spent on the job?

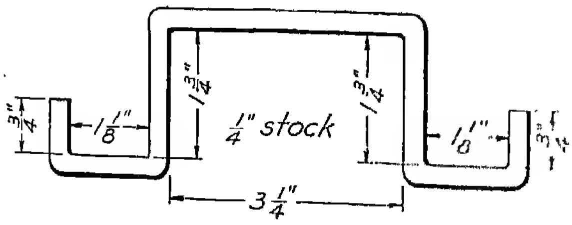
13. Find the sum of the following:

a) $\frac{3}{8}$, $\frac{5}{12}$, $\frac{7}{8}$, $\frac{3}{4}$, $3\frac{5}{6}$.

b) $1\frac{1}{7}$, 4, $\frac{3}{10}$, $\frac{13}{35}$, $\frac{9}{14}$, $12\frac{4}{5}$.

c) $\frac{19}{30}$, $10\frac{5}{6}$, $\frac{14}{15}$, 7.

14. Calculate the length of rod stock needed to make the piece illustrated below.

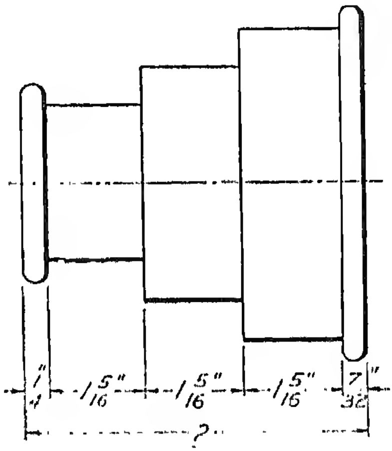


15. A boy attending the stockroom in a machine shop fills out an order for a certain lathe job covering one piece of round

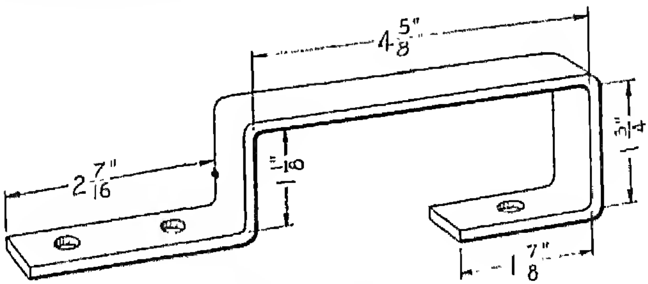
stock $10\frac{3}{16}$ in. long; one piece $4\frac{7}{8}$ in. long, and one piece $9\frac{1}{4}$ in. long. What was the total length of the material given out on this particular job?

16. In checking over his paper stock, a printer finds that he has on hand $2\frac{1}{4}$ reams of letter paper stock, $\frac{3}{4}$ of a ream of blotting paper, $5\frac{1}{2}$ reams of cover stock, and $3\frac{3}{4}$ reams of bristol board. How many reams of paper has he in all?

17. Determine the length of the cone pulley in the following drawing.



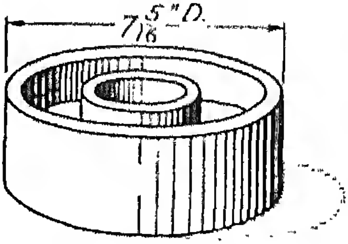
18. What is the length of stock needed to make the wrought-iron handle illustrated below?



Stock $\frac{1}{4}''$ thick

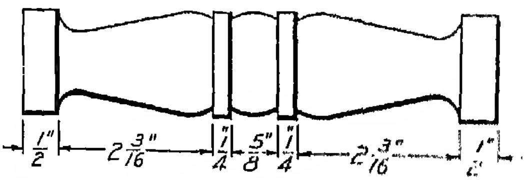
19. During the first week on his new job, a boy worked 5 hr. on Monday, $6\frac{1}{2}$ hr. on Tuesday, $6\frac{3}{4}$ hr. on Wednesday, $7\frac{1}{2}$ hr. on Thursday, and $4\frac{3}{4}$ hr. on Friday. How many hours in all did he work that week?

20. To remodel the pattern of the small pulley as shown in the following drawing, a strip of fiberboard $\frac{1}{8}$ in. thick was wrapped around its circumference in order to increase the diameter to the correct size. If the original diameter was $7\frac{5}{16}$ in. as shown, what was it after the fiber strip was put on?

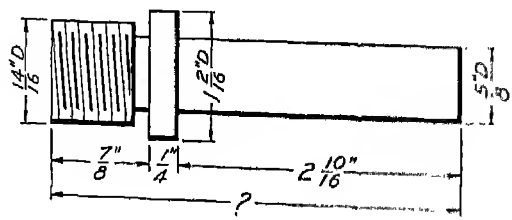


21. To make a guard to protect the operator on a grinding machine, 1 piece of band iron $17\frac{3}{16}$ in. long, 1 piece $8\frac{5}{8}$ in. long, and 1 piece $14\frac{3}{4}$ in. long are required. These are all cut from the one piece of stock. Allowing $\frac{1}{16}$ in. for the thickness of the saw blade, what is the total length in inches of the material cut off?

22. Adding $\frac{3}{4}$ in. on each end for finishing, what will be the amount of material needed to make a spindle like the following?



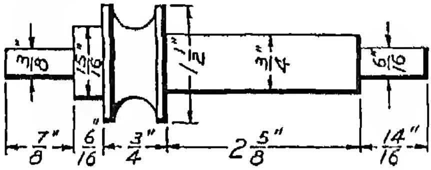
23. Redraw the following sketch of a steel stud shaft,



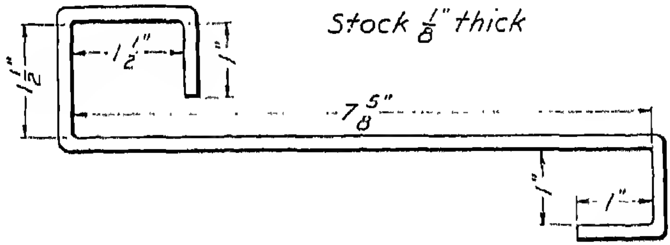
giving all dimensions in their lowest terms. What is the total length as indicated by the question mark?

24. A three-sided piece of land measures 42 ft. on one side, $38\frac{3}{4}$ ft. on another side, and $40\frac{1}{2}$ ft. on the third side. How many feet of wire fence are needed to enclose this plot?

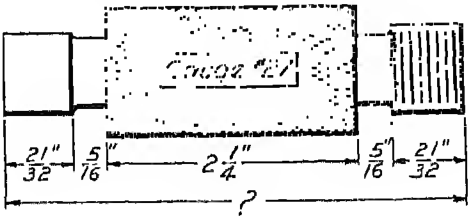
25. Redraw the following spindle giving total length and all detailed dimensions.



26. What is the length of stock needed to make up the ornamental brace as per the following shape and dimensions?



27. What is the length of the gauge illustrated below?

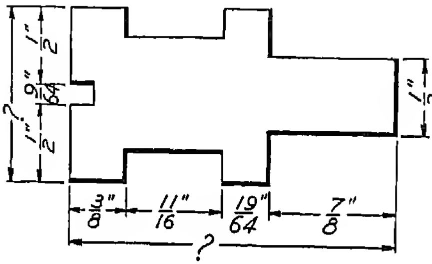


28. A boy working for a neighborhood grocery store after school puts in $1\frac{1}{4}$ hr. on Monday, 2 hr. on Tuesday, $2\frac{1}{4}$ hr.

on Wednesday, $2\frac{1}{4}$ hr. on Thursday, and $6\frac{1}{2}$ hr. on Saturday. How many hours did he work that week?

29. What is the total thickness of a board made by gluing 3 pieces of lumber together if one piece is $\frac{3}{4}$ in. thick, another $\frac{7}{16}$ in. thick, and another $\frac{5}{8}$ in. thick?

30. Determine the width, also the length, of the following piece.



31. In checking over stockroom supplies, it is found that there are 5 kegs of nails which are only partly filled. One keg weighs $19\frac{1}{2}$ lb., another $26\frac{3}{4}$ lb., another 32 lb., another $29\frac{3}{4}$ lb., and another $37\frac{1}{4}$ lb. What is the total weight of all the kegs?

32. After cutting up 4 rolls of strip brass for various jobs, there are left pieces measuring $7\frac{3}{16}$ in. long, $6\frac{1}{4}$ in. long, $4\frac{7}{8}$ in. long, and $4\frac{1}{2}$ in. long. What is the combined length of these pieces that remain?*

SUBTRACTION OF FRACTIONS

To subtract one fraction from another, it is necessary that each of the fractions have the same *common denominator*, or, can be changed to equivalent fractions that *have* the same common denominator. The subtraction is then performed by placing the smaller fraction directly under the larger fraction, and subtracting the *numerators.*

This number is then placed over the common denominator giving a new fraction as a result of this subtraction.

*Answers to these problems will be found on page 47.

Example:

Subtract $\frac{3}{32}$ from $\frac{19}{32}$.

Solution and Explanation:

In this example, the denominators are the same, and the numerators are subtracted directly, as explained.

The result obtained by this subtraction is 16. By placing this over the common denominator 32, there results the fraction $\frac{16}{32}$.

This is arranged as follows:

$$\begin{array}{r} \frac{19}{32} \\ \frac{3}{32} \\ \hline \frac{16}{32} \end{array} = \text{difference}$$

But $\frac{16}{32}$ is not in its lowest terms. When reduced to lowest term it equals $\frac{1}{2}$.

That is, $\frac{3}{32}$ subtracted from $\frac{19}{32}$, equals $\frac{1}{2}$.

Problems Involving Subtraction of Fractions Having Common Denominators

1. Subtract $\frac{13}{40}$ from $\frac{29}{40}$.

2. Taking $\frac{7}{16}$ from $\frac{13}{16}$, leaves what?

3. From $\frac{15}{16}$ take $\frac{3}{16}$.

4. Subtract $\frac{17}{64}$ in. from $\frac{37}{64}$ in.

5. $\frac{19}{32}$ minus $\frac{9}{32}$ equals what?

6. From $\frac{11}{36}$ take $\frac{7}{36}$.

7. What number added to $\frac{19}{64}$ equals $\frac{57}{64}$?

8. After clipping $\frac{7}{32}$ in. from a polished strip of aluminum $\frac{29}{32}$ in. wide, what is the width of the piece remaining?

9. Four brass "shims," or spacers, measure a total of $\frac{11}{64}$ in. thick. After removing one shim, which measures $\frac{3}{64}$ in. thick, what is the combined thickness of the three remaining shims?

10. After subtracting $\frac{32}{125}$ from $\frac{97}{125}$ what remains?

11. Find the difference between:
$\frac{19}{25}$ and $\frac{7}{25}$; $\frac{37}{42}$ and $\frac{13}{42}$; $\frac{35}{64}$ in. and $\frac{19}{64}$ in.; $\frac{29}{32}$ in. and $\frac{7}{32}$ in.; $\frac{15}{16}$ in. and $\frac{3}{16}$ in.

12. In order to use a mahogany board that measures $\frac{15}{16}$ in. thick a young man is obliged to plane $\frac{5}{16}$ in. off one side. What is the final thickness after this planing is done?*

SUBTRACTING FRACTIONS NOT HAVING COMMON DENOMINATORS

When fractions that are to be subtracted have *not* the same common denominator, they should first be changed to equivalent fractions which have the same common denominator. As a result, they become similar fractions, and the subtraction is then carried on as previously explained.

Example 1:

Subtract $\frac{7}{16}$ from $\frac{3}{4}$.

Solution and Explanation:

The common denominator of these fractions is seen to be 16. Changed to equivalent fractions they become $\frac{7}{16}$ and $\frac{12}{16}$.

These being similar fractions, the subtraction takes place as illustrated under the subtraction of similar fractions.

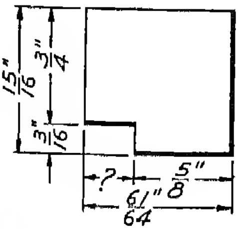
$\frac{7}{16}$ subtracted from $\frac{3}{4}$ works out as:

$$\begin{array}{lcc} & \frac{3}{4} = & \frac{12}{16} \\ & \frac{7}{16} = & \frac{7}{16} \\ \hline \text{Difference} & = & \frac{5}{16} \end{array}$$

Therefore $\frac{7}{16}$ subtracted from $\frac{3}{4}$ equals $\frac{5}{16}$.

Example 2:

Determine the value of the missing dimension in the drawing to the right.



*Answers to these problems will be found on page 48.

Solution and Explanation:

In drawings of this sort the measurements $\frac{3}{16}$ in. and $\frac{3}{4}$ in. are known as *detail dimensions*. They refer to measurements of certain detailed edges or parts.

As seen in previous problems it is the custom to give the total of these detail measurements by a total length dimension, sometimes called an "over-all dimension." Such dimensions are usually placed near the detail dimensions, as illustrated in this drawing.

In this particular drawing the $\frac{15}{16}$-in. dimension is the total of the detail measurements $\frac{3}{16}$ in. and $\frac{3}{4}$ in.

Should either one of these detail measurements be missing, the other measurement may be determined by subtracting the known detail measurement from the $\frac{15}{16}$-in. measurement.

From this explanation it may be seen that the value of the missing dimension, as indicated by the question mark, may be determined by subtracting the detail measurement $\frac{5}{8}$ in. from the total length measurement $\frac{61}{64}$ in.

The first step in this process is to change the fractions to equivalent similar fractions. The subtraction then takes place as previously explained.

This works out as follows:

$$\begin{array}{lr} & \frac{61}{64} = \frac{61}{64} \\ & \frac{5}{8} = \frac{40}{64} \\ \hline \text{Difference} & = \frac{21}{64} \end{array}$$

That is, the missing dimension in the above drawing is $\frac{21}{64}$ in.

Problems in Subtraction of Fractions Having Different Denominators

1. From $\frac{15}{16}$ in. take $\frac{1}{4}$ in.; from $\frac{39}{64}$ in. take $\frac{3}{8}$ in.; from $\frac{3}{4}$ in. take $\frac{5}{64}$ in.

2. Subtract $\frac{17}{32}$ in. from $\frac{3}{4}$ in.; $\frac{19}{27}$ from $\frac{31}{36}$; $\frac{7}{12}$ from $\frac{11}{16}$; $\frac{4}{5}$ from $\frac{19}{20}$.

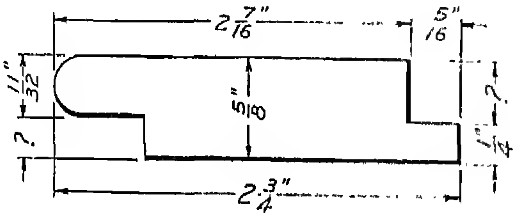
3. After planing $\frac{3}{16}$ in. from a board, which measures $\frac{7}{8}$ in. thick, what is the resulting thickness of the board?

4. What is the difference between $\frac{11}{16}$ and $\frac{13}{24}$?

5. If $\frac{3}{64}$ of an inch is cut from each of the two opposite sides of a steel bar $\frac{29}{32}$ in. wide, how wide would this bar be after the cutting is done?

6. A strip $\frac{3}{16}$ in. wide is cut from a piece of brass that measures $\frac{31}{32}$ in. wide. What is the width of the piece remaining?

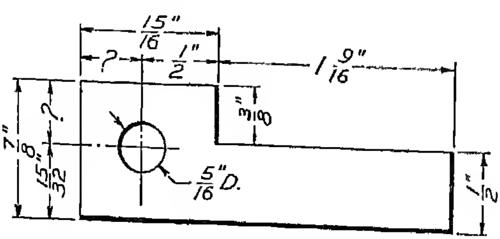
7. Determine the missing dimensions in the following drawing.



8. Subtract $\frac{9}{125}$ from $\frac{11}{25}$; $\frac{7}{64}$ from $\frac{5}{8}$; $\frac{1}{4}$ in. from $\frac{15}{64}$ in.; $\frac{11}{24}$ from $\frac{17}{18}$.

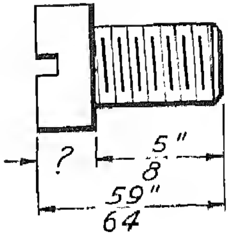
9. A piece of steel measures $\frac{7}{8}$ in. thick before being placed on the table on the surface grinder. What is the thickness after $\frac{5}{64}$ in. is ground off one side?

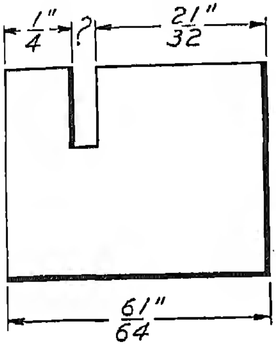
10. In the following drawing locate how far the center of the hole lies from the left side, and from the top.



11. A bronze bushing has an outside diameter of $\frac{1}{2}$ in. The thickness of the metal is $\frac{3}{32}$ in., what is the inside diameter of the bushing?

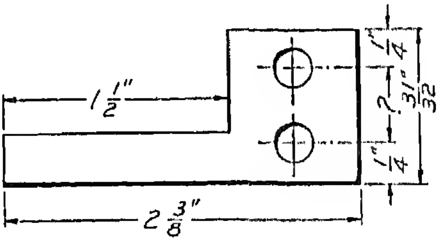
12. What is the thickness of the screw head in the drawing to the right?



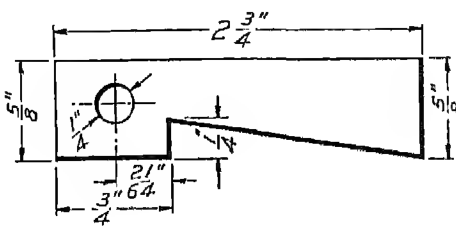


13. The dimension indicating the width of the slot in the block as shown in the illustration to the left is missing. What should it be?

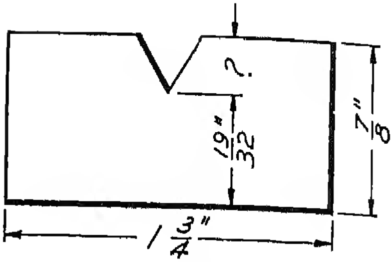
14. Supply the missing measurement in the following sketch.



15. In checking the exact location of the hole in the following piece, how far should it be from the left end?



16. How deep is the "V" cut in the drawing to the right?*



SUBTRACTING A FRACTION FROM A MIXED NUMBER

If a fraction is to be subtracted from a mixed number, the fraction itself is first subtracted from the *fractional* part of the mixed number. The remaining whole number is then placed in front of this fractional difference. It is possible that the result of such subtraction might be either a whole number, a fraction, or a mixed number, depending upon the numbers subtracted.

Example:

After cutting $\frac{7}{16}$ in. from a piece of brass that measures $2\frac{3}{4}$ in. wide, what is the width of the piece remaining?

Solution and Explanation:

These numbers are arranged as in previous examples. The fractional parts are then reduced to equivalent fractions with the lowest common denominator. They are subtracted as explained above.

$$\begin{array}{lrcl} & 2\frac{3}{4} & = & 2\frac{12}{16} \\ & \frac{7}{16} & = & \frac{7}{16} \\ \hline \text{Difference} & & = & 2\frac{5}{16} \end{array}$$

That is, the width of the piece remaining is $2\frac{5}{16}$ in.

SUBTRACTION OF MIXED NUMBERS

Mixed numbers are also subtracted like those above. If the fractional parts are not similar, they must first be changed to

*Answers to these problems will be found on page 48.

similar fractions. After this they are subtracted as already explained.

Then the whole numbers are subtracted, and the resulting whole number, if any, is placed in front of the fraction giving the final result of the subtraction. Such subtraction of mixed numbers might result in either a mixed number, a fraction, or even a whole number, depending upon the numbers subtracted.

Example 1:

Subtract $1\frac{3}{32}$ in. from $4\frac{9}{16}$ in.

Solution and Explanation:

Reduced to similar fractions the fractional parts become,

$$\frac{3}{32} \text{ and } \frac{18}{32}$$

The mixed numbers to be subtracted thus become changed to,

$$1\frac{3}{32} \text{ and } 4\frac{18}{32}$$

According to the above explanation the subtraction is as follows:

$$\begin{array}{lrcl} & 4\frac{9}{16} & = & 4\frac{18}{32} \\ & 1\frac{3}{32} & = & 1\frac{3}{32} \\ \hline \text{Difference} & & = & 3\frac{15}{32} \end{array}$$

That is, $1\frac{3}{32}$ in. from $4\frac{9}{16}$ in. equals $3\frac{15}{32}$ in.

It sometimes occurs in subtracting mixed numbers that the larger mixed number has a fractional part which is less than the fractional part of the smaller mixed number. The subtraction of such numbers takes place as follows.

Example 2:

Subtract $1\frac{17}{32}$ from $3\frac{3}{32}$.

Solution and Explanation:

Before these numbers can be subtracted the fractional part $\frac{3}{32}$ must be increased large enough to permit the subtraction.

This is accomplished by taking 1 unit from the whole number 3 and changing this 1 unit to its equivalent fractional form $\frac{32}{32}$. This fraction is then added to $\frac{3}{32}$ changing it to $\frac{35}{32}$.

As a result, $3\frac{3}{32}$ becomes changed to $2\frac{35}{32}$, its equivalent. The two mixed numbers to be subtracted then become $2\frac{35}{32}$ and $1\frac{17}{32}$. These are subtracted as in the previous example.

$$\begin{array}{lr} & 3\frac{3}{32} = 2\frac{35}{32} \\ & 1\frac{17}{32} = 1\frac{17}{32} \\ \hline \text{Difference} & = 1\frac{18}{32} \end{array}$$

$1\frac{18}{32}$ reduced to lowest terms equals $1\frac{9}{16}$.

That is, $1\frac{17}{32}$ subtracted from $3\frac{3}{32}$ equals $1\frac{9}{16}$.

This method of subtraction also applies where a mixed number or a fraction is to be subtracted from a whole number, as in subtracting $3\frac{7}{16}$ in. from 8 in., or subtracting $\frac{8}{10}$ from 4.

In such cases, 1 unit is taken from the whole number and changed to fractional form of the same denomination as the fraction being subtracted.

The whole number thus becomes reduced by one unit. Combined with the fractional equivalent of this unit the new whole number still remains equal to the original whole number.

The remaining process of subtraction is the same as that explained on page 38. This is illustrated by using the above numbers in the following examples.

Example 3:

Subtract $3\frac{7}{16}$ in. from 8 in.

Solution and Explanation:

One unit taken from the whole number 8 and changed to the denomination of the fraction equals $\frac{16}{16}$. This reduced 8 to 7. Combining the fractional equivalent of this unit with 7 there results $7\frac{16}{16}$. As seen this is the equivalent of the whole number 8.

$$\begin{array}{lr} & 8 \quad = 7\frac{16}{16} \\ & 3\frac{7}{16} = 3\frac{7}{16} \\ \hline \text{Difference} & = 4\frac{9}{16} \end{array}$$

That is, $3\frac{7}{16}$ in. subtracted from 8 in. equals $4\frac{9}{16}$ in.

Example 4:

Subtract $\frac{5}{16}$ from 4.

Solution and Explanation:

According to the rule, 4 is changed to its equivalent mixed number, $3\frac{16}{16}$. The subtraction then becomes:

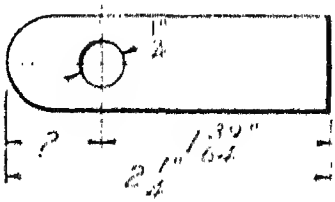
	4	$3\frac{16}{16}$
	$\frac{5}{16}$	$\frac{5}{16}$
Difference		$3\frac{11}{16}$

That is, $\frac{5}{16}$ from 4 equals $3\frac{11}{16}$.

To subtract *mixed numbers* which have *not* similar fractional parts it first becomes necessary to change the fractional parts to equivalent fractions having the same common denominator. *After this is done the subtraction takes place as already* explained. This is illustrated by the following practical example:

Example 5:

In the drawing to the right the dimension which indicates the distance that the center of the hole lies from the left end of the piece is missing. What should it be?



Solution and Explanation:

The missing dimension is equal to the difference between the detail measurement $1\frac{39}{64}$ in. and the total length $2\frac{1}{4}$ in. This difference is found by subtracting $1\frac{39}{64}$ from $2\frac{1}{4}$.

Before such subtraction can take place, however, these fractions must be changed to equivalent similar fractions.

As a result $2\frac{1}{4}$ becomes $2\frac{16}{64}$. In subtracting $1\frac{39}{64}$ from this, the process is the same as previously explained on page 38.

This works out as follows:

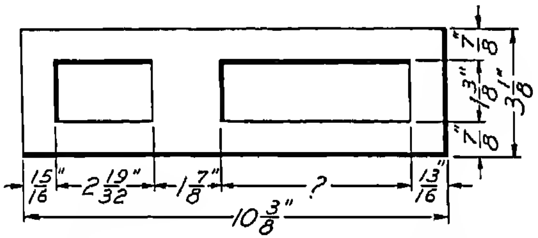
	$2\frac{16}{64}$	$1\frac{80}{64}$
	$1\frac{39}{64}$	$1\frac{39}{64}$
Difference		$\frac{41}{64}$

This indicates that the missing dimension is $\frac{41}{64}$ in.

Problems Involving Subtraction of Fractions and Mixed Numbers

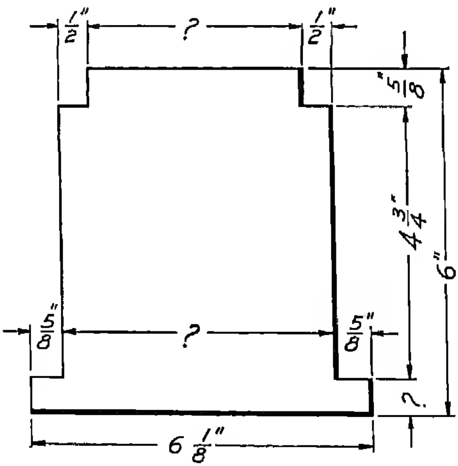
1. *a*) What length added to $5\frac{3}{16}$ in. equals $19\frac{7}{8}$ in.?
   b) After subtracting $1\frac{3}{5}$ from $6\frac{1}{3}$ what remains?
   c) Subtract $2\frac{1}{4}$ from $3\frac{1}{2}$; $8\frac{1}{8}$ from $10\frac{1}{3}$; $15\frac{1}{6}$ from $20\frac{1}{2}$.
   d) What number added to $1\frac{1}{5}$ equals $13\frac{1}{7}$?

2. The following drawing was sent to a machinist to be used in making the piece illustrated. He finds there is one important measurement missing. What should it be?



3. To make two copper pins, one piece $2\frac{13}{16}$ in. long, and another piece $2\frac{9}{32}$ in. long are cut from a rod $13\frac{1}{8}$ in. long. How much of the rod remains after this cutting?

4. In the following drawing of a bookrack end, determine the missing dimensions as indicated by the question marks.



5. From a bundle of paper containing 5 reams, a printer uses $1\frac{1}{2}$ reams in one day, $\frac{1}{2}$ ream on another day, and $\frac{3}{4}$ of a ream on another day. How many reams were there left in the original bundle?

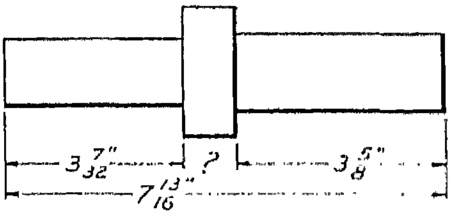
6. A board measuring 4 in. thick is dressed by planing $\frac{3}{8}$ in. off one side and $\frac{3}{16}$ in. off the other side. What is the resulting width of this board?

7. From a coil of lead pipe 50 ft. in length, a plumber uses at different times pieces measuring $6\frac{1}{4}$ ft., $\frac{3}{4}$ ft., $8\frac{1}{2}$ ft., 4 ft., and $9\frac{3}{4}$ ft. How many feet are left in this coil?

8. A grindstone 14 in. in diameter is used in grinding chisels. After one month's use the diameter wears down $\frac{7}{16}$ in. After another month's use it wears down $\frac{5}{8}$ in. more. What is the diameter of the wheel after two months of wear?

9. A tool bit that is used in turning metal on a machine-shop engine lathe measures $2\frac{1}{2}$ in. long. In forming a proper point on this bit the workman grinds away $\frac{3}{16}$ in. After using it for a day he again repoints the tool by grinding away $\frac{5}{32}$ in. more. How long was the tool bit after the second grinding?

10. What is the thickness of the flange on the spindle shown in the drawing below?

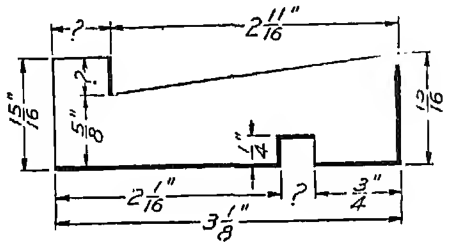


11. After planing $\frac{3}{16}$ in. off the top and $\frac{3}{16}$ in. off the bottom of a block that is $1\frac{1}{4}$ in. thick, how thick will it be after the operation?

12. An apprentice cabinetmaker is directed to surface one side face and the top face of a piece of lumber measuring 3 in.

thick and 6 in. wide. After he finishes surfacing the piece he finds that it measures $2\frac{13}{16}$ in. thick and $5\frac{7}{8}$ in. wide. How much was each of the original dimensions reduced?

13. Redraw the following sketch putting in all dimensions including the ones indicated by the question marks.

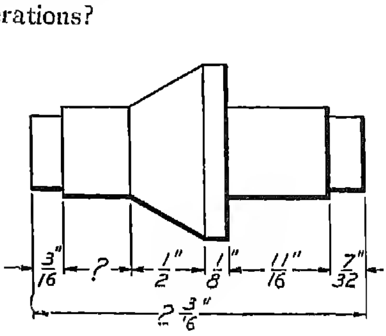


14. The weight of kerosene oil is listed as being $\frac{4}{5}$ that of water. At that rate what would be the weight of one gallon of kerosene when one gallon of water weighs $8\frac{1}{3}$ lb.?

15. From a strip of copper $10\frac{1}{2}$ ft. long the following lengths are cut at various times: $1\frac{1}{4}$ ft.; $\frac{1}{2}$ ft.; $1\frac{1}{2}$ ft.; $2\frac{3}{4}$ ft.; 2 ft. What is the length in feet of the piece remaining?

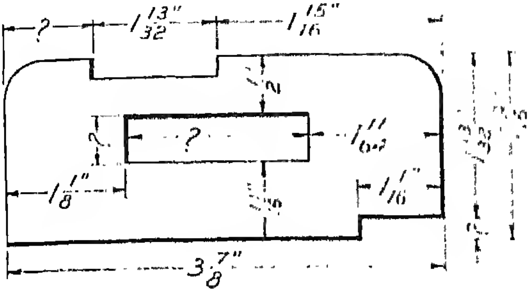
16. A casting weighs $36\frac{1}{2}$ lb. before it is worked upon in the machine shop. When all operations were completed the finished piece weighed $21\frac{3}{4}$ lb. How much metal was machined from this casting in these operations?

17. Determine the length of the part as indicated by the question mark in the drawing to the right.

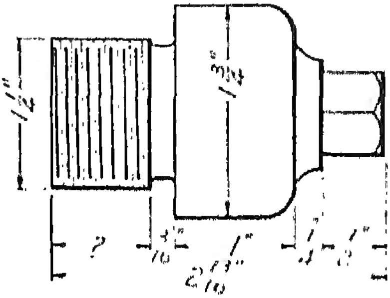


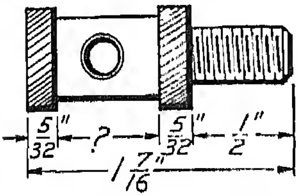
18. After withdrawing $14\frac{3}{4}$ gal. of oil from a tank containing $27\frac{1}{2}$ gal., how much remains in the tank?

19. What are the missing dimensions in the following sketch?



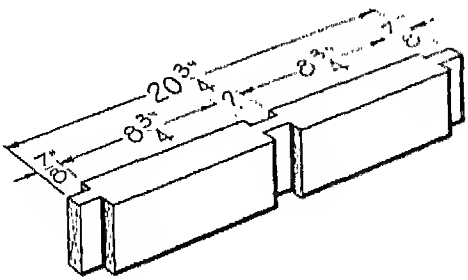
20. What is the length of the threaded portion on the thrust nut in the drawing to the right?





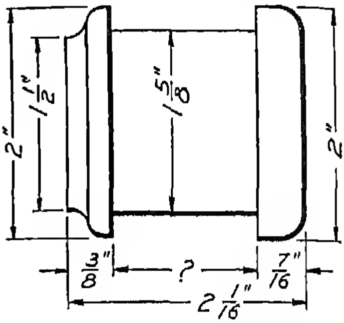
21. Determine the value of the missing dimension as noted by the question mark on the sketch of the binding post to the left.

22. The measurement of the half-lap cut on the following crossrail for a small table is missing. What should it be?

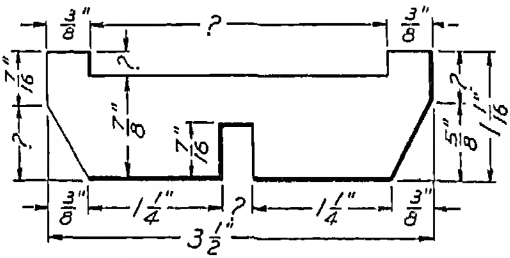


23. From a piece of brass tubing $28\frac{1}{2}$ in. long, two pieces measuring $6\frac{3}{4}$ in. and $9\frac{1}{2}$ in. are cut. What is the length of the piece remaining?

24. An important measurement on the drawing to the right of a bronze bearing is missing. What is the value of the measurement referred to?

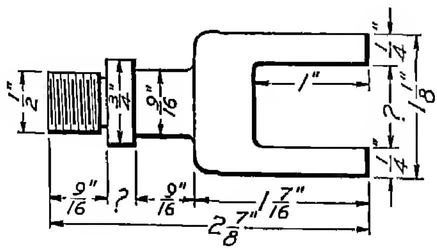


25. What are the missing dimensions in the following?

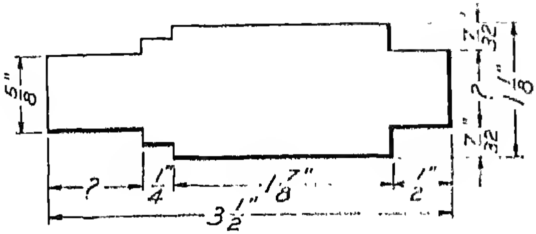


26. A piece of broken belt that was used in a woodworking shop measures $40\frac{1}{2}$ in. In order to be used again, the belt should measure $49\frac{1}{4}$ in. How long a piece must be added to the broken length in order that the belt may be made long enough for use?

27. What are the missing dimensions in the following?



28. What are the values of the dimensions that are missing in the following drawing?*



ANSWERS TO PROBLEMS

Pages 18 to 20.

1. $40\frac{41}{64}$; $22\frac{8}{25}$; 16 in.; $37\frac{3}{4}$.
2. $23\frac{3}{4}$ in.
3. $2\frac{1}{2}$ in.; $\frac{15}{16}$ in.
4. $\frac{7}{8}$ in.
5. 3 ft.
6. $13\frac{3}{4}$ ft.
7. $2\frac{1}{2}$ ft.
8. $3\frac{7}{8}$ in.
9. $40\frac{3}{4}$ ft.
10. $8\frac{5}{16}$.
11. $2\frac{3}{4}$ in.
12. $18\frac{3}{4}$ in.

Pages 25 to 32.

1. $11\frac{11}{16}$; $19\frac{31}{48}$; $25\frac{19}{30}$.
2. $6\frac{5}{64}$ in.
3. $15\frac{5}{16}$.
4. 40 ft.
5. $7\frac{1}{16}$ in.
6. $4\frac{21}{64}$.
7. 8 hr.
8. $17\frac{1}{8}$ in.
9. $16\frac{9}{32}$ in.
10. $70\frac{1}{2}$ ft.
11. $18\frac{1}{16}$ in.
12. $12\frac{1}{2}$ hr.
13. (*a*) $6\frac{1}{4}$; (*b*) $18\frac{29}{35}$; (*c*) $19\frac{2}{5}$.
14. $11\frac{1}{4}$ in.
15. $24\frac{5}{16}$ in.
16. $12\frac{1}{4}$ reams.
17. $4\frac{13}{32}$ in.
18. $12\frac{5}{16}$ in.
19. $30\frac{1}{2}$ hr.
20. $7\frac{9}{16}$ in.
21. $40\frac{3}{4}$ in.
22. 8 in.
23. $3\frac{3}{4}$ in.
24. $121\frac{1}{4}$ ft.
25. $5\frac{1}{2}$ in.
26. $13\frac{15}{16}$ in.
27. $4\frac{3}{16}$ in.
28. $14\frac{1}{4}$ hr.
29. $1\frac{13}{16}$.
30. $2\frac{13}{64}$ in.; $1\frac{9}{64}$ in.;
31. $145\frac{1}{4}$ lb.
32. $22\frac{13}{16}$ in.

*Answers to these problems will be found on page 48.

Pages 33 and 34.

1. $\frac{2}{5}$.
2. $\frac{5}{13}$.
3. $\frac{3}{4}$.
4. $\frac{5}{16}$ in.
5. $\frac{5}{16}$.
6. $\frac{1}{6}$.
7. $\frac{38}{65}$.
8. $\frac{11}{16}$ in.
9. $\frac{1}{8}$ in.
10. $\frac{13}{25}$.
11. $\frac{12}{25}$; $\frac{4}{7}$; $\frac{1}{4}$ in.; $\frac{11}{16}$ in.; $\frac{3}{4}$ in.
12. $\frac{5}{8}$ in.

Pages 35 to 38.

1. $\frac{11}{16}$ in.; $\frac{15}{64}$ in.; $\frac{43}{64}$ in.
2. $\frac{7}{32}$ in.; $\frac{17}{108}$; $\frac{11}{48}$; $\frac{3}{20}$.
3. $\frac{11}{16}$ in.
4. $\frac{5}{72}$.
5. $\frac{13}{16}$ in.
6. $\frac{25}{32}$ in.
7. $\frac{9}{32}$ in.; $\frac{3}{8}$ in.
8. $\frac{61}{125}$; $\frac{4}{9}$; $\frac{7}{64}$; $\frac{35}{72}$.
9. $\frac{51}{64}$ in.
10. $\frac{13}{32}$ in.
11. $\frac{5}{16}$ in.
12. $\frac{19}{64}$ in.
13. $\frac{3}{64}$ in.
14. $\frac{15}{32}$ in.
15. $\frac{27}{64}$ in.
16. $\frac{9}{32}$ in.

Pages 42 to 47.

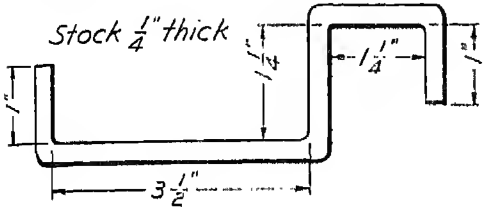
1. (*a*) $14\frac{11}{16}$ in.;
   (*b*) $4\frac{11}{16}$;
   (*c*) $1\frac{1}{4}$; $2\frac{5}{24}$; $5\frac{7}{18}$;
   (*d*) $11\frac{33}{35}$.
2. $4\frac{5}{32}$ in.
3. $8\frac{7}{32}$ in.
4. $3\frac{7}{8}$ in.; $\frac{5}{8}$ in.; $4\frac{7}{8}$ in.
5. $2\frac{1}{4}$.
6. $3\frac{7}{16}$.
7. $20\frac{3}{4}$ ft.
8. $12\frac{15}{16}$ in.
9. $2\frac{5}{32}$ in.
10. $\frac{31}{32}$ in.
11. $\frac{7}{8}$ in.
12. $\frac{3}{16}$ in. off thickness;
    $\frac{1}{8}$ in. off width.
13. $\frac{7}{16}$ in.; $\frac{5}{16}$ in.; $\frac{5}{16}$ in.
14. $6\frac{2}{3}$ lb.
15. $2\frac{1}{2}$ ft.
16. $14\frac{3}{4}$ lb.
17. $\frac{15}{32}$ in.
18. $12\frac{3}{4}$ gal.
19. $\frac{17}{32}$ in.; $1\frac{37}{64}$ in.; $\frac{3}{8}$ in.; $\frac{5}{32}$ in.
20. $\frac{7}{8}$ in.
21. $\frac{5}{8}$ in.
22. $1\frac{1}{2}$ in.
23. $12\frac{1}{4}$ in.
24. $1\frac{1}{4}$ in.
25. $2\frac{3}{4}$ in.; $\frac{7}{16}$ in.; $\frac{1}{4}$ in.; $\frac{3}{16}$ in.; $\frac{5}{8}$ in.
26. $8\frac{3}{4}$ in.
27. $\frac{5}{8}$ in.; $\frac{5}{16}$ in.
28. $\frac{11}{16}$ in.; $\frac{7}{8}$ in.

Review Problems on Addition and Subtraction of Fractions

1. What number added to $17\frac{15}{32}$ equals $35\frac{8}{21}$?

2. What is the difference between $18\frac{15}{16}$ in. and $16\frac{3}{4}$ in.; between $42\frac{17}{64}$ in. and $13\frac{13}{64}$ in.; between $15\frac{19}{32}$ in. and $11\frac{13}{16}$ in.?

3. A steel block $1\frac{11}{64}$ in. thick has $\frac{9}{32}$ in. removed during a shaping operation. What is the resulting thickness after this operation?

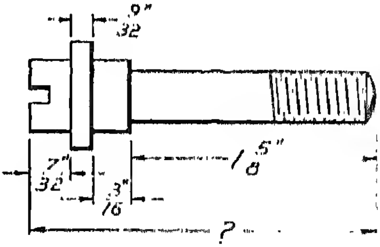


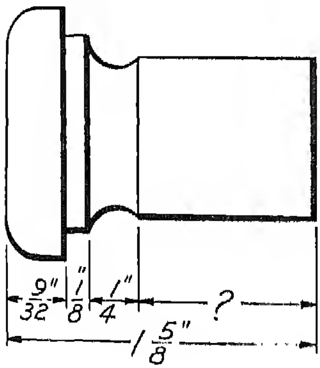
4. Determine the length of metal needed to bend the piece in the drawing at the left.

5. From a sheet of brass $11\frac{5}{8}$ in. wide, there is cut a strip $4\frac{3}{4}$ in. wide. What is the width of the sheet that remains?

6. Determine the length of the special screw below.

7. To paint a small kitchen, a pantry, and a back hall, four boys work 5 hr., $6\frac{1}{4}$ hr., $4\frac{1}{3}$ hr., and $8\frac{3}{4}$ hr. respectively on the job. How many hours all together were spent on the work?

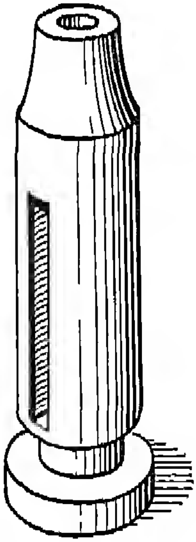


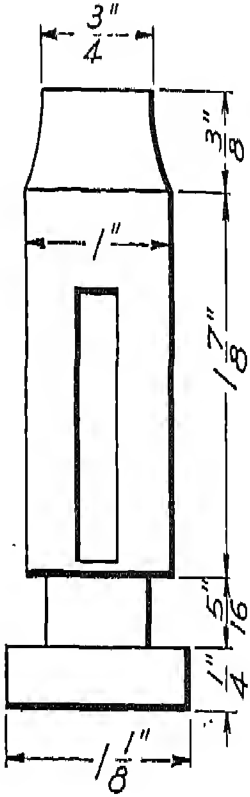


8. The drawing to the left was returned to the drafting room by the foreman of the toolroom because an important dimension was missing. Check it over and redraw it, giving all dimensions.

9. To turn out a certain job on a lathe, the workman is supplied with four bars of stock weighing $19\frac{3}{4}$ lb., $17\frac{3}{4}$ lb., $20\frac{1}{2}$ lb., and $28\frac{1}{4}$ lb. After the job is completed he finds that he has $9\frac{1}{2}$ lb. of stock left. How much did he use?

10. The dimension giving the total length of the tool post shown in the sketch is missing. What should it be?


Tool Post



MULTIPLICATION OF FRACTIONS

Fractions are multiplied by multiplying the numerators for a *new* numerator, and by multiplying the denominators for a *new* denominator. The result of such multiplication may be a *fraction*, a *whole number* or a *mixed number*, as seen in the following examples:

$\frac{3}{4}$ multiplied by $\frac{1}{2}$ becomes $\frac{3}{4} \times \frac{1}{2}$, or $\dfrac{3 \times 1}{4 \times 2} = \frac{3}{8}$.

$\frac{3}{2}$ multiplied by $\frac{4}{3}$ becomes $\frac{3}{2} \times \frac{4}{3}$, or $\dfrac{3 \times 4}{2 \times 3} = \frac{12}{6}$ or 2.

$\frac{3}{2}$ multiplied by $\frac{5}{6}$ becomes $\frac{3}{2} \times \frac{5}{6}$, or $\dfrac{3 \times 5}{2 \times 6} = \frac{15}{12}$ or $1\frac{1}{4}$.

Multiplication, besides being expressed by the sign $\times$ may also be indicated by the word *times*, or *of*, as $3\frac{1}{4}$ times $\frac{5}{8}$, or $\frac{1}{2}$ of $1\frac{11}{16}$.

MULTIPLICATION OF MIXED NUMBERS

To multiply mixed numbers, they should first be changed to *improper* fractions. The multiplication is then carried out as explained above.

This is illustrated in the following examples:

Example 1:

Multiply $2\frac{1}{6}$ by $1\frac{1}{4}$ by $\frac{1}{3}$.

Solution and Explanation:

As explained above the mixed numbers $2\frac{1}{6}$ and $1\frac{1}{4}$ are first changed to improper fractions.

$2\frac{1}{6}$ becomes $\frac{13}{6}$; $1\frac{1}{4}$ becomes $\frac{5}{4}$.

All fractions are then multiplied together as follows:

$2\frac{1}{6} \times 1\frac{1}{4} \times \frac{1}{3}$ equals $\frac{13}{6} \times \frac{5}{4} \times \frac{1}{3}$, which works out as,

$$\frac{13 \times 5 \times 1}{6 \times 4 \times 3} = \frac{65}{72}.$$

That is, $2\frac{1}{6} \times 1\frac{1}{4} \times \frac{1}{3}$ equals $\frac{65}{72}$.

Example 2:

If a printer can "set" $2\frac{1}{4}$ pages of type matter in 1 hour, at this rate how many pages of the same size can he set in $5\frac{1}{2}$ hours?

Solution and Explanation:

Since $2\frac{1}{4}$ and $5\frac{1}{2}$ are mixed numbers they should first be changed to improper fractions as previously explained.

$2\frac{1}{4}$ becomes $\frac{9}{4}$; $5\frac{1}{2}$ becomes $\frac{11}{2}$.

These are multiplied together as in the previous problem.

$2\frac{1}{4} \times 5\frac{1}{2}$ equals $\frac{9}{4} \times \frac{11}{2}$, which in turn becomes:

$$\frac{9 \times 11}{4 \times 2} = \frac{99}{8}, \text{ or } 12\tfrac{3}{8}.$$

That is, the printer should be able to set $12\frac{3}{8}$ pages of such type matter in $5\frac{1}{2}$ hours.

Example 3:

Lead pipe $\frac{5}{8}$ in. in diameter weighs $1\frac{3}{4}$ lb. per foot. What is the weight of a piece of this size pipe that measures $6\frac{1}{2}$ ft. long?

Solution and Explanation:

If one foot of lead pipe weighs $1\frac{3}{4}$ lb., then $6\frac{1}{2}$ ft. of such pipe will weigh $6\frac{1}{2}$ times that amount.

$1\frac{3}{4}$ changed to an improper fraction is $\frac{7}{4}$.

$6\frac{1}{2}$ changed to an improper fraction is $\frac{13}{2}$.

The multiplication then becomes:

$1\frac{3}{4} \times 6\frac{1}{2}$ which equals,

$$\frac{7}{4} \times \frac{13}{2}, \text{ or } \frac{7 \times 13}{4 \times 2} = \frac{91}{8} \text{ or } 11\tfrac{3}{8}.$$

That is: $6\frac{1}{2}$ ft. of $\frac{5}{8}$-in. lead pipe weighs $11\frac{3}{8}$ lb.

MULTIPLYING MIXED NUMBERS OR FRACTIONS BY WHOLE NUMBERS

When a *fraction* or a *mixed number* is to be multiplied by a *whole number*, the whole number is considered as the numerator of a fraction which has 1 for its denominator. The multiplication is then carried out in the same manner as multiplying common fractions.

Example 1:

Water is listed as weighing $8\frac{1}{3}$ lb. per gallon. At this rate what is the weight of water in a can holding 15 gal.?

Solution and Explanation:

Changing $8\frac{1}{3}$ to an improper fraction, it becomes $\frac{25}{3}$.

Following the above rule of using the whole number as the numerator of a fraction that has 1 for the denominator, the multiplication is carried out as follows:

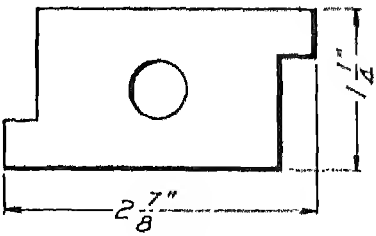
$$15 \times 8\tfrac{1}{3}, \text{ or } \tfrac{15}{1} \times \tfrac{25}{3}, \text{ which equals } 125.$$

That is, the weight of 15 gallons of water is 125 lb.

As a further application of this process of multiplication of fractions to shop problems the following is worked out.

Example 2:

The material out of which the following piece is made weighs $\frac{1}{3}$ lb. per inch of length. In making each piece $\frac{1}{4}$ in. is to be added to the length for waste and finishing. What is the weight of the material required to make up 24 such pieces?



Solution and Explanation:

In calculating problems of this kind the weight of the material required in making one piece should first be determined. After this the number is multiplied by the number of pieces wanted.

The length of the piece as shown is $2\frac{7}{8}$ in. To this is to be added the $\frac{1}{4}$ in. for waste and finishing. This makes the total

length of the material needed for one piece equal to $2\frac{7}{8}$ in. + $\frac{1}{4}$ in., or $3\frac{1}{8}$ in.

Since this material weighs $\frac{1}{6}$ lb. per inch of length then a piece $3\frac{1}{8}$ in. long would weigh $3\frac{1}{8} \times \frac{1}{6}$, or $\frac{25}{48}$ lb.

The weight of 24 such pieces would be 24 times the weight of one piece, or $24 \times \frac{25}{48}$. Following the above rule involving the multiplication of fractions and whole numbers this works out as:

$$\frac{24}{1} \times \frac{25}{48} = \frac{600}{48}, \text{ or } 12\frac{1}{2}.$$

That is, $12\frac{1}{2}$ lb. of material are needed in making 24 pieces according to the drawing on page 53.

CANCELLATION

The multiplication of fractions may be somewhat shortened by the process of *cancellation.* This process eliminates considerable multiplying in both numerator and denominator, and reduces these terms to simple form before multiplying for a new numerator or a new denominator. This is accomplished by striking out, or canceling factors that are common to both numerator and denominator.

How this is done is illustrated in the following problem:

Example 1:

Multiply $\frac{3}{4}$ by $\frac{8}{21}$ by $\frac{7}{12}$.

Solution and Explanation:

Solving this problem by the method of multiplication of fractions as first explained, the result becomes: $\frac{3}{4} \times \frac{8}{21} \times \frac{7}{12}$, with $3 \times 8 \times 7$, or 168 as the new numerator, and $4 \times 21 \times 12$, or 1008 for the new denominator. This produces the fraction $\frac{168}{1008}$.

As this is not in its lowest terms it must be reduced by factoring both the numerator and the denominator.

$$\frac{168}{1008} = \frac{84}{504} = \frac{42}{252} = \frac{21}{126} = \frac{7}{42} = \frac{1}{6}.$$

That is:

$$\frac{3}{4} \times \frac{8}{21} \times \frac{7}{12} = \frac{1}{6}.$$

Working out the same problem by cancellation, the solution becomes:

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{8}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{21}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{6}{\cancel{12}}} = \frac{1}{6}$$

Comparing the two processes, it is readily seen that cancellation is much shorter, and that it is really a *reduction to lowest terms* before multiplying takes place. This reduction to lowest terms is accomplished as explained, by dividing both terms of the fraction by *factors* that *are common* to *each*. The method of cancellation as used in the above problem is explained as follows:

Seven is a factor of 7 in the numerator and 21 in the denominator. These are "canceled" by drawing a line through each number and placing above the numerator and below the denominator the figures which represent the number of times this factor 7 is contained in these numbers. Accordingly 1 would be placed above 7, and 3 below the 21. But this 3 is a factor of the 3 in the numerator, so these are crossed out, or "canceled," and the figure 1 placed as indicated. In the same manner 4 is factor of 4 in the denominator and 8 in the numerator. These are canceled and the quotients are inserted in their proper places. The 2 which results from this last factoring, is a factor of 12 in the denominator, being contained in it 6 times. These numbers are canceled, the figure 1 being placed above the crossed out 2, and the figure 6 being placed below the crossed out 12.

After these operations, it is seen that there is no further opportunity for factoring.

To obtain the result of this cancellation all *uncanceled* terms remaining in the numerator are multiplied together for a *new numerator*, and all *uncanceled* terms remaining in the denominator are multiplied together for a *new denominator*. These give the fraction as the result of the cancellation process.

The only uncanceled terms remaining in the numerator are the three 1's. Their product is 1.

In the denominator, the uncanceled terms are 6 and 1. These multiplied together give 6.

The new fraction then becomes $\frac{1}{6}$, as the result of this cancellation process, which is the same as that obtained by the longer process.

NOTE: It is very important that the terms be crossed out as they are factored, as such care will tend to avoid errors. It is also well to remember that only one number in the numerator and one in the denominator should be canceled at one time. Inasmuch as the product of the quotients, 1, do not effect the result, they may be omitted from the work of cancellation hereafter. This may be illustrated by working out the following problem:

$$\frac{6}{13} \times \frac{8}{11} \times \frac{18}{21} \times \frac{1}{2} \times \frac{7}{8} = ?$$

Canceling the terms as above explained this becomes:

$$\frac{\overset{2}{\cancel{6}}}{13} \times \frac{\cancel{8}}{11} \times \frac{18}{\underset{3}{\cancel{21}}} \times \frac{1}{\cancel{2}} \times \frac{\cancel{7}}{\cancel{8}} = \frac{18}{143}.$$

That is, $\frac{6}{13} \times \frac{8}{11} \times \frac{18}{21} \times \frac{1}{2} \times \frac{7}{8}$ equals $\frac{18}{143}$.

Problems Involving Multiplication of Fractions

1. Multiply: $2\frac{10}{17}$ by 5 by $2\frac{2}{15}$ by $\frac{1}{16}$ by $2\frac{1}{25}$.

2. Solve: $2\frac{1}{3} \times \frac{2}{9} \times \frac{12}{14} \times \frac{3}{4} \times \frac{5}{6} \times 1\frac{1}{2} = ?$

3. What is the weight of 4 cast-iron gear blanks each weighing $5\frac{3}{4}$ lb.?

4. Multiply, $3\frac{7}{16}$ by $6\frac{1}{4}$ by $\frac{1}{5}$, by $\frac{24}{25}$.

5. Water weighs $62\frac{1}{2}$ lb. per cubic foot. What is the weight of water in 6 tanks each containing $5\frac{1}{3}$ cu. ft.?

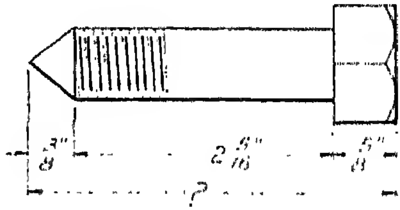
6. If there are $7\frac{1}{2}$ gal. of water in a cubic foot and each gallon weighs $8\frac{1}{3}$ lb., what is the weight of water that will fill a tank containing $3\frac{1}{5}$ cu. ft.?

7. If a standard barrel contains $31\frac{1}{2}$ gal. what is the weight of oil in such a barrel that is $\frac{3}{5}$ full? The oil weighs $7\frac{1}{4}$ lb. per gallon.

8. If there are $7\frac{1}{2}$ gal. to the cubic foot, how many gallons of oil will 4 tanks hold, each tank containing $4\frac{1}{2}$ cu. ft.?

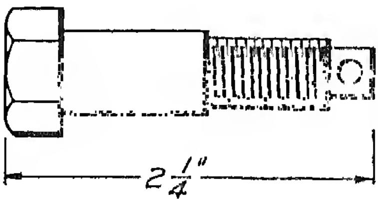
9. Calculate the weight of 16 bars of cold-rolled steel, each bar being $11\frac{1}{2}$ ft. long. This steel weighs $2\frac{1}{4}$ lb. per foot length.

10. What is the weight of the material needed to make 64 bolts like the one to the right, $\frac{1}{4}$ in. being allowed on each piece for finishing up. The stock out of which the bolts are to be made weighs $\frac{3}{4}$ lb. per inch of length.



11. If a 3-in. wastepipe is listed as weighing $9\frac{1}{2}$ lb. per foot of length, what is the weight of 4 such pipes each $4\frac{3}{4}$ ft. long?

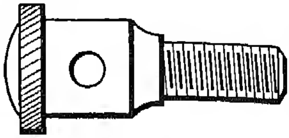
12. How many pounds of stock weighing $\frac{1}{5}$ lb. per inch of length, are needed to make 200 bolts according to the following drawing? There is to be added to each bolt $\frac{1}{8}$ in. for cutting off and finishing.



13. What is the weight of a $20\frac{1}{2}$-ft. coil of lead pipe, if the pipe weighs $1\frac{1}{4}$ lb. per running foot?

14. Round steel stock $\frac{3}{4}$ in. in diameter weighs $1\frac{1}{2}$ lb. per running foot. Calculate the weight of 12 bars each $8\frac{1}{2}$ ft. long.

15. A machinist used $7\frac{1}{3}$ ft. of a steel bar $1\frac{1}{8}$ in. in diameter to make up 25 pieces like those in the sketch on the right. If this stock weighs $3\frac{3}{8}$ lb. per foot of length, what is the weight of the material used in making these screws?



16. Brass rod $\frac{3}{4}$ in. in diameter is listed as weighing $1\frac{2}{3}$ lb. per foot. Determine the weight of 16 pieces of this stock each $6\frac{3}{4}$ ft. long.

17. Find the weight of material needed to make 48 pins, each requiring $6\frac{3}{4}$ in. of stock. The material weighs $\frac{5}{16}$ lb. per running inch.*

DIVISION OF FRACTIONS

To divide one fraction by another, invert the terms of the divisor, that is, turn the fraction upside down, and then proceed as in multiplying fractions. In this division, the cancellation method should be used as much as possible.

Example:

Divide $\frac{3}{4}$ by $\frac{7}{8}$.

Solution and Explanation:

$\frac{7}{8}$ is the divisor. Inverted, this becomes $\frac{8}{7}$.

According to the above rule:

$$\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{3}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{7} = \frac{6}{7}$$

That is, $\frac{3}{4} \div \frac{7}{8} = \frac{6}{7}$.

DIVISION INVOLVING MIXED NUMBERS

In dividing mixed numbers, first change the mixed numbers to *improper fractions*. After this, the terms of the *divisor* are *inverted*, and the process becomes one of multiplication.

*Answers to these problems will be found on page 64.

Example 1:

Divide $1\frac{3}{4}$ by $2\frac{5}{8}$.

Solution and Explanation:

As the first step $1\frac{3}{4} = \frac{7}{4}$, and $2\frac{5}{8} = \frac{21}{8}$.

$$\frac{7}{4} \div \frac{21}{8} = \frac{7}{4} \times \frac{8}{21}, \text{ which becomes } \frac{\cancel{7}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{21}}}, \text{ or } \frac{2}{3}.$$

That is: $1\frac{3}{4}$ divided by $2\frac{5}{8}$ equals $\frac{2}{3}$.

Example 2:

How many pieces $\frac{9}{16}$ in. long can be cut from a piece $6\frac{3}{4}$ in. long?

Solution and Explanation:

This is determined by dividing $6\frac{3}{4}$ by $\frac{9}{16}$.

As a first step, $6\frac{3}{4}$ is changed to the improper fraction $\frac{27}{4}$.

Proceeding as above, $6\frac{3}{4} \div \frac{9}{16}$ equals $\frac{27}{4} \times \frac{16}{9}$, which works out as $\frac{\overset{3}{\cancel{27}}}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{\cancel{9}} = 12.$

This indicates that 12 pieces each $\frac{9}{16}$ in. long can be cut from the above piece which measures $6\frac{3}{4}$ in. long.

Example 3:

Divide 6 by $8\frac{2}{3}$.

Solution and Explanation:

Changing $8\frac{2}{3}$ to an improper fraction, it equals $\frac{26}{3}$.

The division is then carried on as explained above.

$6 \div 8\frac{2}{3} = \frac{6}{1} \div \frac{26}{3}$, or $\frac{6}{1} \times \frac{3}{26}$, which becomes,

$$\frac{\overset{3}{\cancel{6}}}{1} \times \frac{3}{\underset{13}{\cancel{26}}}, \text{ or } \frac{9}{13}.$$

That is, $6 \div 8\frac{2}{3}$ equals $\frac{9}{13}$.

Should the divisor be a *whole number*, then this whole number is considered as a fraction which has 1 as the denominator

and the whole number for the numerator. The division is then carried out the same as in the division of ordinary fractions.

Example 4:

In slitting a piece of thin brass that measures $1\frac{5}{16}$ in. wide into 2 equal parts how wide will each piece be, assuming there is no loss in cutting?

Solution and Explanation:

This problem is solved by dividing $1\frac{5}{16}$ by 2.

Following the above procedure, $1\frac{5}{16}$ changed to an improper fraction equals $\frac{21}{16}$.

$1\frac{5}{16} \div 2$ then becomes $\frac{21}{16} \div 2$, or $\frac{21}{16} \times \frac{1}{2}$ which is $\frac{21}{32}$.

This indicates that each piece should be $\frac{21}{32}$ in. wide.

Problems Involving the Division of Fractions

1. Divide: $32\frac{13}{16}$ by $3\frac{1}{8}$; $7\frac{1}{3}$ by $\frac{4}{9}$.

   Divide $2\frac{13}{16}$ in. into 5 equal parts.

2. What is the weight of a coil of wire measuring 250 ft., there being $31\frac{1}{4}$ ft. to the pound?

3. A special size bolt weighs $\frac{1}{4}$ lb. How many such bolts are there in a box which weighs $80\frac{1}{4}$ lb., if the box when empty weighs $3\frac{1}{4}$ lb.?

4. How many cubic feet of oil in a tank containing $487\frac{1}{2}$ gal., if one cubic foot is the equivalent of $7\frac{1}{2}$ gal.?

5. If 4 brass pins can be made from each foot length of rod stock weighing $1\frac{1}{4}$ lb. per foot, how many pins can be made from $72\frac{1}{2}$ lb. of this stock?

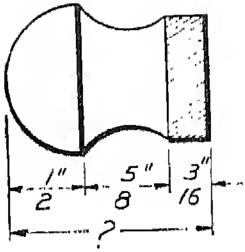
6. Solve the following:

$$3\tfrac{3}{16} \times \tfrac{4}{9} \times 6\tfrac{3}{4} \div \tfrac{1}{6} \times \tfrac{4}{17} \div \tfrac{1}{2}.$$

$$16 \div 1\tfrac{1}{7} \div 2\tfrac{1}{3} \div 4 \div 1\tfrac{1}{2} \times 6.$$

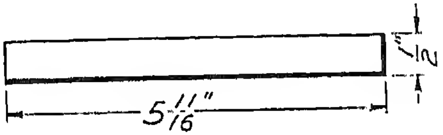
$$5 \div 2\tfrac{1}{2} \times \tfrac{1}{4} \div 6 \times 15 \div 8\tfrac{1}{3}.$$

7. Allowing $\frac{3}{16}$ in. for finishing up each piece, how many knobs like the one to the right can be cut from a bar $19\frac{1}{2}$ in. long?



8. A cubic foot of copper is listed as weighing 550 lb. and a cubic foot of water weighs $62\frac{1}{2}$ lb. How many times heavier is the copper than the water?

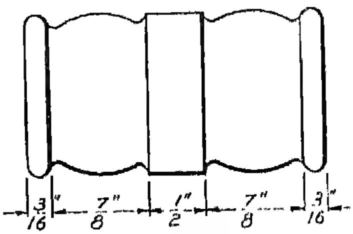
9. Redraw the sketch below, and then draw lines across it showing the length of the piece divided into 7 equal parts. How far apart should these lines be?



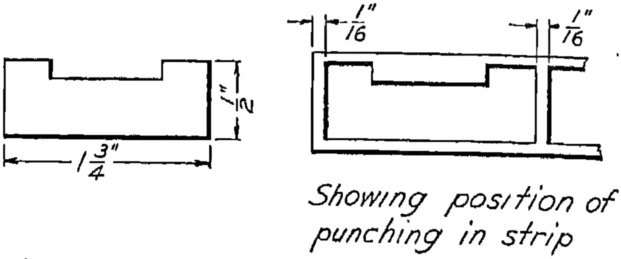
10. How many pins $1\frac{3}{16}$ in. long can be cut from a piece of drill rod $28\frac{3}{4}$ in. long? To allow for cutting off and finishing each pin $\frac{1}{16}$ in. is to be added to its length.

11. A machine apprentice is asked to cut a copper bar that measures $32\frac{15}{16}$ in. long into 8 equal lengths. The cutting saw on the machine which he uses is $\frac{1}{16}$ in. thick. What should be the length of each piece when cut?

12. A young man learning how to do wood turning is given the drawing to the right of a gavel head. He is also given a piece of mahogany $23\frac{1}{4}$ in. long, and told to turn up as many gavel heads as possible from this piece. How many gavel heads should he be able to get out of this material allowing a waste of $1\frac{1}{4}$ in. for each head due to fastening it in the lathe? What is the length of the piece remaining, if any?

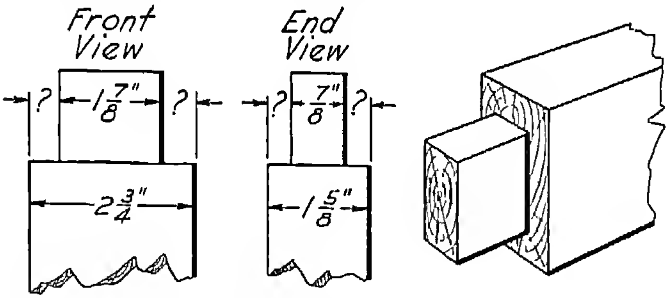


13. Allowing $\frac{1}{16}$ in. between each punching and at each end of the strip, how many blanks like the following can be punched from a strip that measures $23\frac{5}{8}$ in. long?



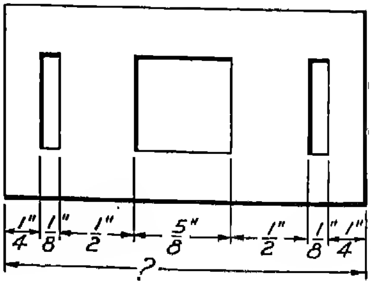
14. A thin copper strip $10\frac{11}{16}$ in. long is to be divided into 9 equal pieces. What is the length of each of these pieces?

15. As shown in the drawing below, the tenon is to be cut exactly in the middle of the piece illustrated. Determine the measurements indicated by the question marks.

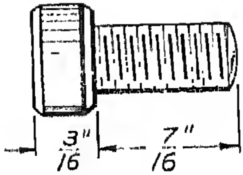


16. A board that measures $10\frac{1}{2}$ in. wide is to be "ripped up" on a table saw into strips $1\frac{1}{2}$ in. wide. How many such strips can be cut from this board adding $\frac{1}{8}$ in. for the width of the saw cut? What is the width of the strip that remains?

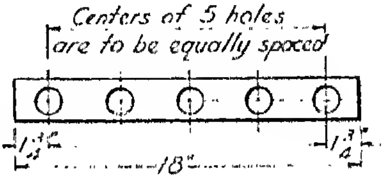
17. How many pieces according to the dimensions to the right can be made from a strip of fiber $1\frac{1}{4}$ in. wide and $21\frac{1}{4}$ in. long?



18. Determine the number of binding-post screws like the one shown that can be turned from a brass rod $19\frac{1}{2}$ in. long. To provide for cutting off and finishing $\frac{1}{8}$ in. should be added to the length of each screw.

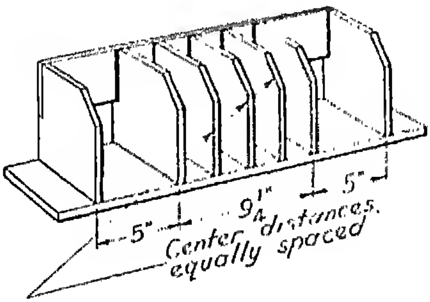


19. As specified, the strip of fiber illustrated below is to have the centers of the five screw holes spaced at equal distances. What should this measurement be?



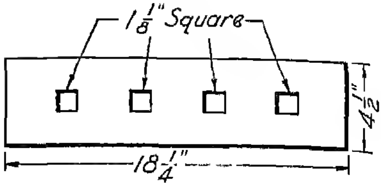
20. A piece of brass tubing measuring $5\frac{1}{4}$ in. long is to be cut into 7 equal parts. Neglecting the width of the saw blade how long will the length of each piece be?

21. Referring to the following sketch of a stationery rack, what should be the center distance between each of the upright partitions?

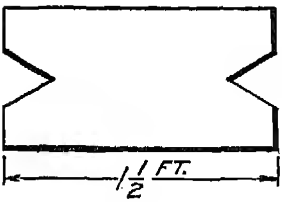


22. With each turn of the hand wheel on a large valve, the valve stem moves $\frac{5}{32}$ in. How many turns of this hand wheel are necessary to move the stem $1\frac{1}{4}$ in.?

23. The sketch to the right is to be used in laying out an iron plate as per the directions listed below. Redraw the sketch giving all the dimensions needed for this job.



DIRECTIONS: Place four $1\frac{1}{8}$ in. square holes so that the spaces between the holes will measure the same as the spaces at the ends. These holes are also to be the same distance from the top edge as from the bottom edge.



24. A board $13\frac{3}{4}$ ft. long is used in making an order of pieces like the one to the left. A crack in one end of the board necessitates cutting off $1\frac{1}{4}$ ft. from that end. Working from this newly cut end how many such pieces can be laid out on the board? What is the length of the piece remaining?

25. After cutting off $3\frac{1}{4}$ ft. from a board 12 ft. long the remainder of the board is cut into 5 equal lengths. Neglecting the width of the saw cut what is the measurement of each of these 5 equal lengths?*

ANSWERS TO PROBLEMS

Pages 56 to 58.

1. 4.
2. $\frac{5}{12}$.
3. 23 lb.
4. $4\frac{1}{8}$.
5. 2000 lb.
6. 200 lb.
7. $162\frac{3}{4}$ lb.
8. 135 lb.
9. 414 lb.
10. 57 lb.
11. 171 lb.
12. 95 lb.
13. $25\frac{5}{8}$ lb.
14. 153 lb.
15. $24\frac{3}{4}$.
16. 180 lb.
17. $101\frac{1}{4}$ lb.

Pages 60 to 64.

1. $10\frac{1}{2}$; $16\frac{1}{2}$; $\frac{9}{16}$ in.
2. 8 lb.
3. 308 bolts.
4. 65 cu. ft.
5. 232 pins.
6. 27; 6; $\frac{3}{20}$.
7. 13 knobs.
8. $8\frac{4}{5}$.
9. $\frac{13}{16}$ in.
10. 23 pins.

*Answers to these problems will be found on this page and page 65.

11. $4\frac{1}{16}$ in.
12. 6 heads, nothing remaining.
13. 13 blanks.
14. $1\frac{3}{16}$ in.
15. $\frac{7}{16}$ in.
16. 6 strips; $\frac{3}{4}$ in. wide.
17. 8 whole pieces.
18. 26 pieces.
19. $3\frac{5}{8}$ in.
20. $\frac{3}{4}$ in.
21. $2\frac{5}{16}$ in.
22. 8 turns.
23. $2\frac{3}{4}$ in.; $1\frac{11}{16}$ in.
24. 8 pieces; $\frac{1}{2}$ ft.
25. $1\frac{3}{4}$ ft.

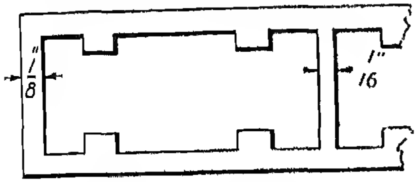
Review Problems in Multiplication and Division of Fractions

1. Determine the answers to the following:

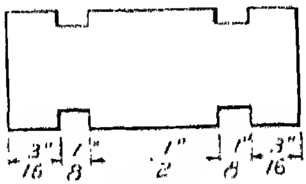
$$\frac{1}{2} \times 7\frac{1}{3} \times \frac{5}{11} \div 2\frac{1}{2} \times 6 = ?$$
$$\frac{14}{15} \div 2\frac{1}{3} \times 4 \times \frac{1}{5} \div \frac{1}{4} = ?$$

2. A job being done in a sheet-metal shop requires that a strip of sheet iron $13\frac{3}{4}$ in. long be measured off into 5 equal spaces. How wide should each of these spaces be?

3. How many full pieces like the following can be "punched out" from a strip of sheet brass 21 in. long? The first punching is to be made $\frac{1}{8}$ in. from the end and the remaining punchings are to lie $\frac{1}{16}$ in. apart as noted.

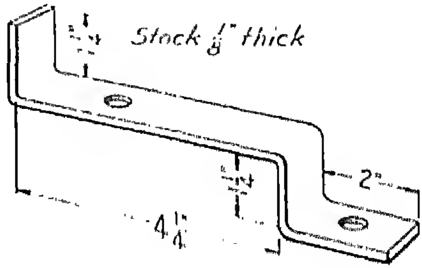


Showing position of punchings in strip



Punching

4. Calculate the length of stock required to make 4 pieces according to the shape and dimensions to the right.

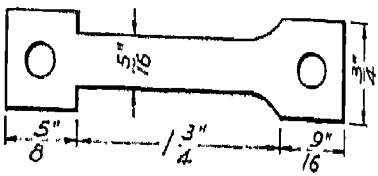


5. An oil tank empties through a faucet at the rate of $18\frac{3}{4}$ gal. per minute. How long will it take to empty this tank if it contains $31\frac{1}{4}$ cu. ft. of oil, there being $7\frac{1}{2}$ gal. to the cubic foot?

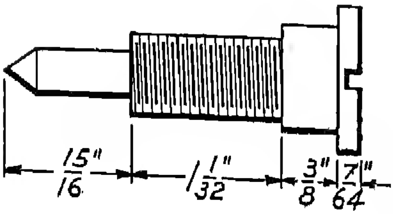
6. How many strips each $2\frac{7}{8}$ in. wide can be cut from a sheet of tin plate $16\frac{5}{8}$ in. wide?

7. A coil of lead pipe weighs $58\frac{1}{2}$ lb. If this pipe is listed at $2\frac{1}{4}$ lb. to the foot, how many feet are in this coil?

8. How many pieces like the one to the right can be made from a strip of metal $\frac{3}{4}$ in. wide and $49\frac{1}{2}$ in. long? No allowance is to be made for cutting off each piece. What is the length of the piece remaining?



9. The stock from which the screw to the left is to be made weighs $\frac{1}{8}$ lb. per inch of length. What is the weight of stock needed to make up 64 screws, allowing $\frac{1}{16}$-in. waste for cutting off and finishing up each screw?



10. A shelf that measures 14 ft. long is to be divided into 6 spaces as follows: The first space is to be marked off $1\frac{1}{3}$ ft. from the right end. The remaining length is to be divided into 5 equal spaces. How long will each of these 5 spaces be?

DECIMAL FRACTIONS

Interpretation; addition; subtraction; multiplication; division; conversion; application to daily problems in business, industry, and everyday shop conditions.

Decimal Fractions

Decimal fractions are more commonly referred to as decimals. They are fractions whose denominators consist of 10 or some power of 10, as 100, 1,000, 10,000, 100,000, etc.

These denominators, however, are only expressed in reading the decimal, and are *not* expressed in writing it. As for example:

.7 is read "seven tenths."
.42 is read "forty-two hundredths."
.325 is read "three hundred twenty-five thousandths."
.9827 is read "nine thousand eight hundred twenty-seven ten-thousandths."

The *denominator*, or name, of the decimal, whether it be tenths, hundredths, or thousandths, is determined by the position of a point (.) called the *decimal point*. The position of this decimal point, in turn, is governed by the *value* of the denominator of the fraction.

For example, in the fraction $\frac{277}{1000}$, the decimal form is .277. In this the *value* of the denominator is thousandths, and the *third* place to the right of the decimal point indicates thousandths.

In the same manner, $\frac{3}{10}$, $\frac{28}{100}$, $\frac{36}{1000}$, $\frac{3}{10000}$, and $\frac{271}{100000}$ when changed to decimal form become, .3, .28, .036, .0003, and .00271.

By examining the above it may be seen that there is a direct relation between the number of figures *after* the decimal point and the number of zeros in the denominator of the fraction.

The denominator *tenths*, having *one* zero, indicates that there is but *one* figure after the decimal point. *Hundredths*, having *two* zeros, indicates that there are *two* figures after the decimal point. *Thousandths*, having *three* zeros, indicates that there are *three* figures after the decimal point, and so on.

That is, there are as many places following the decimal point as there would be zeros in the denominator of the fraction, if it were written. As further illustrations:

Twenty-five hundredths written as a decimal becomes, .25.

One hundred forty thousandths written as a decimal becomes, .140.

One thousand sixty-five ten-thousandths inches written as a decimal becomes, .1065 in., or, as sometimes written, 0.1065 in.

The practice of placing a zero before the decimal point in such decimal measurements is sometimes followed to indicate directly that *no* whole number precedes the decimal. This is of special value in writing dimensions for machine work and toolmaking, where extra precautions are observed in such matters.

When there are *not* as many figures in the numerator as there would be zeros in the denominator, if written, the decimal equivalent is expressed by placing *in front of* the figures representing the numerator, enough zeros to make up this difference. The decimal point is then placed directly in front of the last zero.

For example, in the number thirty-six thousandths, there is one more zero in the denominator of the fraction, $\frac{36}{1000}$, than there are figures in the numerator. This is expressed as a decimal by writing the 36 as an ordinary number, and then placing the one zero in front of it. The decimal becomes, .036.

In the same manner, three thousandths becomes, .003; forty-one ten-thousandths becomes, .0041; and so on.

From the foregoing, it is seen that the *first place* to the right of the decimal point indicates tenths; the *second place* indicates *hundredths;* the *third place* indicates *thousandths;* the *fourth place, ten thousandths;* the *fifth, hundred thousandths;* and so on.

The following chart gives the relative positions of the various places with regard to the decimal point. Numbers to the *left* of the decimal point are *whole* numbers, while numbers to the *right* are *decimals.*

7 — Millions	6 — Hundreds of thousands	5 — Tens of thousands	4 — Thousands	3 — Hundreds	2 — Tens	1 — Units	.	1 — Tenths	2 — Hundredths	3 — Thousandths	4 — Tens of thousandths	5 — Hundreds of thousandths	6 — Millionths
		Whole number					*and*			Decimal			

Referring to this chart, decimals similar to those below are readily interpreted.

.04 is read "four hundredths."
.004 is read "four thousandths."
.4 is read "four tenths."
.251 is read "two hundred fifty-one thousandths."
200.0162 is read "two hundred and one hundred sixty-two ten-thousandths."
36.65 is read "thirty-six and sixty-five hundredths."
1.250 in. is read "one and two hundred fifty thousandths inches."
.0125 in. is read "one hundred twenty-five ten-thousandths inches."

Where a whole number is combined with a decimal, as in the number 36.65, the number to the left of the decimal point is read as an ordinary whole number, "thirty-six," but the word "*and*" is *always* used to indicate the *decimal* point between the whole number and the decimal. The part following the decimal point is read as already explained.

The number 8.065 is read in the same manner, as "eight *and* sixty-five thousandths," there being a slight pause after the word *and*.

From the above decimals, it may be readily appreciated how important is the position of the decimal point.

This might be illustrated by the numbers, .4, .04, and .004.

Although the one figure 4 is used, the *value* of the decimal *changes* according to the relation of that figure with the decimal point. Moving the decimal point one place to the *left*, *divides* the value of the number by 10, while moving it one place to the *right*, *multiplies* the value of the number by 10.

This important relation is further illustrated in the following arrangement of the number 2.041. Each succeeding time it is written, the decimal point is moved one place toward the right.

2.041
20.41 This is ten times the value of the number above it.
204.1 This is ten times the value of the number above it.
2041. This is ten times the value of the number above it.

An important fact about decimals is that the *value of the decimal does not change when zeros are added after it.* For example:

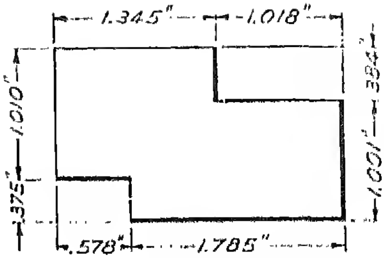
0.25, 0.250, 0.2500 are all of the *same* value because by factoring the numerators and the denominators of their fractional forms: $\frac{2500}{10000} = \frac{250}{1000} = \frac{25}{100}$.

In reading decimals, or *calling off* numbers that have a decimal point, it is sometimes the practice to omit naming the decimal place and to refer to it by the one word *point*. For example, the decimals 18.75 and .251 might be read as one eight point seven five, and point two five one. It is recommended however, that before the student follows this practice, he become entirely familiar with the more common method previously explained.

Problems Involving the Expression of Decimals

1. Express in figures:

   Four hundred and eight thousandths.
   Four hundred eight thousandths.
   Four hundred eight and eight hundredths.
   One thousand and fourteen ten-thousandths.
   One thousand fourteen ten-thousandths.
   One thousand fourteen and one thousandth.
   One and fourteen thousandths.



2. In the drawing of a template to the right, write out the value of the given dimensions.

3. An order from the drafting room to the machine shop calls for a round copper disk two and two hundredths inches in diameter, with a round hole eight hundred seventy-five thousandths inches in diameter through its center. Make a sketch of such a disk placing on the dimensions indicated.

4. In checking over a drawing, it is found that a dimension which should have read fifteen thousandths inches, is written 0.15". How should this have been written?

5. Read aloud the following:

"A cubic foot of water weighs 62.425 pounds."

"The circumference of a circle is always equal to 3.1416 times the length of the diameter."

"Cast iron weighs 0.26 lb. per cubic inch."

6. Ice is ninety-two hundredths times as heavy as water. Express this amount in figures.

7. Cork is 0.24 times as heavy as water. Express this decimal in words.

8. Express as decimals the following:

34/1000; 4 25/10,000; 625/1000; 41/100; 3/100; 7/10; 5 8/10; 18/1000.

9. Write in figures the following:

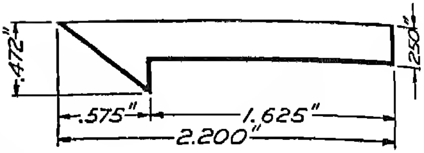
a) Two hundred and seventy-three thousandths.

b) Two hundred seventy-three thousandths.

c) Seven thousand and six ten-thousandths.

d) Seven thousand six ten-thousandths.

10. Write in words the dimensions that are noted on the drawing to the right.*



ADDITION OF DECIMALS

In adding decimals the same method is followed as in adding whole numbers. The numbers to be added should be so arranged that like terms are in the same vertical column. As a result of this, the decimal points of these numbers will lie in a vertical line, and tenths will lie under tenths, hundredths under hundredths, thousandths under thousandths, and so on.

How this is done is illustrated in the following problem.

Example 1:

Find the sum of 5.072, 0.64, 0.0023, and 15.1.

Solution and Explanation:

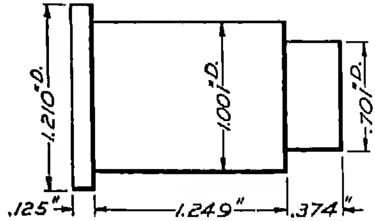
As explained above these numbers should be so written that the decimal points and like places lie in vertical lines.

$$\begin{array}{rl} & 5.072 \\ & 0.64 \\ & 0.0023 \\ & \underline{15.1} \\ \text{Sum} = & 20.8143 \end{array}$$

That is, the sum of 5.072, 0.64, 0.0023, and 15.1 equals 20.8143.

Example 2:

The drawing to the right represents a steel pin. The dimension giving the total length of this pin has been omitted from this drawing. Redraw the pin placing in position all dimensions including the total length.



*Answers to these problems will be found on page 85.

Solution and Explanation:

The total length of this pin is equal to the sum of the three dimensions: 0.125 in., 1.249 in., and 0.374 in., which indicate the lengths of different portions making up the *total* length.

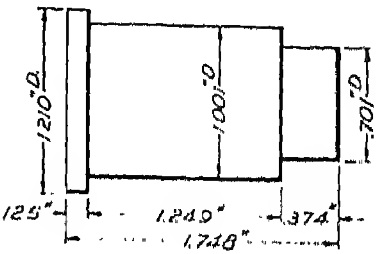
Their sum is found by arranging the decimals as previously explained and adding them as shown below.

$$\begin{array}{rr} & 0.125 \\ & 1.249 \\ & \underline{0.374} \\ \text{Sum} = & 1.748 \end{array}$$

That is, the total length of this pin is 1.748 in.

The redrawn pin is shown below with all dimensions including the total length.

It is customary to place the dimension indicating the total length in a position near the detail measurements that make up this total length. Accordingly in the redrawn pin the total length dimension is placed just below the detail dimensions as shown.



The abbreviations "D." indicate that this pin is cylindrical, the numbers giving the diameters of the respective portions.

Problems Involving Addition of Decimals

1. Find the sum of each of the following:

   a) 7.2; 0.076; 1.09; 3.

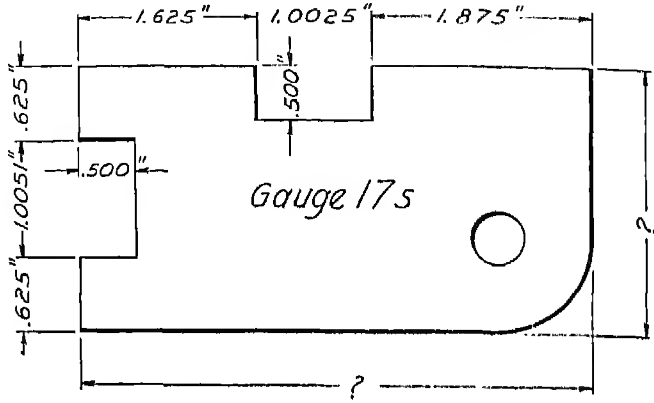
   b) 9.4329; 0.06; 4.0025.

   c) 6; 3; 5.0002; 11.8; 0.4.

2. Determine the sum of:

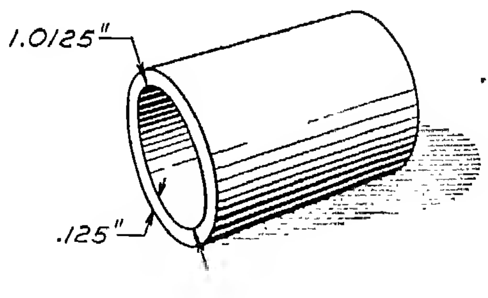
Two and seventy-three ten-thousandths; four and one hundred five thousandths; sixty-three hundredths; fifty-two thousandths; eight; fourteen; seven and two hundredths; sixteen ten-thousandths.

3. Redraw the measuring gauge illustrated in the following sketch, writing in the positions indicated by the question marks, the dimensions that give the total width and total length.

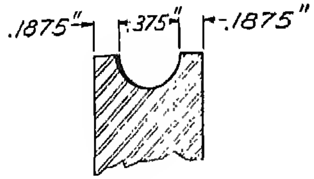


4. In grinding a steel plate, the grinding wheel reduces the thickness of the plate 0.015 in. during the first minute of grinding, 0.0215 in. the second minute, and 0.0195 in. the third minute of grinding. What is the total amount removed in these three minutes?

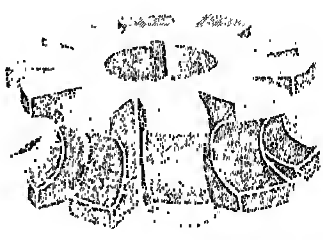
5. The drawing of the special steel tube illustrated does not have the outside diameter given. From the sketch shown, determine what this diameter should be.



6. The milling-machine cutter in the following sketch has its dimensions indicated in a separate detail drawing. From these dimensions determine the total width of this cutter.

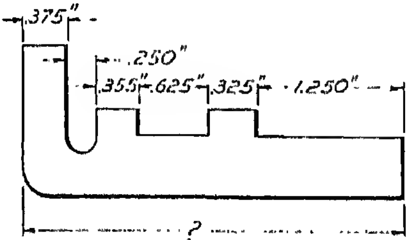


Section showing tooth outline

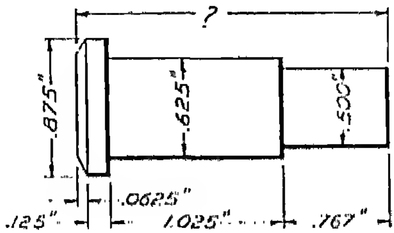


7. The inside diameter of a special brass tube is 0.6875 in. The thickness of the metal is 0.049 in. What is the outside diameter?

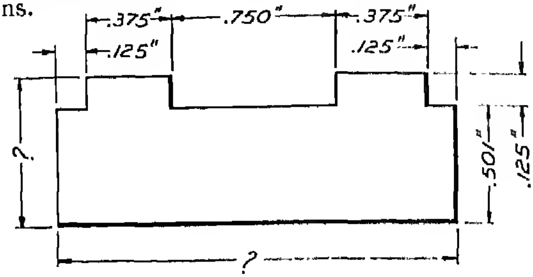
8. Determine the total length of the template illustrated below.



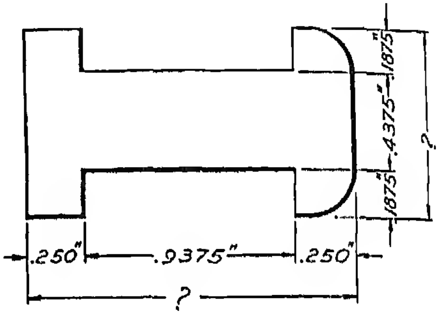
9. From the following drawing of a crankpin, calculate its total length. Make a sketch of this pin, placing in position all dimensions, including the one giving the total length.



10. What is the total length and the total width of the steel punching shown below? Redraw the piece putting in all dimensions.

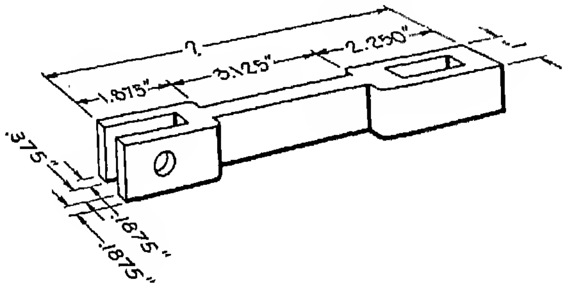


11. Two important dimensions as noted by the question marks are missing from the drawing to the right. What are they?

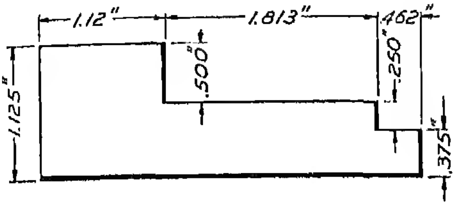


12. Three strips of sheet brass each measuring in thickness 0.032 in., 0.035 in., 0.025 in. are bound together in one piece by placing the strips on top of each other and clamping them together. What is the combined thickness of these strips when so bound together?

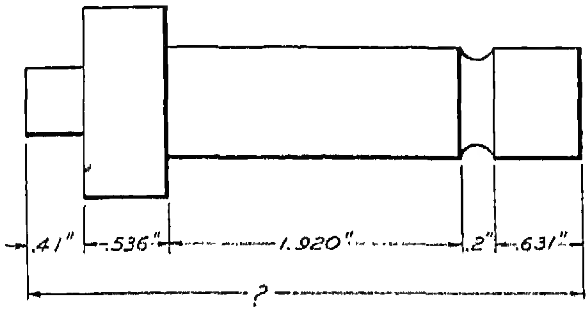
13. What is the total length of the link illustrated below? What is the total width of the open end?



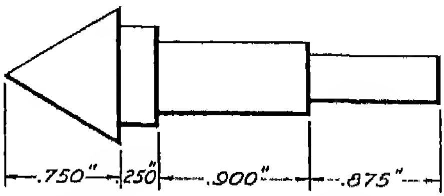
14. Redraw the piece in the following drawing placing on it all the dimensions, including the dimension that gives the total length.



15. Supply the missing dimension as noted by the question mark on the drawing below.



16. The young man who submitted the following drawing to his foreman neglected to place on it the over-all dimension. What should it be?*



*Answers to these problems will be found on page 85.

SUBTRACTION OF DECIMALS

The same method is used in subtracting decimals as is followed in subtracting whole numbers. That is, the numbers are arranged according to their denominations. This results in bringing the decimal points in the same vertical line, with units under units, tenths under tenths, hundredths under hundredths, thousandths under thousandths, and so on.

The following example illustrates how this rule is applied.

Example 1:

Subtract 2.0018 from 4.1372.

Solution and Explanation:

As shown below the decimal points lie in the same vertical line thus lining up the numbers as explained.

$$\begin{array}{r} 4.1372 \\ \underline{2.0016} \\ 2.1356 \end{array} = \text{difference.}$$

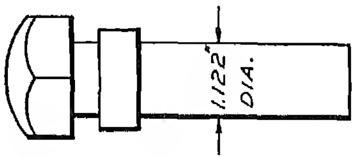
That is, 2.0016 subtracted from 4.1372 equals 2.1356.

Should the larger decimal not extend to as many decimal places as the smaller decimal, then zeros must be added *after* the larger decimal to bring it out as far as the other decimal, so that the subtraction may take place. As previously explained such addition of zeros after a decimal number does not change the value of that decimal.

This is illustrated in the following practical problem.

Example 2:

The pin to the right is too large to fit into the hole for which it was designed. If the hole measures 1.0375 in. in diameter how much must the diameter of the pin be reduced in order that it may be used as planned?



Solution and Explanation:

As shown, the diameter of the pin measures 1.122 in. The hole into which it is supposed to fit measures 1.0375 in. in diameter. From these measurements it is seen that the pin is *too* large to fit into the hole.

The diameter of this pin must therefore be reduced by an amount which is equal to the difference between 1.122 in. and 1.0375 in. This works out as follows:

$$\begin{array}{r} 1.1220 \\ \underline{1.0375} \\ 0.0845 \end{array} = \text{difference.}$$

That is, the diameter of 1.122 in. must be reduced by 0.0845 in.

In this subtraction it will be noted that it was necessary to add one zero after 1.122 in order to bring it out far enough to perform the necessary subtraction. This addition of the zero did not change the value of the original number 1.122 in.

It frequently occurs that a decimal is to be subtracted from a whole number. In such cases enough zeros are placed after the decimal point, which follows the whole number, to bring the decimal places out far enough for the required subtraction. This done, the subtraction takes place as in the following example.

Example 3:

Subtract 0.7854 from 5.

Solution and Explanation:

In order to carry out this subtraction four zeros are added after the decimal point, which follows the number 5.

The subtraction then takes place in the usual manner.

$$\begin{array}{r} 5.0000 \\ \underline{0.7854} \\ 4.2146 \end{array} = \text{difference.}$$

Therefore 0.7854 subtracted from 5 equals 4.2146.

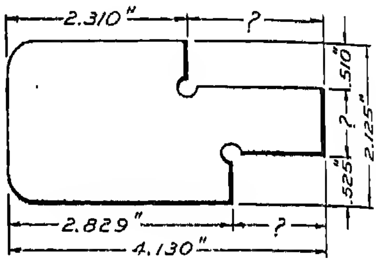
Problems Involving Subtraction of Decimals

1. Solve the following:

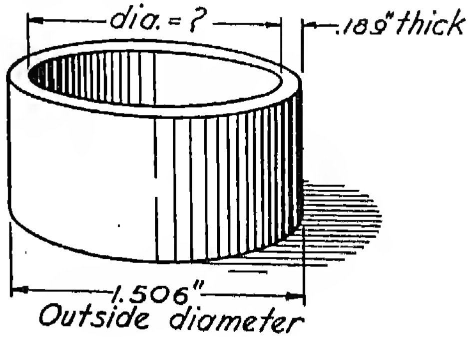
   a) $9.842 - 6.35 = ?$
   b) $4.125 - .046 = ?$
   c) $2.106 + 1.304 - 1.5 = ?$
   d) $.52 + 6.195 - 4.1 - 1.31 = ?$

2. In reply to an inquiry, a price of 8.45 cents per pound is quoted on a certain grade of steel. The following day it is quoted at 8.72 cents per pound. What is the change in the price per pound?

3. The drawing to the right was given to a toolmaker to be used in making a special gauge for measuring parts of a machine that is to be made in a small manufacturing plant. The foreman finds that three important measurements are missing. Draw a new sketch of this gauge placing on it all measurements.

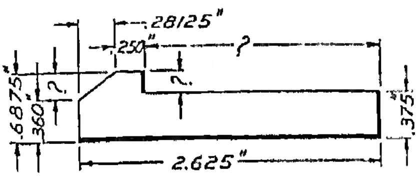


4. Find the inside diameter of the piece of tubing in the following drawing.

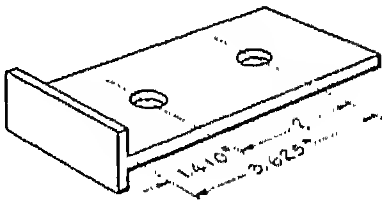


5. To reduce a measuring gauge to its proper thickness, 0.0025 in. must be ground off one flat side. Before this grinding takes place, however, the gauge measures 0.375 in. thick. What should be the measurement when the above grinding is done?

6. The drawing of a special machine key as shown below, was returned to the drafting room with question marks regarding three missing dimensions. Redraw this sketch putting in all dimensions.

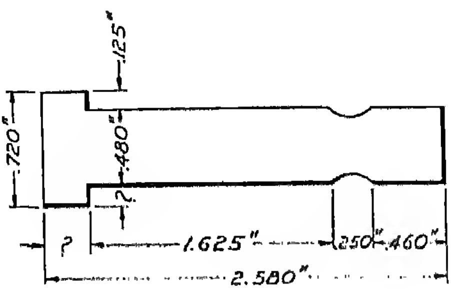


7. Determine the distance between the centers of the holes in the drill jig illustrated to the right.



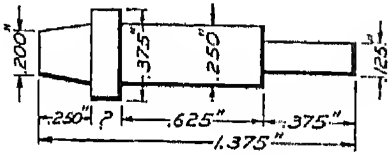
8. A steel bar $1\frac{1}{8}$ in. in diameter and 1 ft. long weighs 3.379 lb. The same size aluminum bar weighs only 1.15 lb. What is the difference in weight between these two bars?

9. Redraw the following sketch and put in all dimensions.



10. After milling off 0.126 in. from each of the two flat sides of a circular plate which measured 1.370 in. thick before the operation, what is the resulting thickness?

11. The draftsman who made the following drawing omitted an important dimension. Redraw the piece and place in position all of the dimensions.



12. In order to use a piston pin which measures one and twenty-one thousandths inches in diameter, the mechanic must reduce the diameter to nine hundred eighty-five thousandths inches. In performing this task how much would the present diameter be reduced?

13. A liquid quart, such as is used in measuring milk, contains 57.75 cu. in. A dry quart, used in measuring vegetables, contains 67.2 cu. in. How many cubic inches difference between these two measures?

14. A copper rod 1 in. in diameter and 3 ft. long weighs 9.09 lb., while a brass rod of the same dimensions weighs 8.55 lb. What is the difference in the weight of these two rods?

15. The actual outside diameter of a certain piece of wrought-iron pipe is 2.875 in. The thickness of the metal is .204 in. Find the inside diameter.

16. The actual diameter of a given shaft is 3.2478 in. The measurement of the inside diameter of the bearing in which this shaft runs is 3.249 in. Find the clearance.*

*Answers to these problems will be found on page 85.

ANSWERS TO PROBLEMS

Pages 72 to 74.

1. 400.008; 0.408; 408.08; 1000.0014; 0.1014; 1014.001; 1.014.
2. One and three hundred forty-five thousandths inches; one and eighteen thousandths inches; three hundred eighty-four thousandths inches; one and one thousandth inches; one and seven hundred eighty-five thousandths inches; five hundred seventy-eight thousandths inches; three hundred seventy-five thousandths inches; one and ten thousandths inches.
3. 0.875 in.; 2.02 in.
4. 0.015 in.
5. "A cubic foot of water weighs sixty-two and four hundred twenty-five thousandths pounds"; "The circumference of a circle is always equal to three and one thousand four hundred sixteen ten-thousandths times the length of the diameter"; "Cast iron weighs twenty-six hundredths pounds per cubic inch."
6. .92.
7. Twenty-four hundredths.
8. 0.034; 4.0025; 0.625; 0.41; 0.03; 7; 5.8; 0.018.
9. (*a*) 200.073; (*b*) 0.273; (*c*) 7000.0006; (*d*) 0.7008.
10. Two and two hundred thousandths; two hundred fifty thousandths; one and six hundred twenty-five thousandths; five hundred seventy-five thousandths; four hundred seventy-two thousandths.

Pages 75 to 79.

1. (*a*) 11.366; (*b*) 13.4954; (*c*) 26.2002.
2. 35.8159.
3. 4.5025 in.; 2.2551 in.
4. 0.056 in.
5. 1.2625 in.
6. 0.75 in.
7. 0.7855 in.
8. 3.180 in.
9. 1.9795 in.
10. 1.750 in.; 0.626 in.
11. 1.4375 in.; 0.8125 in.
12. 0.092 in.
13. 7.250 in.; 0.75 in.
14. 3.396 in.
15. 3.697 in.
16. 2.775 in.

Pages 82 to 84.

1. (*a*) 3.492; (*b*) 4.079; (*c*) 1.91; (*d*) 1.305.
2. 0.27 cents.
3. 1.820 in.; 1.301 in.; 1.090 in.
4. 1.128 in.
5. 0.3725 in.
6. 2.09375 in.; 0.3125 in.; 0.3275 in.
7. 2.215 in.
8. 2.229 lb.
9. 0.245 in.; 0.115 in.
10. 1.118 in.
11. 0.125 in.
12. 0.036 in.
13. 9.45.
14. 0.54 lb.
15. 2.467 in.
16. .0012 in.

Review Problems in Addition and Subtraction of Decimals

1. Write as decimals the following: Ten and ten-thousandths; four and seventy-two ten-thousandths; seven hundredths inches; eight hundred forty-two hundred-thousandths; two and nine tenths.

2. Add the following: Seventy-two thousandths; four and sixteen hundredths; ninety and ninety-one hundredths; forty-six and two ten-thousandths; and thirty-seven.

3. From eighteen and six ten-thousandths, subtract four and thirteen hundredths. From seventy-two, subtract ninety-four thousandths.

4. A cubic inch of steel weighs 0.282 lb. A cubic inch of aluminum weighs 0.0924 lb. What is the difference in the weights of these metals per cubic inch?

5. In trying to fit a gear on a spindle, an auto mechanic finds that the hole in the gear measures 0.738 in. in diameter while the spindle diameter measures 0.752 in. How much will the diameter of the spindle have to be reduced in order to just fit into the hole in the gear?

6. What amount added to 8.032 in. will equal 11.7 in.?

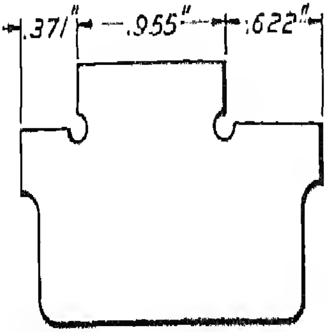
7. The thickness of a special ground-steel tubing measures 0.113 in. while the inside diameter measures 0.811 in. What is the outside diameter of this tubing?

8. Four gauge blocks used by a toolmaker for accurate measuring are 0.285 in.; 0.750 in.; 0.1875 in.; and 0.365 in. respectively. What is their combined thickness?

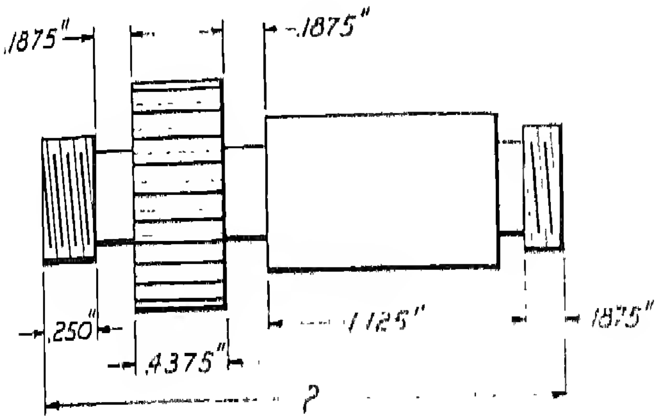
9. A notation on the drawing of a steel block that measures 1.520 in. thick states that "this thickness is to be reduced 0.272 in." What should be the thickness of the block after this operation?

10. A piece of flat steel, measuring 1.016 in. thick, is placed on a surface grinder in order to reduce the thickness to .987 in. During this operation how much is the thickness reduced?

11. Determine the length of the gauge in the following drawing.



12. The draftsman overlooked giving the total length of the gear spindle in the following drawing. What should it be?



MULTIPLICATION OF DECIMALS

In multiplying decimals, multiply as in whole numbers, forgetting about the decimal point until the multiplication is completed.

Locating the decimal point, or "pointing off," as it more often is called, is then done by counting from the right-hand figure in the product as many places as there are decimal places in both the multiplier and the multiplicand.

The decimal point is then placed directly in front of the last figure so counted off.

The following example shows how this rule is applied.

Example 1: Multiply 2.2 by 0.061.

Solution and Explanation:

```
    2.2
   0.061
   -----
    2 2
  132
  -----
 .134 2   = product
```

Since there are 3 decimal places in the multiplier, and 1 place in the multiplicand, their sum is 4. Therefore, the decimal point is placed in front of the fourth figure, counting from the right end of the product. That is, in front of the figure 1.

Sometimes it so happens that the sum of the decimal places in both terms to be multiplied is greater than the number of figures in the resulting product. In such cases, enough zeros must be placed *in front of* the product to make up the number of decimal places needed. This is seen in the following example.

Example 2: Determine the product of 5.12 and .0011.

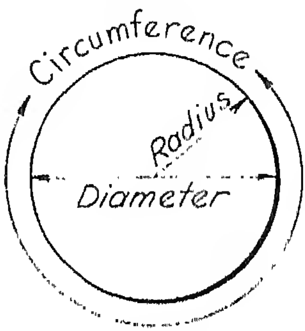
Solution and Explanation:

```
     5.12
     .0011
     -----
     5 12
    51 2
  -------
  .0056 32   = product
```

In this case the sum of the decimal places in the multiplier and the multiplicand is 6. The product, however, has only the four figures: 5, 6, 3, 2. *Two zeros*, therefore, must be placed in front of the figure 5 in order to give the product the *required* number of places. The decimal point is then placed in front of the sixth figure from the right end, and the result of the multiplication reads .005632 as shown.

A common shop application of the above processes is found in calculations relating to circles.

Referring to the drawing below, the *circumference* of a circle is the distance *around* the circle.



The *radius* is the distance from the center of the circle to the circumference.

The *diameter*, as seen from previous problems, is the distance across the circle *through* the center. It is equal to *twice* the length of the radius.

The circumference of a circle is always equal to 3.1416 times the length of the diameter.

How these terms are used is shown in the following problem.

Example 3:

Calculate the length of the circumference of a circular steel disk that has a radius of 1.475 in.

Solution and Explanation:

Since the circumference of a circle is equal to 3.1416 multiplied by the diameter, the first step in this problem is to determine the length of the diameter.

As previously explained the diameter equals twice the length of the radius.

In this problem the diameter equals 2×1.475 or 2.95 in.

The circumference of a circle having a diameter of 2.95 in. is, 3.1416×2.95, or 9.26772 in.

That is, the circumference of a steel disk with a radius of 1.475 in. is 9.26772 in.

Problems Involving the Multiplication of Decimals

1. $2.5 \times .7854 = ?$ $3.1416 \times 4.26 = ?$

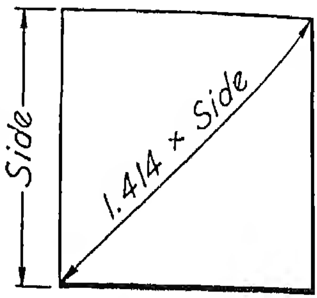
   Multiply 4.125 by 1.51 by 0.22.

   0.306 multiplied by 0.071 equals what?

   $44.002 \times 21.01 = ?$ $1002.5 \times 0.65 = ?$

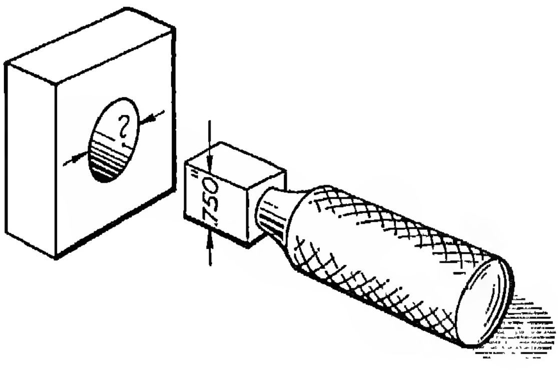
2. Gasoline is listed as weighing 0.90 times that of water. At this rate what is the weight of 15 gal. of gasoline if one gallon of water weighs 8.336 lb.?

3. The distance across the corners of a square, as shown in the drawing to the right, is always equal to 1.414 times the length of one side of the square.

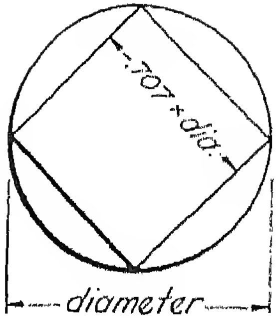


According to this rule, what is the diameter of the circle that will just touch the corners of a 2.1 in. square?

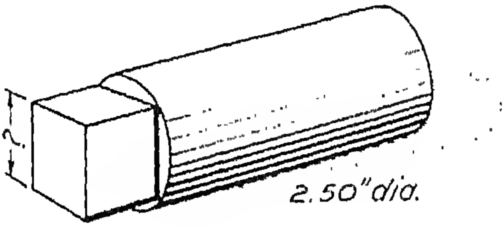
4. How large in diameter must the hole be drilled to just receive a square plug gauge which measures 0.75 in. on the edge as shown in the following drawing?



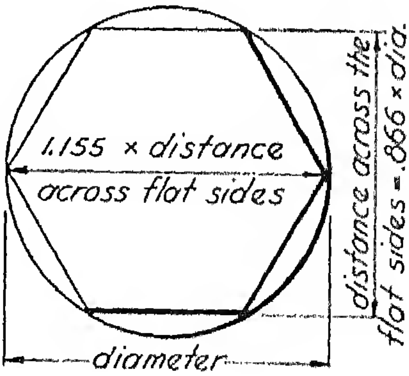
5. The largest square that can be drawn inside of a circle has a side whose length equals .707 times the diameter of that circle. This is shown in the illustration to the right.



According to this rule, what is the largest square that can be machined on the end of a shaft which measures 2.500 in. in diameter? How this might look is illustrated below, the size of the square being carried to the third decimal place only.



6. The distance across the corners of a hexagon equals 1.155 times the distance across the flat sides. This is also equal to the diameter of the circle which passes through the corners of the hexagon.

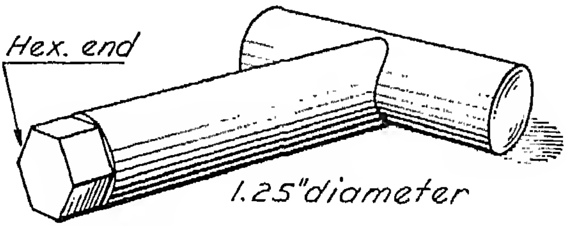


Using this rule, determine the distance across the corners of a hexagonal-shaped steel bar which is 1.62 in. across the flat sides.

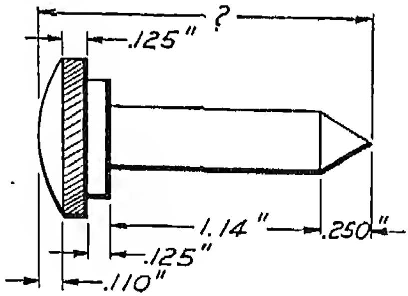
7. A plug wrench resembling that in the sketch on page 92 is to be made from a piece of round stock which measures

1.25 in. in diameter. The size of this hexagon is limited by the diameter as noted in the previous drawing.

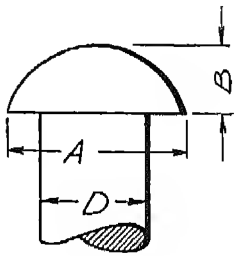
What is the distance between the flat sides of the largest hexagon that can be machined on the end of this stock?



8. How much stock is needed to make 20 special pins like the following. These are made on a machine that allows no waste in the length of the material.



9. Using the following rule for round-headed rivet proportions, determine the values of dimensions A and B in rivets that have a diameter of .375 in. and .500 in.



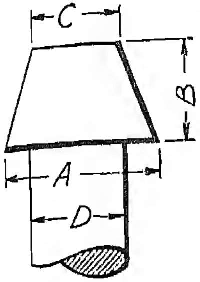
D = diameter of rivet

A = diameter of rivet head

B = thickness of rivet head

$A = 1.75 \times D$

$B = 0.75 \times D$



10. Determine the proportions of a .500-in. rivet, using the following rules for a rivet with a cone-shaped head.

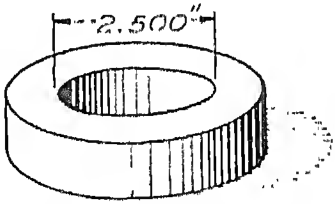
D = diameter of rivet
A = bottom diameter of head
C = top diameter of head
B = thickness of head

$A = 1.75 \times D$
$B = 0.9375 \times D$
$C = 0.875 \times D$

11. Determine the length of the side of the largest square that can be cut from a circular piece of galvanized iron that measures 5 in. in diameter.

12. What should be the inside diameter of a brass pipe in order to have it just fit over the end of a hexagonal bar, measuring 2 in. across the flats?

13. What are the dimensions of a square block of metal that will just fit into the hole in the steel ring shown to the right? Make a drawing of a 2.500-in. circle and lay out such a square.



14. If a cubic foot of water weighs 62.425 lb., what is the weight of water in a tank containing 120 cu. ft. of water?

15. A quart of water weighs 2.08 lb. What is the weight of water that will fill a 12-qt. pail?

16. A wooden flagpole measuring 7 in. in diameter is to be reinforced by fastening an iron band around its circumference. Allowing 2 in. for lapping the ends of this band, how long should the strip of iron be?

17. Lead is 11.37 times heavier than water. What is the weight of a cubic foot of lead when a cubic foot of water weighs 62.425 lb.? The answer to this should be carried to the third decimal place.

18. A sheet-metal apprentice has the job of making an 8-in. smoke pipe for a furnace. This pipe is to be made of galvanized sheet iron and 1 in. is to be allowed for the lap joint. What should be the length of this material before it is bent into the 8-in. circle? Use one decimal place in the answer.

19. Calculate the weight of the following shipment of material:

12 pcs. $\frac{3}{4}$-in. round steel $14\frac{1}{2}$ ft. long
5 pcs. $\frac{1}{2}$-in. square steel 9 ft. long
2 pcs. $\frac{1}{4}$-in. round brass $8\frac{3}{4}$ ft. long

The $\frac{3}{4}$-in. round steel lists at 1.502 lb. per foot. The $\frac{1}{2}$-in. square steel lists at .85 lb. per foot. The $\frac{1}{4}$-in. round brass lists at .175 lb. per foot. The answer to the above problem should be carried to the third decimal place.

20. A cubic foot is listed as containing 7.48 gal. On this basis calculate the number of gallons in a tank that contains $12\frac{1}{2}$ cu. ft.

21. What is the weight of a bar of 1-in. square iron that measures $6\frac{3}{4}$ ft. long when this stock weighs 3.33 lb. per foot?

22. Flat brass stock that measures 1 ft. wide is listed as weighing .34 lb. per foot of length. At this rate what is the weight of a roll that contains 55 ft. of this stock?

23. After slitting 3 pieces each 1.1875 in. wide from a metal strip 6 in. wide, what is the width of the piece remaining?*

DIVISION OF DECIMALS

In dividing decimals, divide the same as in whole numbers, forgetting about the decimal point until the operation of division is completed.

To properly locate the decimal point, count or point off from the right-hand figure in the quotient, as many places as the number decimal places in the dividend exceed those in the divisor.

*Answers to these problems will be found on page 106.

Example 1:

Divide 92.307 by 8.7.

Solution and Explanation:

```
8.7)92.307(10.61
    87
    ----
     5 30
     5 22
     ----
        87
        87
        --
```

Dividing these as ordinary whole numbers the result obtained is 1061. It is seen that there are 2 more decimal places in the dividend than there are in the divisor. Consequently two places are pointed off from the right of the last figure in quotient. This gives 10.61 as the correct quotient.

In cases where there are not as many decimal places in the dividend as in the divisor, add zeros after the last figure in the dividend *until* the decimal places in the dividend *equal* those in the divisor. Then proceed with the division as above explained.

Should there be still not enough figures in the dividend to begin the division, add more zeros. Such additional zeros after the last figure in the decimal, as has been explained, does not alter the value of the decimal.

Example 2:

Divide 3.14 by 5.024.

Solution and Explanation:

```
5.024)3.140000(.625
       3 0144
       ------
        12560
        10048
        -----
         25120
         25120
         -----
```

The first step in this division is to add one zero after the figure 4 in order that there may be as many places in the dividend as in the divisor. But even with this addition of one zero there are still not enough figures in the dividend to enable the division to be started. Accordingly more zeros are added as shown.

After the division has been completed it is found that there are six places in the dividend and three in the divisor. That is, there must be three places pointed off in the quotient, counting from the right. This locates the decimal point in front of the figure 6 as shown.

As a result, 3.14 divided by 5.024 equals .625.

Should the divisor be a whole number, the division is carried out in the same manner as above, care being given to the proper location of the decimal point. Such division is illustrated in the following example.

Example 3:

Divide 3.24 by 90.

Solution and Explanation:

```
90)3.240(.036
   2 70
   ----
    540
    540
    ---
```

Since there are three places in the final dividend and none in the divisor, then there are three places to be pointed off in the quotient. To provide for these three places, one zero must be inserted in front of the figure 3. The decimal point is then placed in front of this zero, giving the final quotient—.036 as the result of the division.

The correctness of divisions of this kind may readily be proven by multiplying the quotient by the divisor. The product obtained by this multiplication should equal the dividend.

In the problems so far, the division has come out evenly. Frequently the division does not work out this way, and the

student may be at a loss to know just where to stop the process and still have a result that is sufficiently accurate.

The number of decimal places to which a quotient should be carried is largely determined by the trade practice to which the problem relates. For example, it would be difficult to work to a thousandth part of an inch on woodworking jobs. On the other hand it is common practice to work to the thousandth and to even the ten-thousandth part of an inch in the machine and toolmaking trades.

Sometimes it is stated definitely in the problem how many places are required in the quotient. In any case, the farther the division is carried the more accurate will be the result.

However, in the problems that follow it will be sufficiently accurate to continue the division until there are *three* decimal places in the quotient, unless otherwise stated.

How such division affects the accuracy of the result may be more clearly understood by a discussion of the following.

Example 4:

Divide 1.5 by 8.724, carrying the quotient to 4 decimal places.

Solution and Explanation:

```
8.724)1.5000000(.1719
       8724
       ─────
       62760
       61068
       ─────
        16920
         8724
        ─────
         81960
         78516
         ─────
          3444
```

When a remainder occurs after the last division it is the general practice to increase the last figure in the quotient by 1 if that remainder is equal to one half, or more than one half, of the divisor. This slight increase brings the result nearer the exact quotient.

If the remainder is less than one half of the divisor, the original quotient is not changed.

The application of this rule may be further illustrated by assuming that the above division were to be carried to only the *third* decimal place.

After the third division has been completed it is noted that the remainder is 8196. As this is greater than one half of the divisor, the figure in the third decimal place of the quotient would be increased by 1. The original quotient .171 would then become changed to .172 which as may be seen is very nearly equal to the quotient that was carried to the fourth decimal place.

If only two decimal places were required in the quotient, the answer would be .17. This would be the correct answer to two places as the remainder after the second division is less than one half of the divisor.

How this practice applies to the division of whole numbers is illustrated in the following problem where the quotient is to be carried beyond the usual number of places.

Example 5:

Divide 215 by 46, having the quotient correct to 4 decimal places.

Solution and Explanation:

```
46)215.0000(4.6739
   184
   ----
    31 0
    27 6
    ----
     3 40
     3 22
     ----
       180
       138
       ---
        420
        414
        ---
          8
```

In beginning this division a decimal point is placed after figure 5 in the dividend and enough zeros are added to the dividend to give the four decimal places required in the quotient.

After carrying out the division as requested there results 4.6739 as the quotient. As the final remainder, 8, is less than one half of the divisor, 46, the last place in the quotient remains in its original form.

The quotient obtained in dividing 215 by 46 therefore is 4.6739 when carried to four decimal places.

Problems Involving Division of Decimals

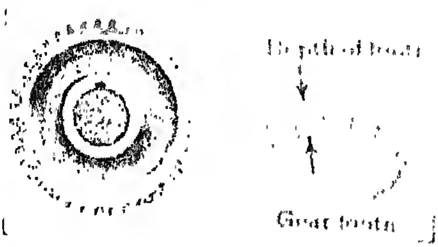
1. Solve the following, giving results correct to the third decimal place:

Divide two tenths by eight and two tenths.

Divide four and ninety-five hundredths by fifteen thousandths.

935.06 ÷ 2.25; .064 ÷ 1.28; 3.3 ÷ .005.

2. A gallon of machine oil weighs 7.56 lb. Allowing 16 lb. for the weight of the can in which it is kept, how many gallons would there be in this can if partly filled with such oil and weighing 128.5 lb.? Give the answer correct to 2 decimal places.



3. In spur gears such as that shown, the depth of the cut which forms the teeth of the gear is equal to 2.157 divided by the pitch of the gear. This depth is expressed in inches and is usually carried out to the fourth decimal place because of the accuracy required in constructing gears.

According to the previous rule, what is the depth of the teeth in a set of lathe gears which have a pitch of 16? What would be the depth if the pitch were 18?

4. What is the depth of the teeth on a special gear of 12 pitch as used on a milling-machine attachment?

5. No. 10 copper wire measures 31.82 ft. to the pound. How many pounds of wire are there in a coil containing 125 ft. of such wire? The answer to this problem is to be correct to two decimal places.

6. A box of $\frac{1}{4}$-in. bolts weighs 89.5 lb. The box alone weighs 4 lb. To determine how many bolts there are in the box, the boy who is assigned to the job of counting them finds that 5 of them weigh 0.3 lb. From this weight he calculates the number of bolts in the box. How many are there?

7. Octagon steel as used in making cold chisels, weighs 1.86 lb. to the foot. Use this figure in determining how many feet there are in an octagon bar weighing 21 lb. and carry the result to 2 decimal places.

8. Calculate the diameter of a water tank that measures 24 ft. in circumference, giving the result correct to 2 decimal places.

9. What is the diameter of the largest circular hoop that can be made from a strip of thin band iron 36 in. long? An allowance of 1 in. is to be made for joining the two ends together. The answer to this is to be correct to two decimal places.

10. A tank partly filled with oil weighs 216 lb. At various times during the week amounts were drawn from the tank so that at the end of the week it weighed 134 lb. If this oil is listed as weighing 7.75 lb. per gallon, how many gallons were used during that time? Give the quotient correct to 2 decimal places.

11. Brass rod 1 in. in diameter weighs 2.85 lb. per foot of length. From this calculate the length in feet of a shipment

of this stock weighing 128.25 lb. The quotient is to be carried to only one decimal place.

12. Steel that measures $\frac{1}{4}$ in. thick and $1\frac{1}{2}$ in. wide is listed as weighing 1.28 lb. per foot of length. Two brackets made from this stock weigh 32 lb. How many feet of material are there in these brackets?

13. No. 14 copper wire is listed as weighing 12.44 lb. per 1,000 ft. Determine how many feet of this size wire there are in a coil that weighs 20 lb. One decimal place is sufficient in this answer.

14. How many feet of 1-in. brass tubing in a bundle of odd lengths of such tubes that weighs 31.5 lb.? This size tube weighs .70 lb. per foot.

15. Cold-rolled steel one inch in diameter, weighs 2.68 lb., per foot length. At this rate, how many feet are there in a bundle of such bars weighing 201 lb.?

16. A piece of $\frac{3}{8}$-in. seamless brass tubing weighs $3\frac{1}{4}$ lb. If this tubing weighs .36 lb. per foot of length what is the length of this particular piece?*

CHANGING FRACTIONS TO DECIMALS

In certain calculations it sometimes becomes necessary to change fractions to their equivalent decimal forms. This is readily accomplished by dividing the numerator of the fraction by the denominator.

The process is carried out by placing a decimal point after the number which represents the numerator and adding zeros beyond the point. This number is then divided by the denominator. The quotient is pointed off in the usual manner.

How this change is made is illustrated in the following problem.

---

* Answers to these problems will be found on page 106.

Example:

Change $\frac{1}{16}$ in. to its equivalent decimal form.

Solution and Explanation:

$$\begin{array}{r l} 16)1.0000(.0625 \\ \underline{96} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \end{array}$$

That is $\frac{1}{16}$ in. changed to decimal form equals .0625 in.

Should the division not come out evenly when reducing fractions to decimals the same process is followed as explained on page 97.

CHANGING DECIMALS TO FRACTIONS

To change a decimal to its equivalent fractional form write the decimal as the numerator omitting the decimal point. The denominator of this fraction would then be equal to a number having the denomination of the original decimal, whether it be tenths, hundredths, thousandths, or such. The fraction is then reduced to lowest terms.

This is illustrated in the following example.

Example 1:

What is the fractional equivalent of .625 in.?

Solution:

$.625 = \frac{625}{1000}$
Reducing $\frac{625}{1000}$ to lowest terms, it becomes $\frac{5}{8}$.
That is, .625 in. equals $\frac{5}{8}$ in.

If a whole number is included with the decimal, treat only the decimal part as above, and combine the resulting fraction with the whole number. This will give a mixed number as the result of this change.

Example 2:

Reduce 5.375 to fractional form.

Solution and Explanation:

Changing the decimal portion to a fraction and reducing to lowest terms it becomes: $.375 = \frac{375}{1000} = \frac{3}{8}$.

Combining $\frac{3}{8}$ with the whole number 5, there results $5\frac{3}{8}$, as the fractional equivalent of 5.375.

Problems Involving Changing Decimal and Fractional Forms

1. Change to their equivalent fractional form:

   0.625; 1.750 in.; 4.125; 0.09375 in.; 6.03125 in.

2. Change the decimal .375 to a fraction whose denominator is 56.

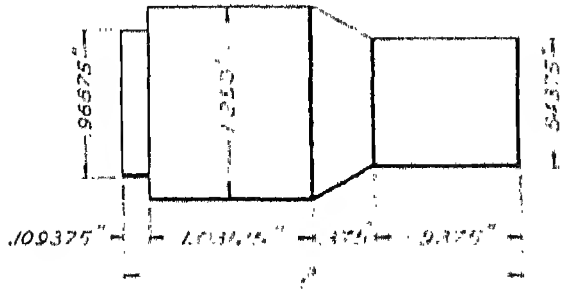
3. How many 32nds of an inch in .875 in.? How many 16ths of an inch in 3.125 in.? How many 64ths of an inch in .9375 in.?

4. Reduce the following to fractional form:

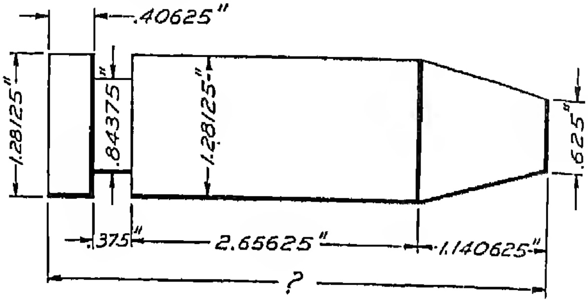
   3.875; 1.03125 in.; 8.450; 12.250 in.; 4.5025; 7.48.

5. Change .125 to 72nds; .875 to 96ths; .750 to 28ths; .8 to 35ths; .43750 to 80ths; .28 to 75ths.

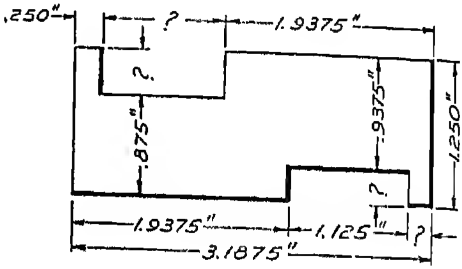
6. Redraw the following changing all dimensions, including the total length, to fractional form.



7. In the following drawing change all decimal dimensions, including the total length, to equivalent fractional form.

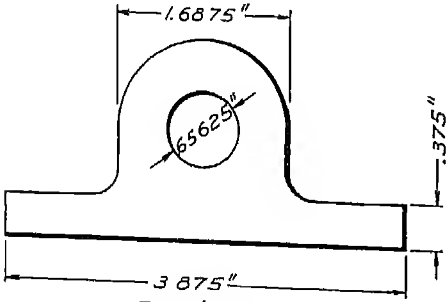


8. Redraw the template below giving all dimensions, including those indicated by question marks, in lowest fractional form.

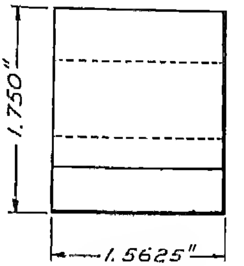


9. A small steel pin has its diameter expressed as 0.234375 in. What is the equivalent of this dimension in fractional form?

10. The following drawing shows a front view and a side view of a bearing. This was sent to a patternmaker so that



Front View



End View

he might build a wooden pattern from which several castings were to be made. The large decimal dimensions somewhat confused the patternmaker and he was obliged to reduce them to fractions. What is the fractional equivalent of these dimensions?

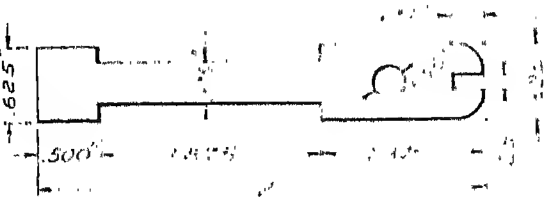
11. Wrought iron $1\frac{1}{4}$ in. in diameter is listed as weighing 4.09 lb. per foot of length. What is the weight of a bar of such iron that measures $12\frac{3}{4}$ ft. long?

12. A boy who is learning his trade in a machine shop is given a piece of stock $\frac{3}{4}$ in. in diameter to turn down to $\frac{5}{8}$ in. in diameter. His first cut across the piece reduces the diameter by 0.08 in. He then hands the piece to his foreman as finished. Is the job done correctly? Why?

13. A special arbor measuring 1.425 in. in diameter is placed in a cylindrical grinder where its diameter is reduced $\frac{1}{32}$ in. What is the diameter of this arbor after the grinding operation?

14. Add the following: $3\frac{1}{2}$; $7\frac{1}{4}$; 2.093; 1.46; 4; 4.0075; $5\frac{1}{4}$.

15. Redraw the following sketch giving all dimensions in their correct fractional form.



16. Change the following decimal dimensions to the nearest 32nd of an inch:

.837; .215; .484 .358; .712

Do not use a decimal equivalent table.*

*Answers to these problems will be found on page 306

ANSWERS TO PROBLEMS

Pages 90 to 94.

1. 1.9635; 13.383216; 1.370325; .021726; 924.48202; 651.625.
2. 112.536.
3. 2.9694 in.
4. 1.0605 in.
5. 1.7675 in.
6. 1.8711 in.
7. 1.0825 in.
8. 35 in.
9. (*a*) 0.65625 in.; (*b*) 0.28125 in. (*a*) 0.875 in.; (*b*) 0.375 in.
10. (*a*) 0.875 in.; (*b*) 0.46875 in.; (*c*) 0.4375 in.
11. 3.535 in.
12. 2.31 in.
13. 1.7675 in.
14. 7491 lb.
15. 24.96 lb.
16. 23.9912 in.
17. 709.772 lb.
18. 26.1 in.
19. 302.661 lb.
20. 93.5 lb.
21. 22.4775 lb.
22. 18.7 lb.
23. 2.4375 in.

Pages 99 to 101.

1. .024; 330; 415.582; .050; 660.
2. 14.88 gal.
3. 0.1348 in.; 0.1198 in.
4. 0.1798 in.
5. 3.93 lb.
6. 1425 bolts.
7. 11.29 ft.
8. 7.64 ft.
9. 11.14 in.
10. 10.58 gal.
11. 45 ft.
12. 25 ft.
13. 1607.7 ft.
14. 45 ft.
15. 75 ft.
16. 9.03 ft.

Pages 103 to 105.

1. $\frac{5}{8}$ in.; $1\frac{3}{4}$ in.; $4\frac{1}{8}$ in.; $\frac{3}{32}$ in.; $6\frac{1}{32}$ in.
2. $\frac{21}{56}$.
3. 28; 50; 60.
4. $3\frac{7}{8}$ in.; $1\frac{1}{32}$; $8\frac{9}{20}$; $12\frac{1}{4}$; $4\frac{201}{400}$; $7\frac{12}{25}$.
5. $\frac{9}{72}$; $\frac{84}{96}$; $\frac{21}{28}$; $\frac{28}{35}$; $\frac{35}{80}$; $\frac{21}{75}$.
6. $\frac{31}{32}$ in.; $1\frac{1}{4}$ in.; $\frac{27}{32}$ in.; $\frac{7}{64}$ in.; $1\frac{1}{32}$ in.; $\frac{3}{8}$ in.; $\frac{15}{16}$ in.; $2\frac{29}{64}$ in.
7. $1\frac{9}{32}$ in.; $\frac{27}{32}$ in.; $1\frac{9}{32}$ in.; $\frac{5}{8}$ in.; $1\frac{9}{64}$ in.; $2\frac{21}{32}$ in.; $\frac{3}{8}$ in.; $\frac{13}{32}$ in.; $4\frac{37}{64}$ in.
8. $\frac{1}{4}$ in.; 1 in.; $1\frac{15}{16}$ in.; $1\frac{1}{4}$ in.; $1\frac{15}{16}$ in.; $1\frac{1}{8}$ in.; $\frac{1}{8}$ in.; $\frac{3}{8}$ in.; $\frac{7}{8}$ in.; $\frac{15}{16}$ in.; $\frac{5}{16}$ in.; $3\frac{3}{16}$ in.
9. $\frac{15}{64}$ in. dia.
10. $1\frac{11}{16}$ in.; $1\frac{3}{4}$ in.; $1\frac{9}{16}$ in.; $3\frac{7}{8}$ in.; $\frac{3}{8}$ in.; $\frac{21}{32}$ in.
11. 52.15 lb.
12. No. Still 0.045 in. too large.
13. 1.39375 in.
14. 25.4355.
15. $\frac{1}{4}$ in.; $\frac{1}{8}$ in.; $\frac{5}{8}$ in.; $1\frac{3}{8}$ in.; $1\frac{7}{8}$ in.; $\frac{1}{2}$ in.; $\frac{5}{8}$ in.; $\frac{3}{8}$ in.; $\frac{1}{4}$ in.
16. $\frac{27}{32}$; $\frac{7}{32}$; $\frac{15}{32}$; $\frac{11}{32}$; $\frac{23}{32}$ in.

Review Problems in Multiplication and Division of Decimals

1. Divide 3.872 by $9\frac{1}{4}$.

   Divide 8.75 by .063, to four decimal places.

   Divide $4\frac{1}{4}$ by .935.

   Divide 1 by 87.1, to five decimal places.

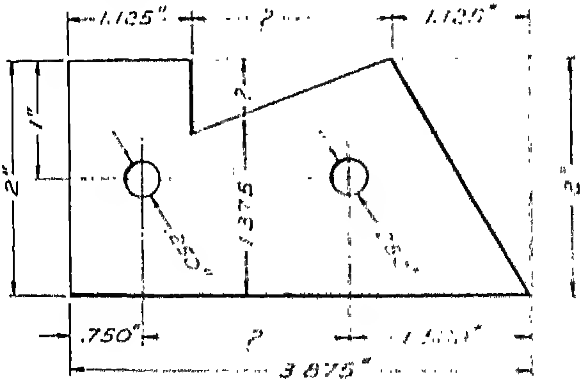
   Divide .01875 by .625.

   Divide 1.125 by 10.3125.

2. Reduce to their complete decimal equivalents, the fractional parts of an inch from 0 in. to $\frac{1}{2}$ in. by sixteenths, as found on the ordinary foot rule.

3. The following drawing was sent to a toolmaker with certain necessary dimensions missing. On this sketch which was returned to the drafting room for correction, these missing dimensions are indicated by the question marks as shown.

Redraw this giving all dimensions in fractional form and supplying the indicated missing dimensions.

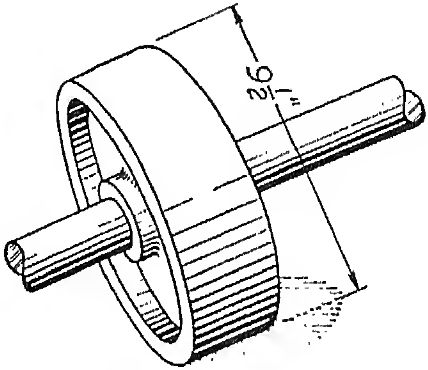


4. A cubic foot of water is listed as weighing 62.425 lb. At this rate what will be the weight of a bucketful of water if the bucket alone weighs $3\frac{1}{4}$ lb. and contains $\frac{1}{2}$ cu. ft.?

5. A steel bar 1 in. square measures $9\frac{5}{8}$ in. long. From it are cut 5 pieces, each measuring 1.120 in. in length. Allowing

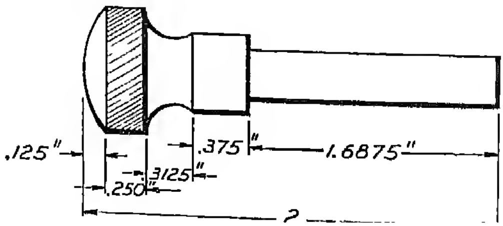
$\frac{1}{8}$ in. for the width of the cutting tool, how much stock is left of the original bar after the cutting is done?

6. The circumference or the distance around a circle equals 3.1416 times the diameter of the circle. According to this what is the length of a thin sheet of fiber needed to cover the entire face of the pulley in the following drawing? The answer to this should be carried to the second decimal place.



7. A machine apprentice who has the task of checking weights of stock finds one bundle of flat strips of iron that weighs 117 lb. This stock is listed as weighing 0.52 lb. per foot of length. How many feet of stock should there be in this bundle?

8. A shop order calls for 320 pins as per the following drawing. These pins are to be made on a turret lathe, and $\frac{1}{8}$ in. is added for finishing and cutting off each pin. The stock for these weighs .125 lb. per inch of length. What is the total weight of the stock used in making these pins?



MONEY

Wages, purchases,
costs, estimates,

Money

Calculations involving the use of money enter into the daily lives of practically every one of us as we go about our affairs, whether we are students, workmen, householders, or businessmen.

Because of this it is very important that one have at least a working knowledge of those elementary calculations commonly used in buying, selling, business transaction, and employment.

The unit of the money system in the United States is the dollar. This is subdivided into the following smaller units, each amount being represented by its own coin:

The 1-cent piece, or penny, which equals $\frac{1}{100}$ of a dollar.

The 5-cent piece, or nickel, which equals $\frac{5}{100}$ of a dollar.

The 10-cent piece, or dime, which equals $\frac{10}{100}$ of a dollar.

The 25-cent piece, or quarter, which equals $\frac{25}{100}$ of a dollar.

The 50-cent piece, or half dollar, which equals $\frac{50}{100}$ of a dollar.

There are also larger multiples of the dollar, as two dollars, five dollars, ten dollars, twenty dollars, and so on. These larger divisions make it convenient in handling large sums. They are usually issued in the note or bill form.

The symbol used in writing amounts of money is the \$, known as the dollar sign. It is placed just in front of the figure, or figures, representing the number of dollars, as \$25, which is read "twenty-five dollars."

The writing of amounts of money resembles the writing of decimals. The decimal point is used to separate dollars from cents, and is placed directly after the number representing dollars, as in \$5.15.

If the amount is less than one dollar, the decimal point is placed in front of the "cents" only when the dollar sign is

expressed, as in the amount forty-eight cents, expressed \$0.48; or the amount six cents, expressed \$0.06. The use of the zero before the decimal point is to indicate that there are "no dollars" expressed.

As seen, the first figure to the right of the decimal point represents tenths of a dollar, or dimes. The second figure to the right of the point represents hundredths of a dollar, or cents.

The correct manner of expressing amounts of money is illustrated below:

Two hundred four dollars and seven cents is written: \$204.07.

Twenty-one dollars and twelve cents is written: \$21.12.

Four dollars and thirty-two cents is written: \$4.32.

Five dollars is written: \$5.00, or \$5.

Fifty-six cents is written: \$0.56, or 56¢.

Nine cents is written: \$0.09, or 9¢.

Twelve and one-half cents is written: \$0.12½, or 12½¢.

The sign ¢ indicates cents, and is quite often used in amounts less than one dollar where it is desired not to express the dollar sign. However, when the sign ¢ is used, neither the dollar sign, \$, nor the decimal point should be used.

It is not correct to express seventy-two cents as \$0.72¢ because the cent sign (¢) when used, takes the place of the dollar sign, the decimal point, and the zero before the decimal point.

Correctly expressed the above amount should be either \$0.72 or 72¢.

For the same reason amounts like six and one-half cents would not be expressed as \$0.06½¢, but rather \$0.06½ or 6½¢, the latter being no doubt preferred.

Problems Involving Expression of Money

1. Write in words the following amounts:
   \$5.32; \$0.19; \$364.08; \$0.07; 23¢; \$1.12½.

2. Examine the following amounts and write correctly those that are not properly expressed:

$.72; $0.19¢; $3.24; $.39¢; 36¢.

3. A plumbing *apprentice is sent out by his employer to* purchase a small brush and a coil of copper wire. He asks the boy to write out the cost of each item on a slip of paper and place it on his desk when he returns. The brush costs seven cents and the wire costs twenty-three cents. Write out a statement similar to what the apprentice should give his employer using figures to express the amount of each item as purchased.

4. Upon accepting a job as a helper in a garage a boy is told that he will be paid at the rate of two dollars and twenty cents per day. How would he express this amount in figures?

5. Express in figures: four hundred eight dollars and seventeen cents; thirty-two and one-half cents; nine dollars and nine cents; eighty-seven and one half cents; six cents.

6. A boy, who works in a shoe store after school hours, is paid one dollar and eighty cents for working two afternoons. How is this amount expressed in figures?

7. After being employed one week as an apprentice, a young man received seven dollars and eighty four cents. How was this amount expressed in figures on his pay envelope?

8. In purchasing items for a short-wave radio set an amateur operator finds that one tube costs ninety-five cents; a transformer costs one dollar and a quarter; a condenser costs eighteen cents; another condenser costs thirty cents; and a trimming condenser one dollar and a half. How would these prices be expressed in figures?

9. A hardware price catalogue shows the following prices: hinges seventeen cents; screws forty cents per box; window lock thirty-five cents; special door lock one dollar and twenty cents; screw driver a quarter. List these prices in figures.

10. A young man at work in a factory received the following pay during the four days he worked there: 78¢; $2.35; $1.90; $2.05. How would each of these amounts be expressed in writing?*

Addition, Subtraction, Multiplication, and Division Involving Money

Numbers representing amounts of money are added, subtracted, multiplied, and divided the same as decimal numbers.

In adding and subtracting, care must be taken to see that the decimal points and figures line up properly. The decimal points should lie in a vertical line, thus placing dollars under dollars, and cents under cents, as shown in the examples below.

Example Illustrating Addition and Subtraction Involving Money:

To equip his chest of tools a young man spent $1.50 for two planes, 34 cents for a hammer, and $2.25 for two saws. What was the total cost of these items? A five-dollar bill was given in payment for these tools. How much change should the young man receive?

Solution:

Cost of planes	$1.50
Cost of hammer	.34
Cost of saws	2.25
	$4.09 = total cost.

The amount of change to be returned would be equal to the difference between the cost $4.09, and $5. This works out as follows:

$5.00 = amount of bill given in payment
$4.09 = cost of tools
$0.91 = change to be received from the $5 bill after paying for the above tools.

Example Illustrating Multiplication Using Money:

To complete a job of tinning a roof, a tinsmith purchases 5 lb. of solder at 35¢ per lb. If he pays for this with a $2 bill how much should he receive in change?

*Answers to these problems will be found on page 118.

Solution and Explanation:

If 1 lb. of solder costs 35 cents then 5 lb. would cost 5 times that amount. This works out as,

$$\begin{array}{r} .35 \\ 5 \\ \hline \$1.75 \end{array} = \text{cost of the solder.}$$

The amount of change to be returned is equal to the difference between this amount, \$1.75, and the \$2.00, or:

$$\begin{array}{r} \$2.00 \\ 1.75 \\ \hline \$\ .25 \end{array}$$

That is, the amount of change returned from the \$2 bill in making the above purchase is \$0.25, or 25¢.

Example Illustrating Division Using Money:

A young man building a short-wave radio set finds that he can purchase from a secondhand dealer the kind of wire he needs at 8¢ per pound. After school he goes to this dealer with a 50-cent piece intending to purchase all he can for that amount. How many pounds of this wire can he purchase?

Solution and Explanation:

If each pound cost 8 cents then he can buy as many pounds for 50¢, as 8¢ is contained in 50¢.

This would be expressed as .50 ÷ .08, and would be worked out as follows:

$$\begin{array}{l} .08).50\,00(6.25 \\ \quad\;\; \underline{48} \\ \quad\;\;\; 20 \\ \quad\;\;\; \underline{16} \\ \quad\;\;\;\; 40 \\ \quad\;\;\;\; \underline{40} \end{array}$$

That is, he can buy 6.25 lb., or $6\frac{1}{4}$ lb., for the 50¢.

General Problems Involving Money Calculations

1. An apprentice in a machine shop is paid at the rate of 22 cents per hour. On Monday and Tuesday he put in $8\frac{1}{2}$ hr. each day. Wednesday he worked 7 hr., and Thursday he worked $9\frac{1}{2}$ hr. He did not work on Friday but put in 4 hr. on Saturday. How much pay should he get in his envelope for that week?

2. A bar of iron $\frac{3}{4}$ in. in diameter weighs $1\frac{1}{2}$ lb. per foot length. What is the cost of 40 ft. of such stock at 3.7¢ per pound?

3. During his summer vacation a boy earns $5.75 each week for nine weeks. When he is about to return to school in the fall he spends $3.50 for shoes, 75¢ for a cap, $14.50 for a suit of clothes, and after paying out $9.28 for school-books and supplies, he puts what is left in the bank. How much money did he earn during his vacation? How much money did he spend out of his earnings? How much money did he put in the bank?

4. The iron castings used in making a small steam engine weigh 26 lb. What is the cost of this material if the castings are bought at the rate of $8\frac{1}{2}$¢ per pound?

5. To excavate for the building of a shed, 14 cartloads of dirt were hauled away. The excavating and hauling costs $1.75 per load, but each load was sold at $2.25 to a contractor who used it for filling in a piece of land. How much profit was there in this transaction?

6. A special job of printing requires $5\frac{1}{2}$ reams of paper. This paper is sold at $2.50 per ream, but an extra charge of 25 cents is made for breaking up a ream in order to get the extra $\frac{1}{2}$ ream required. What is the total cost of this material?

7. On a plumbing job 6 bars of solder each weighing $1\frac{1}{2}$ lb. per bar were used. If the solder sold at 42¢ per pound, what did the total cost of the 6 bars amount to?

8. If white lead for painting purposes is quoted at 14¢ per pound, what is the cost of a can containing 150 lb.?

9. A boy aspiring to become a carpenter purchases 42 ft. of lumber with which to build some shelves in his mother's kitchen. The lumber costs him 9½¢ per foot. To pay for this the mother gave the boy two 2-dollar bills. How much change is there coming to the boy after he pays for the lumber?

10. The daily time-clock card of a workman shows that he has put in 46 hours of work during the week. He is being paid for this at the rate of 37½¢ per hour. When he opened his weekly envelope he found it contained $16.85. Although he could not figure out the proper amount, he felt that this was not the correct pay. What should he have received in his envelope?

11. An apprentice in the machine shop is paid at an hourly rate of 22½¢. In one week he works 3 days, 8 hours each day. The balance of the week he works on piecework and turns out 148 machine bolts at 2¾¢ a bolt. How much should there be in his envelope for that week?

12. To paper 3 rooms in a house there are needed 26 rolls of wallpaper. Eight of these rolls cost 22½¢ each, 10 cost 16¢ each, and the balance cost 12½¢ each. What is the total cost of the paper used on this job?

13. On an automobile trip, 8 gallons of gasoline were used up to the time of making the first stop. At this stop 12 gallons more were taken on. The 8 gallons cost 17¢ per gallon, and the 12 gallons cost 18½¢ per gallon. What was the total cost of the gasoline purchased?

14. A carpenter takes 5½ hours to build a small cabinet for use in a kitchen. He charged for his work at the rate of 80¢ per hour. The material used in building the cabinet cost $2.18. What is the total cost of the cabinet as completed?

15. An apprentice is given a $5 bill with which to purchase for shop use, 4 dozen bolts at 15¢ per dozen, and 8 ft. of lead

pipe weighing 14 lb. at $9\frac{1}{2}$¢ per pound. How much does his purchase amount to? How much change should he return to his foreman?

16. At the rate of $8\frac{1}{2}$¢ per pound, how much will 5 reams of paper cost, each ream containing 60 lb.?

17. A young man working in a machine shop is paid at the rate of 46¢ per hour. During one week he works on regular day jobs 32 hours, and in addition he put in 8 hours overtime for which he is paid time and one half. He also works on piecework and turns out 850 machine pins at 32 cents per hundred. What are his wages for this particular week?

18. A carpenter is paid at the hourly rate of $1.05 per day of 8 hours. At this rate, what are his weekly wages if he puts in 44 hours per week?

19. At $4.40 per day of 8 hours, to what will a workman's equivalent hourly rate of pay be equal?

20. On an automobile trip from Newark to Atlantic City, a young man purchased 2 quarts of oil at 24¢ per quart and 12 gallons of gasoline at $16\frac{1}{2}$¢ per gallon. What was the total cost of oil and gasoline purchased on this trip?

21. A price of $27 is quoted for the construction of a 60-ft. fence. What is the price per foot of length?*

ANSWERS TO PROBLEMS

Pages 112 to 114.

1. Five dollars and thirty-two cents; nineteen cents; three hundred sixty-four dollars and eight cents; seven cents; twenty-three cents; one dollar and twelve and one-half cents.
2. $0.19 or 19¢; $0.39 or 39¢.
3. Brush 7¢; copper wire 23¢.
4. $2.20.
5. $408.17; $32\frac{1}{2}$¢ or $0.32½; $9.09; $87\frac{1}{2}$¢ or $0.87½; 6¢ or $0.06.
6. $1.80.
7. $7.84.

*Answers to these problems will be found on page 119.

8. 95¢; $1.25; 18¢; 30¢; $1.50.
9. 17¢; 40¢; 35¢; $1.20; 25¢.
10. Seventy-eight cents; two dollars and thirty-five cents; one dollar and ninety cents; two dollars and five cents.

Pages 116 to 118.

1. $8.25.
2. $2.22.
3. $51.75; $28.03; $23.72.
4. $2.21.
5. $7.00.
6. $14.00.
7. $3.78.
8. $21.00.
9. 1¢.
10. $17.25.
11. $9.47.
12. $4.40.
13. $3.58.
14. $6.58.
15. $1.93; $3.07.
16. $25.50.
17. $22.96.
18. $46.20.
19. 55¢.
20. $2.46.
21. 45¢.

Review Problems Involving Money Calculations

1. How would you express in figures the following:

Sixty-four dollars and nine cents; thirty and one-half cents; one hundred two dollars and seven cents; five dollars and two and one-half cents; six cents.

How would you express in writing the following amounts: 42¢; $0.09; ½; $1.01; $110.71; $25.25.

2. How much change should a boy get in presenting a $5 bill in payment for a pair of overalls at $1.35, and a work shirt at 75¢?

3. A boy's uncle gave him $2 and told him to divide it equally among his three younger brothers after taking out 65¢ for himself. How much did each of the three younger brothers get?

4. A carpenter, paid at the rate of 95¢ per hour for an 8-hour day, is assigned to a job that is scheduled to run 24 days. What should be his earnings for this time?

5. At 4¢ per pound, how many pounds of nails may be bought for 75¢?

6. A boy working after school hours keeps the following record for one week's work. What are his net earnings for the week?

Monday —Runs 4 errands @ 15¢ each.
Tuesday —Works in grocery 3 hr. @ 20¢ per hour.
Wednesday—Mows lawn and trims hedge: 75¢.
Thursday —Works in grocery 3 hr. @ 20¢ per hour.
Friday —Waxes and polishes automobile $1.25 (paid 50¢ for polishing materials out of this).
Saturday —Works $9\frac{1}{2}$ hr. in grocery @ 20¢ per hr.

7. In checking over the material needed to make a special size screw, it is found that it takes exactly 1 lb. of stock to make six of these screws. If this material costs $5\frac{1}{2}$¢ per pound, what is the cost of the material necessary to make 120 screws?

8. What change does a boy receive from a five-dollar bill, after purchasing 14 knobs at 5¢ each; a coil of wire 12¢; 6 cleats at 3¢ each; and a soldering iron $1.30?

9. A factory worker turned into the time office the following time ticket. What does he receive in his weekly envelope?

Monday —8 hours @ 38¢ per hour.
Tuesday —$9\frac{1}{2}$ hours @ 38¢ per hour.
Wednesday—Piecework—480 pieces @ 75¢ per hundred.
Thursday —Piecework—410 pieces @ 80¢ per hundred.
Friday —Piecework—400 pieces @ 75¢ per hundred.
Saturday —4 hours @ 38¢ per hour.

10. A boy wishing to learn the patternmaking trade accepts a job, at $2.40 per day of 8 hours. The first day of his employment he starts to work at 9:30 a.m. and quits at 5 o'clock in the afternoon, having taken one hour from 12 to 1 for dinner. What is the amount of his first day's wage?

PERCENTAGE

Discounts, profits,
trade discounts,
losses, averages,
efficiencies, ratings.

Percentage

Calculations in percentage have to do with determining amounts or rates of increases, decreases, gains, losses, discounts, averages, efficiencies, and the like. The process is one of computing by hundreds and is somewhat like processes already covered in fractions and decimals. It is frequently referred to by the word *per cent* which means *by the hundred.* The symbol for per cent is %.

To illustrate:

Twenty-four per cent is expressed as 24%.
Three and one-half per cent is expressed as $3\frac{1}{2}$%.

To be used in calculations, the number representing the per cent should be changed to decimal form. This is done by substituting the word *hundredths* for the per cent sign, %.

Thus, 24% would first be changed to 24 hundredths. Expressed as a decimal 24 hundredths is .24. In this manner 24% changed to decimal form becomes .24. In this final form it may be used in calculations.

Should there already be a decimal point in the given per cent, the change to decimal form would be accomplished by moving the decimal point *two places* to the *left* of its *present* position.

For example, in changing 37.4% to decimal form it may first be expressed as 37.4 hundredths (thirty-seven and four tenths hundredths). The decimal point being already located in front of the figure 4, is then moved two places to the left, as explained. This puts it in front of the figure 3. As a result 37.4% changed to decimals is .374.

In the same manner 125% changed to decimal form becomes 125 hundredths, expressed in figures as 1.25. Also, 5% becomes .05; 2% becomes .02; $\frac{1}{2}$% becomes $.00\frac{1}{2}$ or .005.

After being so changed to decimal form these numbers are used in calculations the same as ordinary decimals. This is illustrated in the following examples.

Example 1:

After a printer's apprentice had printed an order of 1200 announcement cards, his foreman told him that it was necessary to reject 15% of the order because of faulty workmanship. How many cards were rejected?

Solution and Explanation:

There were a total of 1200 cards printed. Of that number 15% were rejected. The problem is to find what number of the 1200 cards was rejected.

Expressed simply, this amount is 15% of 1200.

The word *of* as previously explained means *times* and indicates multiplication. The answer to this would therefore be found by multiplying 1200 by 15%.

15% expressed as a decimal equals .15.

Then, 1200 × .15 becomes:

$$\begin{array}{r} 1200 \\ .15 \\ \hline 60\ 00 \\ 120\ 0 \\ \hline 180.00 \end{array}$$

That is, 180 cards were rejected because of faulty workmanship.

Example 2:

A carpenter desiring a two-foot rule finds one listed in a tool catalogue at 60¢, with a discount of 15%. With this discount what is the net cost of the rule?

Solution and Explanation:

The discount above referred to means that the price of 60¢, as listed, is to be reduced by 15%.

To find the amount of this reduction, take 15% of .60. This expressed in multiplication form becomes, .60 × .15, or:

$$\begin{array}{r} .60 \\ .15 \\ \hline 300 \\ 60 \\ \hline .0900 \end{array} \text{ — or } 9¢ \text{ reduction.}$$

That is, the price of 60¢ reduced by 15%, is changed by 9¢, making the net cost of the rule to the carpenter equal to 60¢ − 9¢, or 51¢.

Example 3:

A hardware supply catalogue shows that a certain grade of a brass screw lists at $2 per gross. The discount sheet which accompanies the catalogue states that hardware dealers are allowed discounts of 60%, 25%, and 10% from this list. With these discounts what is the net price of these screws to the hardware dealer?

Solution and Explanation:

This form of discount is referred to as *trade discount.* But it does not mean that there is a total discount of 95% (equal to the sum of 60%, 25%, and 10%) on the material in question. In such cases where trade discounts are allowed, the list price remains as originally stated, and the second discount is made after the first discounted price has been established. In the same way the third discount is made after the second discounted price has been established.

The discounts in the above problem are used in calculating the net price by first taking from the list price the first discount of 60%. After this is done, the second discount of 25% is taken from this net amount, giving as a result the second discounted price. The net amount obtained as a result of this is the final, or *net price*, to the dealer.

This works out as follows:

Determining the first discounted price:

2.00 × .60 = 1.20, or $1.20 as the first discount.

$2.00 − $1.20 = $0.80 — the net cost after the first discount is taken.

Determining the second discounted price:

.80 × .25 = .20, or $0.20 as the second discount.
$0.80 − $0.20 = $0.60 the net cost after the first and the second discounts have been taken.

Determining the third or final discounted price:

.60 × .10 = .06, or $0.06 as the third discount.
$0.60 − $.06 = $0.54 — the net cost after the first, second, and third discounts have been taken.

This gives $0.54, or 54¢, as the net price per gross of the above brass screws listing at $2.00 per gross with discounts of 60%, 25%, and 10%.

Example 4:

After estimating the cost of painting a house to be $240, the contractor adds 25% for profit. How much does this profit amount to and what is the final "figure" on this job?

Solution and Explanation:

Since 25% is added to the figure of $240 for *profit*, that means that 25% of $240 is the amount of the profit.

Expressed in simple form this would be 25% of $240, or 240 × .25.

This works out as:

$$\begin{array}{r} 240 \\ \underline{.25} \\ 12\,00 \\ \underline{48\,0} \\ 60.00 \end{array}$$

That is, a profit of 25% would amount to $60.

If this profit of \$60 is to be made on a job which costs \$240, then the estimate of \$240 must be increased by \$60. This makes the final price equal \$240 + \$60, or \$300.

From these problems it may be seen that when a certain per cent of a number is to be obtained, whether it be in the form of a *discount*, a *profit*, an *increase* or a *decrease*, the process is one of multiplication.

CHANGING PER CENT TO FRACTIONAL FORM

In some calculations a per cent may best be expressed and handled as a fraction. This is done by changing the per cent to a decimal which in turn is used as the numerator of a decimal fraction.

The decimal fraction is then reduced to lowest terms, giving the simplest fraction which is the equivalent of the per cent.

This is illustrated in the following examples:

Example 1:

Express 45% as a fraction.

Solution and Explanation:

45% changed to a decimal equals .45.
The decimal .45 equals the fraction $\frac{45}{100}$.
$\frac{45}{100}$ reduced to lowest terms equals $\frac{9}{20}$.
45% changed to fractional form equals $\frac{9}{20}$.

Example 2:

Change $16\frac{2}{3}\%$ to fractional form.

Solution:

$16\frac{2}{3}\%$ changed to a decimal equals $.16\frac{2}{3}$

$$.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{\frac{50}{3}}{100} = \frac{50}{3} \div 100 = \frac{50}{300}$$

$\frac{50}{300}$ reduced to lowest terms equals $\frac{1}{6}$

$16\frac{2}{3}\%$ changed to fractional form equals $\frac{1}{6}$.

Example 3:

Express $12\frac{1}{2}\%$ as a fraction.

Solution:

$12\frac{1}{2}\%$ changed to a decimal equals .125
$.125 = \frac{125}{1000}$
$\frac{125}{1000}$ reduced to lowest terms equals $\frac{1}{8}$
$12\frac{1}{2}\%$ changed to fractional form becomes $\frac{1}{8}$.

When such per cents are changed to fractions they are more conveniently used in calculations. However, unless the fractional form can be handled to better advantage, it is more advisable to use the decimal equivalent.

Per cents like $33\frac{1}{3}\%$; $66\frac{2}{3}\%$; $16\frac{2}{3}\%$; and $11\frac{1}{9}\%$ are of this type. When changed to fractional form these become:

$33\frac{1}{3}\%$ or $\frac{100}{300}$, which equals $\frac{1}{3}$.
$66\frac{2}{3}\%$ or $\frac{200}{300}$, which equals $\frac{2}{3}$.
$16\frac{2}{3}\%$ or $\frac{50}{300}$, which equals $\frac{1}{6}$.
$11\frac{1}{9}\%$ or $\frac{100}{900}$, which equals $\frac{1}{9}$.

In such fractional forms these per cents are readily used in calculations.

Problems Involving Calculations in Percentage

1. Express in decimal form $82\frac{1}{2}\%$; 9%; .4%; 132%; $2\frac{1}{2}\%$; 1.3%. Express in per cent form .52; 4.2; 6; .001; 8.07; 37.5. What is 19% of \$5; 3% of 11; $8\frac{1}{2}\%$ of 20; .6% of 43; 124% of 55; 1.2% of \$150?

2. Reduce to fractional form:

140%; 2%; 62.5%; 22%; 20%; 225%;
110%; 85%; $\frac{1}{2}\%$; $14\frac{2}{7}\%$.

3. During his summer vacation a young man put his savings, which amounted to \$33.60, in the bank. At the opening of school he withdrew $12\frac{1}{2}\%$ of this to buy clothes. How much did he draw out of the bank? How much remains in the bank on his account?

4. A leaflet that advertises an attachable turret for standard machine-shop lathes, lists this tool at \$300. A notation is also made on this leaflet that, on a certain date, discounts of 30%,

30%, and 10% had been allowed on this price. What is the cost of this tool after these discounts have been made?

5. An apprentice patternmaker who was being paid at the rate of $1.20 per day, received a notice in his pay envelope stating that because of his good work, his daily rate of pay would be increased 10%. What will be his daily wage after this increase?

6. By paying cash for a load of lumber which is listed at $76, a discount of $2\frac{1}{2}$% was granted. How much was saved by paying cash?

7. A 12-in. file listed at $4 per dozen sells at wholesale with discounts of 30%, 25%, and 2% for cash. Determine the wholesale price of these files per dozen.

8. Lumber to be used for concrete forms is quoted at $60 with discounts of $7\frac{1}{2}$% and 2% for cash. If this lumber is paid for in cash, what is its net cost?

9. A machine department in a manufacturing plant uses each month about 20% of a 50-gal. drum of cutting compound. At this rate how long should 50 gallons of this cutting compound last?

10. Reports for the month of April show that out of 80 men at work in the machine department of a local shop, 15% were out of work because of sickness. How many men were out sick?

11. The total accidents in a garment factory over a period of three months were 36. Of these accidents 75% were charged to direct carelessness. How many accidents were due to carelessness?

12. A cubic foot of ice is 91.7% as heavy as a cubic foot of water. If water weighs 62.425 lb. per cubic foot, what is the weight of a cubic foot of ice?

13. Brass is listed in a shop catalogue as containing 65% copper and 35% zinc. At this rate, what is the weight of zinc and copper in a piece of brass that weighs 45 lb.?

14. A jewelry catalogue quotes a price of \$8 on school emblem rings. It allows a discount of 25% with 2% off for cash if 50 or more rings are purchased at one time. What would be the net cost per ring if 50 students agreed to pay cash for these rings?

15. To raise a heavy casting from the floor to a workbench a set of hoisting pulleys was used. The pulleys, not having been previously used in some time, did not work well and required a pull of 75 lb., to move the casting. After oiling the pulleys well, however, this pull was reduced 20%. What was the final pull on the rope needed to raise the casting?

16. By keeping his tools properly ground a workman on piecework finds that he can increase his production 8%. At that rate what will be his day's pay if before using this precaution he earned \$4.25 on the same kind of work?

17. A young man learning the plumbing trade was told by his employer that his wages for the week would be advanced $12\frac{1}{2}$%. The boy at the time was being paid at the rate of 24 cents per hour for a 44-hour week. What will his weekly wage be when this change takes place?

18. When John got his first job he immediately laid out a weekly budget plan. He planned to give 60% to his mother; 30% was to be for clothes and savings; 10% was to be put aside for emergency. The job paid him \$15 per week. What was the amount of each item on John's budget?

19. Fifteen pounds of babbitt are required for rebabbitting machine bearings. If this babbitt has 15% tin, 25% antimony, 60% lead, how much of each of these metals is used in this composition?

20. An automobile which was originally purchased for \$950 is offered for resale as a used car at 30% less than the original price. What is the resale price?

21. A newspaper advertisement explains that a popular make of radio which formerly sold at $85 is now offered at 20% less than that price. What is the cost of this radio?

22. In advising a class of high-school seniors about improving their report-card marks, the principal explained that out of last year's senior class of 120 students, 15% failed to graduate because of low records. How many students were graduated last year?

23. The following notice appeared on a shop bulletin board:

> "Shop accidents for the 6 months' period ending June 30th, dropped 15% below that of the previous 6 months' period."

If there were 80 accidents during the previous 6 months' period referred to, how many accidents were there during the 6 months' period ending June 30th?

24. A shoe factory that regularly employed 250 workers announces that due to business improvements it has increased its working force by 18%. How many workers are now employed in this factory?

25. Brass hexagon nuts for ½-in. brass screws are listed in a manufacturer's catalogue at $9.60 per hundred. The discount sheet shows that there are discounts of 60% and 10% on this price. What is the net price on 500 of these nuts?

26. An advertisement concerning a certain make of wood screws states that size No. 10 screw is listed at 70¢ per gross, with trade discounts of 60%, 20%, and 10%. At this rate what would be the cost of 100 gross of these screws?

27. A standard make of twist drill lists at $7.60 per dozen. The trade-discount sheet allows 50% and 5% off for cash. What is the net cost of these drills per dozen?

28. A dealer in automobile supplies advertises that he is closing out a certain make of automobile tire, which formerly sold for $7, at 20% and 10% off the original price. If you were

to purchase two of these tires what would you have to pay for them?

29. During a department-store sale, a small table that listed at \$12 is marked, "$33\frac{1}{3}\%$ off because of slight damage." What is the selling price of this table?

30. In making a batch of concrete which weighed approximately 1200 lb., about $16\frac{2}{3}\%$ of the weight is cement, $33\frac{1}{3}\%$ is sand, and 50% is gravel. How many pounds of each item are used in this mixture?*

CHANGING FRACTIONS TO PER CENT FORM

Fractions may be readily changed to per cent form by the same process as that followed in changing a fraction to a decimal.

This is done as previously explained by dividing the numerator by the denominator. After the decimal is obtained the per cent sign (%), is added after the number, and the decimal point is moved two places to the right. In case the decimal point comes *directly* after the number as a result of this change, it is not expressed when the per cent sign is added.

This is illustrated in the following examples:

Example 1:

Change the fraction $\frac{1}{5}$ to per cent form.

Solution and Explanation:

$\frac{1}{5}$ becomes $1 \div 5$, or,

$$\begin{array}{r} 5)1.00(.20 \\ \underline{1\ 0} \\ 0 \end{array}$$

Expressed as a decimal $\frac{1}{5}$ equals .20.
Expressed as a per cent .20 equals 20%.
That is, $\frac{1}{5}$ changed to per cent form equals 20%.

Example 2:

What is the per cent form of the fraction $\frac{3}{8}$?

*Answers to these problems will be found on page 140.

Solution and Explanation:

$\frac{3}{8}$ becomes $3 \div 8$, or,

```
8)3.000(.375
  2 4
  ---
    60
    56
    --
     40
     40
     --
```

Expressed as a decimal $\frac{3}{8}$ equals .375.

Expressed as per cent .375 equals 37.5%.

In per cent form $\frac{3}{8}$ becomes 37.5%.

Example 3:

Change $\frac{1}{6}$ to per cent form.

Solution and Explanation:

$\frac{1}{6}$ becomes $1 \div 6$, or,

```
6)1.00(.16 4/6
  6
  --
  40
  36
  --
   4
```

Expressed as a decimal, $\frac{1}{6}$ equals $.16\frac{4}{6}$, or $.16\frac{2}{3}$.

Expressed as a per cent, $.16\frac{2}{3}$ equals $16\frac{2}{3}\%$.

The fraction $\frac{1}{6}$, expressed in per cent form, is $16\frac{2}{3}\%$.

Example 4:

Determine the per cent form of the fraction $\frac{5}{4}$.

Solution and Explanation:

$\frac{5}{4}$ becomes $5 \div 4$, or,

```
4)5.00(1.25
  4
  --
  1 0
    8
   --
   20
   20
   --
```

Expressed as a decimal, $\frac{5}{4}$ equals 1.25.

Expressed as a per cent, 1.25 equals 125%.

That is, $\frac{5}{4}$ changed to per cent form equals 125%.

From these calculations it may be seen that fractions less 1 are equal to less than 100%. Fractions greater than 1 are equal to more than 100%. Fractions equal to 1 are equal to 100%.

HOW TO DETERMINE PER CENT

Previous to this, consideration has been given principally to calculations involving the use of percentage, and to methods of changing fractional and per cent forms. Sometimes it is desired to know what per cent one number is of another number, or how a per cent may be determined when certain facts are given.

For example, what per cent is 45 of 60?

In fractional form 45 is $\frac{45}{60}$ of 60. However, $\frac{45}{60}$ is not in per cent form. From previous explanations it is seen that to change this fraction to per cent form, 45 should be divided by 60, after which the quotient may be readily changed to per cent.

This fractional form is changed as follows:

```
60)45.00(.75
   42 0
   ----
    3 00
    3 00
    ----
```

.75 expressed in per cent form becomes 75%.

Therefore, 45 is 75% of 60.

The correctness of this result may be determined by calculating 75% of 60. This is 75% × 60, or .75 × 60, which works out as,

```
   60
  .75
 ----
  3 00
 42 0
 -----
 45.00, or 45.
```

This proves that 45 is 75% of 60.

DETERMINING EFFICIENCIES, PROFITS, AND RATINGS

The foregoing method is also used in determining efficiencies, profits, and ratings. Typical conditions involving these are illustrated in the following examples.

Example 1:

It is estimated that on a certain job a journeyman printer can print 1,000 folders on a printing press in 3 hours. To do the same work, however, will take an apprentice printer 4 hours. How efficient is the apprentice on this particular job?

Solution and Explanation:

The journeyman's time on the job is 3 hours.

The apprentice's time on the job is 4 hours.

The apprentice's rating would be found by dividing the journeyman's time, which is sometimes called the *estimated* time, by the *actual* time it took to do the job, 4 hours. This works out as: $3 \div 4$ or

$$\begin{array}{r} 4)3.00(.75 \\ \underline{2\,8} \\ 20 \\ \underline{20} \end{array}$$

.75 expressed as a per cent equals 75%.

That is, the apprentice is 75% efficient on this particular job when compared with the journeyman printer.

Example 2:

A young man purchases a jackknife for 50¢. Later on he sells the same knife to a chum for 60¢. How much does he gain in the transaction? What is his per cent profit on this sale?

Solution and Explanation:

The profit in cash would be the difference between the selling price and the cost. This would be 60¢ − 50¢ or 10¢.

This 10¢ profit was gained on an investment of 50¢. Therefore, the fractional gain would be $\frac{10}{50}$ or $\frac{1}{5}$.

$\frac{1}{5}$ expressed as a decimal equals .20;

$$\begin{array}{r} 5)1.00(.20 \\ \underline{1\,0} \\ 00 \end{array}$$

.20 expressed as a per cent equals 20%.

That is, the boy made 20% profit on this sale.

Example 3:

During the first day of his employment a young workman assembled 240 units, which he was told were to fit into mechanical toys that were being manufactured in that shop. However, the inspector on that work accepted but 228 of these units, and rejected the balance as not correctly assembled.

What per cent of his work was acceptable this first day of employment?

Solution and Explanation:

An examination of the above figures shows that out of 240 units 228 were acceptable. In other words, 228 of the 240, or $\frac{228}{240}$, were acceptable.

This fractional form $\frac{228}{240}$ is readily changed to per cent form by dividing the numerator by the denominator as previously explained.

This works out as: $228 \div 240$, or,

$$
\begin{array}{r}
240)228.00(.95 \\
\underline{216\ 0} \\
12\ 00 \\
\underline{12\ 00}
\end{array}
$$

.95 expressed in per cent form becomes 95%.

That is, 95% of the young man's work was acceptable the first day of his employment.

The above problems illustrate that in order to determine an *efficiency*, a *profit*, or a *rating*, the process is really one of finding what per cent one number is of another. This relation is expressed in fractional form and changed to a decimal, which in turn is changed to per cent form as already explained.

Problems Involving Ratings, Profits, and Efficiencies

1. Change to per cent form the following fractions and mixed numbers:

$\frac{5}{32}$; $\frac{7}{8}$; $1\frac{1}{8}$; $\frac{3}{16}$; $2\frac{1}{4}$; $\frac{3}{8}$; $4\frac{1}{6}$; $10\frac{3}{5}$.

2. Determine what per cent,

36 is of 48; 12 is of 50; 18 is of 15;
$\frac{1}{2}$ is of 4; 5 is of 2; 132 is of 96.

3. With the baseball season less than half finished the St. Louis National League Baseball Team had won 43 games out of 68 games played. Its nearest rival, the Chicago team, during the same period had won 41 games out of 66 games played. What is the rating, or standing, of these two baseball teams at that date?

4. In its drive to reduce accidents, a shop safety committee recommended that certain guards be installed on several machines. The month following the installation of these guards there was a total of 12 accidents throughout the shop. The previous month's record showed there were 18 accidents. What was the percentage reduction?

5. Twenty-seven pounds of strip brass were needed to blank out an order of punchings on a foot press. The scrap remaining after this job was done weighed 3 lb. What per cent of waste resulted from this operation?

6. By a slight change in the pattern a saving of $7\frac{1}{2}$ lb. is made in a casting that formerly weighed 60 lb. What per cent of the former weight does this equal?

7. The calculated speed of a pulley on a printing press is 250 revolutions per minute. However, by actual check it is found that the pulley makes only 230 revolutions per minute. An investigation shows that this loss is due to belt slippage. What is the per cent slippage in this particular case?

8. Due to a leaky faucet, $1\frac{1}{2}$ gal. of oil were lost from a can containing 25 gal. What is the per cent loss?

9. In machining a brass casting weighing 36 lb. there is removed in the process $4\frac{1}{2}$ lb. of material. Determine the per cent reduction in the weight of this casting as a result of this machining.

10. A shop that had been running on a 40-hour week agrees to change to a 36-hour week. The workmen are to receive the same weekly rate as formerly. What is the equivalent per cent increase in the hourly rate of pay?

11. Out of 1,080 pieces blanked on a foot press, the inspector rejects 27 pieces that were damaged in the operation. What per cent spoilage does this equal?

12. Loose knots and checks in a 16-ft. board necessitate scrapping $1\frac{1}{2}$ ft. off one end and 2 ft. off the other end. What is the per cent waste in this piece of lumber?

13. A learner on a spot-welding machine was able to weld 15 pieces per hour at the end of the first day of his employment. At the end of the second day he was able to weld 24 pieces per hour. At the end of the next day this jumped to 33 pieces. Calculate the per cent of hourly increase in production for each day over the first day.

14. As a result of a local safety campaign, the automobile accidents in an eastern city during a 3-month period dropped from 32 to 20. What was the per cent reduction in accidents during that period?

15. To lay out a drawing board and build it to specifications, it is estimated that it will take a cabinetmaker $2\frac{1}{2}$ hours. However, an apprentice does the job in $4\frac{1}{2}$ hours. What is the per cent efficiency of the apprentice on this job?

16. A lathe hand engaged in turning a small pivot pin makes 75 pieces the first day on the job. The second day he increases the speed of his machine and makes 90 pieces. Determine his per cent gain over the first day's output.

17. To paint a two-story wooden house there is a price quoted of $180. The paint and other materials used on this job, together with the cost of labor, cartage, and other expenses, amounted to $135. What was the per cent profit on this painting job?

18. A young man employed in a woodworking shop finds that it takes him 3 hours to turn up a wooden handle. After sharpening his wood-turning tools he does the same job in $2\frac{1}{4}$ hours. Calculate the per cent gain in time as a result of this precaution.

19. Butter, which sold at 30¢ per pound, is advanced to 36¢ as a result of a drought in the farming districts. What per cent increase in price does this equal?

20. Due to improved business conditions a factory employing 120 men engages 15 extra men. What is the per cent increase in employment?

21. A boy at work in a novelty workshop earns $15 per week. Out of this amount, $1.80 is kept out by the company for his weekly savings account, while 20¢ is withheld for his factory insurance policy and medical attention. If he gives $10 to his mother as his share of the household expenses, what per cent of his weekly pay does he have left?

22. To meet the demands for early delivery of orders for the Christmas trade, a toy factory employed 84 men. After the rush was over 36 of the workers were laid off. What per cent of the working force remained?

23. An apprentice in the pressroom of a printshop spoils 60 sheets in a run of 1500 billheads. What per cent spoilage does this equal?

24. During the week a young man employed at $18.75 a week is "docked" 75¢ for being late for work three times. What per cent of his weekly wage does this equal?

25. In checking over a quantity of $\frac{1}{4}$-in. bolts the inspector found that out of a total of 160 bolts, 6 were undersize and 4 were oversize. What per cent of the total number does each of these represent?

26. On his final examination in arithmetic a student had 42 problems correct out of 50. What is his per cent rating?

27. During the first month of his employment a young man saved \$8. The following month he saved \$9.60. What does this increase equal when expressed in per cent form?

28. The daily time card of a machine-shop apprentice shows the following:

$2\frac{1}{2}$ hours shaper work
$1\frac{1}{2}$ hours milling-machine work
4 hours benchwork

What per cent of the day was given to each branch of work?

29. Due to competition among service-station managers, the price of gasoline dropped during one week from 16.4¢ per gallon to 13.9¢ per gallon. What is the per cent drop in price?

30. Near the close of one of his most successful baseball seasons Charlie Gehringer of the Detroit Tigers led all players in both the National League and the American League in batting. Records show that out of 482 times at bat he made 187 hits. What was his batting average at that date?*

ANSWERS TO PROBLEMS

Pages 128 to 132.

1. .825; .09; .004; 1.32; .025; .013; 52%; 420%; 600%; .1%; 807%; 3750%; 95¢; .33; 1.7; .258; 68.2; \$1.80.
2. $1\frac{2}{5}$; $\frac{1}{50}$; $\frac{5}{8}$; $\frac{11}{50}$; $\frac{1}{8}$; $2\frac{1}{4}$; $1\frac{1}{10}$; $\frac{17}{20}$; $\frac{1}{200}$; $\frac{1}{7}$.
3. \$4.20 clothes; \$29.40 bank.
4. \$132.30.
5. \$1.32.
6. \$1.90.
7. \$2.06.
8. \$54.39.
9. 5 months.
10. 12.
11. 27.
12. 57.24 lb.
13. 29.25 lb. copper; 15.75 lb. zinc.
14. \$5.88.
15. 60 lb.
16. \$4.59.
17. \$11.88.
18. \$9.00; \$4.50; \$1.50.
19. $2\frac{1}{4}$ lb. tin; $3\frac{3}{4}$ lb. antimony; 9 lb. lead.
20. \$665.00.
21. \$68.00.
22. 102.
23. 68.
24. 295.
25. \$17.28.
26. \$20.16.
27. \$3.61.
28. \$10.08.
29. \$8.00.
30. 200 lb. cement; 400 lb. sand; 600 lb. gravel.

*Answers to these problems will be found on page 141.

Pages 136 to 140.

1. $15\frac{5}{8}\%$; $77\frac{7}{9}\%$; $112\frac{1}{2}\%$; $18\frac{3}{4}\%$; 225%; $62\frac{1}{2}\%$; $416\frac{2}{3}\%$; 1060%.
2. 75%; 24%; 120%; $12\frac{1}{2}\%$; 250%; $137\frac{1}{2}\%$.
3. 63.23%; 62.12%.
4. $33\frac{1}{3}\%$.
5. $11\frac{1}{9}\%$.
6. $12\frac{1}{2}\%$.
7. 8%.
8. 6%.
9. $12\frac{1}{2}\%$.
10. $11\frac{1}{9}\%$.
11. $2\frac{1}{2}\%$.
12. 21.9%.
13. 60%; 120%.
14. $37\frac{1}{2}\%$.
15. $55\frac{5}{9}\%$.
16. 20%.
17. $33\frac{1}{3}\%$.
18. 25%.
19. 20%.
20. $12\frac{1}{2}\%$.
21. 20%.
22. $57\frac{1}{7}\%$.
23. 4%.
24. 4%.
25. $3\frac{3}{4}\%$; $2\frac{1}{2}\%$.
26. 84%.
27. 20%.
28. $31\frac{1}{4}\%$; $18\frac{3}{4}\%$; 50%.
29. 15.24%.
30. 388%.

Review Problems Involving Percentage Calculations

1. Change to fractional form:

Twenty per cent; one hundred thirty per cent; four per cent; thirty-seven and one-half per cent; seventy-five per cent.

Reduce to decimal form: $12\frac{1}{2}\%$; 2%; 140%; $.6\%$; 270%.

2. What per cent of,

64 is 16; 12 is 18; 125 is 10; $12\frac{1}{2}$ is $37\frac{1}{2}$; .36 is .009?

3. Determine the per cent equivalent of:

$$\tfrac{16}{80};\ \tfrac{3}{25};\ \tfrac{7}{35};\ \tfrac{1}{3};\ \tfrac{9}{18};\ \tfrac{6}{11};\ \tfrac{40}{20};$$
$$\tfrac{1}{15};\ \tfrac{17}{12};\ \tfrac{72}{60};\ \tfrac{13}{4};\ \tfrac{5}{125}.$$

4. Pure block tin costs 58¢ per lb., and pig lead $5\frac{1}{2}$¢ per lb. At these prices what is the cost of 80 lb. of solder containing 60% lead and 40% tin?

5. A young man at work in a printshop runs off 1800 circulars on a printing press. Of these, $3\frac{1}{2}\%$ were spoiled through carelessness. What was the total number of circulars spoiled?

6. A well-known twist drill which lists at $9 per set, has a discount of 30%, 10%, and 5%. What is the net price of this set of drills?

7. An auto mechanic's helper found a slip in his weekly pay envelope stating that beginning the following week his pay would be raised 12%. His present wage is \$14.50. What pay should he receive on the new basis?

8. A textbook on mathematics is listed in the catalogue at \$2. Students, however, are allowed a discount of 20%. What is the net price to students?

9. In checking a cast-iron pulley before it was machined, it was found to weigh $22\frac{1}{2}$ lb. After the machining operations were complete it weighed exactly $19\frac{1}{2}$ lb. Calculate the per cent reduction in weight as a result of these operations.

10. On February 1st, white lead used in mixing paint, sold at $12\frac{1}{2}$¢ per lb. in 100-lb. lots. On March 1st, it was quoted at 14¢ per lb. in the same amounts. What was the per cent increase in price?

11. A boy learning the printing trade was given the job of "running off" circulars on the printing press. His foreman estimates that it would take a journeyman pressman 3 hours to do the job. The boy begins work at 1:00 p.m. and finishes at 5:30 p.m. What is his rating on this job?

12. Baseball records show that during one of his banner seasons with the Philadelphia Athletics Jimmie Foxx made 180 base hits out of 539 times at bat. During that same season Lou Gehrig of the New York Yankees made 210 hits out of 579 times at bat. What were the batting averages of these two great baseball players for that particular season?

LINEAR MEASURE

Lengths, distances,
amounts, quantities,
estimates.

Linear Measure

Linear measure has to do with measurements in one direction only, as for example: the length, or the width of an object, or the distance to a certain place. Each of these involves a measurement in one direction.

The units of linear measure most common in shop use are the inch and the foot. Both of these are divisions of the English standard unit of length, the yard.

The original English yard, later adopted by the United States, was established by England some years ago as being the length between two fixed points on a special metal bar which has been kept in the Government Bureau of Standards. Duplicates of this bar are also in the United States Bureau of Standards at Washington.

For convenience in working out smaller measurements, the yard has been divided into thirty-six equal parts called inches. Twelve of these inches make one foot.

The following table shows how these different units are related to each other and to the standard unit, the yard.

12 inches	=	1 foot			
3 feet	=	1 yard	= 36 inches		
$5\frac{1}{2}$ yards or					
$16\frac{1}{2}$ feet	=	1 rod			
5,280 feet	=	1 mile	= 320 rods	=	1,760 yards

The word *inch* or *inches* as previously explained is indicated by the abbreviation in., or by the symbol ("), as in the measurement 5 inches, expressed as 5 in., or 5".

The unit of measure, foot, also its plural, feet, is abbreviated ft., the symbol for this being ('). Accordingly, the measurement 3 feet would be expressed as 3 ft., or 3'.

A measurement combining feet and inches, as 6 feet 8 inches, would be expressed either as 6 ft. 8 in., or 6'—8".

Where the symbols (′) and (″) are used, it is recommended that the numbers be separated by a long dash as shown in 6′—8″. This will avoid crowding and confusing the symbols used.

The measurement *yard* or *yards* is abbreviated yd. Combined with feet, or inches, the measurement would be abbreviated as above expressed. For example, 5 yards and 2 feet would be written 5 yd. 2 ft.; 4 yards 5 inches would be written as 4 yd. 5 in.

Rod or *rods* is abbreviated as rd., similar to yd. in yard or yards. The rod is usually used in measurements relating to land.

CHANGING FROM ONE DENOMINATION TO ANOTHER

The many uses to which these units of measure are put, make it necessary sometimes to change units of one denomination to equivalent units of another denomination, as to reduce $7\frac{1}{3}$ yards to its equivalent number of feet; or to find out how many feet there are in 70 inches.

This is accomplished through the use of the table on linear measurements as shown in the following example:

Example 1:

In determining the amount of lumber required to build a kitchen shelf, a boy finds that there is needed a total of 90 inches of white pine of a given width and thickness. What is the equivalent number of linear feet in this amount?

Solution and Explanation:

Since there are 12 in. in one foot, then there are as many feet in 90 in. as 12 is contained in 90. This becomes,

$$90 \div 12, \text{ or } 7\tfrac{1}{2}$$

That is, there are $7\frac{1}{2}$ feet in 90 inches.

Hence, $7\frac{1}{2}$ ft. of lumber are needed to build the shelf referred to.

From this it is seen that to change smaller units to equivalent units of a higher denomination, one should divide the

amount which represents the smaller unit by the number of these smaller units it takes to make *one* of the larger unit.

The following problems illustrate the method of changing larger units to their equivalent number of smaller units.

Example 2:

Reduce 4 yards $2\frac{1}{2}$ feet to inches.

Solution and Explanation:

The number of yards is first reduced to inches. This equals

$$4 \times 36, \text{ or } 144 \text{ inches.}$$

The number of feet is then reduced to inches, which becomes:

$$2\tfrac{1}{2} \times 12, \text{ or } 30 \text{ inches.}$$

The yards and feet reduced to inches become:

$$\begin{array}{rcr} 4 \text{ yards} & = & 144 \text{ inches} \\ 2\frac{1}{2} \text{ feet} & = & 30 \text{ inches} \\ \hline 4 \text{ yards } 2\frac{1}{2} \text{ feet} & = & 174 \text{ inches} \end{array}$$

That is, 4 yards $2\frac{1}{2}$ feet reduced to inches equals 174 inches.

Example 3:

How many feet of wire fence are needed to enclose a plot of land that measures 28 rods around?

Solution and Explanation:

Rods are reduced to feet by multiplying the number of rods by $16\frac{1}{2}$ as there are $16\frac{1}{2}$ feet in one rod. This becomes:

$$28 \times 16\tfrac{1}{2}, \text{ or } 462$$

That is, 462 feet of wire fence are needed to enclose the plot mentioned above.

The preceding problems illustrate that in order to change larger units to their equivalent number of smaller units, multiply the number which represents the larger unit by the number of the smaller units it takes to make one of this larger unit.

As an aid in changing units of one denomination to units of another denomination, the following summary is arranged:

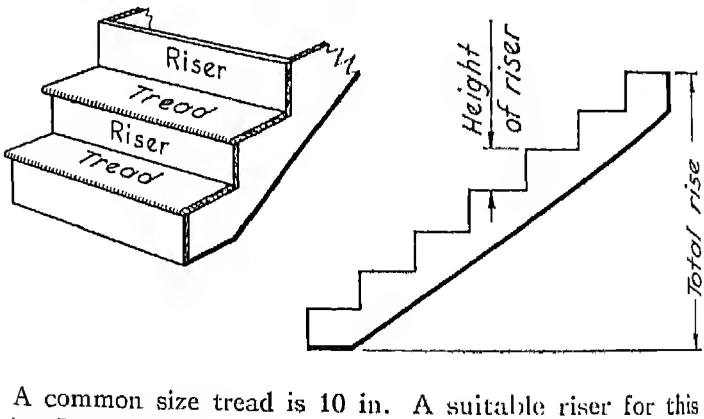
To reduce miles to feet, multiply the number of miles by 5,280.

To reduce feet to miles, divide the number of feet by 5,280.

To reduce miles to rods, multiply the number of miles by 320.
To reduce rods to miles, divide the number of rods by 320.
To reduce miles to yards, multiply the number of miles by 1,760.
To reduce yards to miles, divide the number of yards by 1,760.
To reduce rods to yards, multiply the number of rods by $5\frac{1}{2}$.
To reduce yards to rods, divide the number of yards by $5\frac{1}{2}$.
To reduce rods to feet, multiply the number of rods by $16\frac{1}{2}$.
To reduce feet to rods, divide the number of feet by $16\frac{1}{2}$.
To reduce yards to feet, multiply the number of yards by 3.
To reduce feet to yards, divide the number of feet by 3.
To reduce yards to inches, multiply the number of yards by 36.
To reduce inches to yards, divide the number of inches by 36.
To reduce feet to inches, multiply the number of feet by 12.
To reduce inches to feet, divide the number of inches by 12.

APPLICATION TO A SHOP PROBLEM

Two common terms used in carpentry work are "riser" and "tread." These terms apply to stair building and are illustrated in the following diagram.



A common size tread is 10 in. A suitable riser for this 7 in. Long flights of stairs as a rule have a shorter riser with a longer tread.

To determine the number of risers for a given flight of stairs, the height of the stairs, or the total rise, is divided by the width of the riser. In the same manner the width of the riser may be found by dividing the total rise by the number of risers desired.

Calculations involving these terms are similar to the following:

Example 1:

Determine the number of risers needed for a flight of stairs where the total rise is $10\frac{1}{2}$ ft. The riser recommended is 7 in.

Solution and Explanation:

This problem would be worked out by reducing the total rise $10\frac{1}{2}$ ft. to inches and then dividing that by 7 in., the width of the riser.

$10\frac{1}{2}$ feet reduced to inches equals $10\frac{1}{2} \times 12$, or 126 inches.

Dividing this amount by 7 there results: $126 \div 7$, which equals 18.

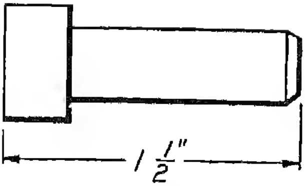
That is, 18 risers are needed for the above flight of stairs.

A further application is found in calculations that involve determining how many parts of a specified length may be made from a piece of stock of a given length; or in determining how many blanks of a certain size may be stamped from a strip of metal of a known length. Where waste, due to thickness of saw cut, or to the space between the stamped parts, or to the finishing up of a piece, has to be considered, such waste should be added to the length of the part in working out the calculation.

This same type of calculation may also be used in determining the length of stock needed in making a specified number of parts of a given size.

It will aid the student in working out these calculations if he will make a simple sketch illustrating how the various details enter into the solution of the problem.

This type of calculation is illustrated in the following problem.



Example 2:

How many pins like that shown can be made from a piece of brass that measures 3 ft. 8 in. long?

There is a waste of $\frac{1}{4}$ in. due to cutting off and finishing up each pin.

Solution and Explanation:

Problems like this are solved by first changing the length of the piece of brass into inches. After this is done, it is determined how many times the length of this pin *plus* the waste on each pin, are contained in the total length of the rod.

If the result of this division is a mixed number, only the *whole* number in this is used and the fractional part is disregarded. The whole number gives the number of full-length pieces.

As the first step in the above problem, 3 ft. 8 in. changed to inches becomes, 3×12 or 36 in., plus 8 in. which makes 44 in.

The length of each pin, *plus* the waste on each pin due to cutting off and finishing up equals:

$$1\frac{1}{2} \text{ in.} + \frac{1}{4} \text{ in., or } 1\frac{3}{4} \text{ in.}$$

The remaining part of the problem is to determine how many times the length $1\frac{3}{4}$ in. is contained in the length 44 in.

This is worked out as follows:

$$44 \div 1\frac{3}{4}, \text{ or } 44 \times \frac{4}{7}, \text{ which equals } \frac{176}{7}, \text{ or } 25\frac{1}{7}.$$

That is 25 full-length pins according to the above dimensions can be made from a piece of brass rod that measures 3 ft. 8 in.

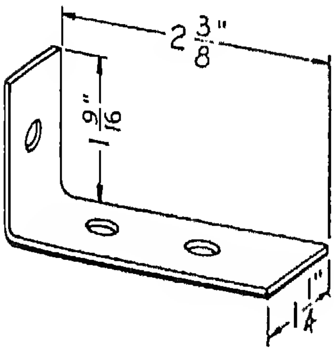
Problems Involving Reduction of Measurements

1. A ball of twine is advertised as containing "twine enough to extend $\frac{1}{8}$ of a mile." Of how many inches is this the equivalent?

2. To measure the size of the top of a table, a boy uses a yardstick and finds that it is $1\frac{1}{4}$ yd. long and $\frac{3}{4}$ of a yard wide. What would these measurements be if expressed in inches?

3. What is the cost of wire fence material needed to enclose a plot of land which measures $\frac{1}{4}$ of a mile around? This fencing sells at 38¢ per running rod.

4. In laying out measurements for a concrete sidewalk a young man used a stick which is exactly $\frac{3}{4}$ of a yard long. He finds that the length of the walk is equal to 28 lengths of the measuring stick, and the width is equal to exactly 2 lengths of the measuring stick. What are the dimensions of this walk in feet?



5. A thin brass strip 4 ft. long, is used in making 10 small angle braces as per the sketch to the left. The stock is first cut into smaller strips each long enough for one brace and then bent to the shape as shown in the following drawing. After cutting enough pieces for 10 braces, what will be the length in inches of the strip that remains?

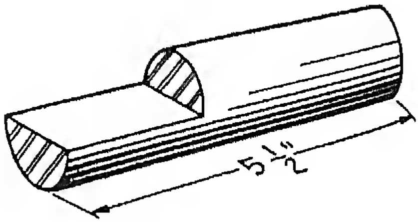
6. A plumbing apprentice is given a piece of sheet metal 2 ft. 3 in. long and told to cut it up into 12 equal lengths. Neglecting the width of the saw cut, how many inches long should each piece be?

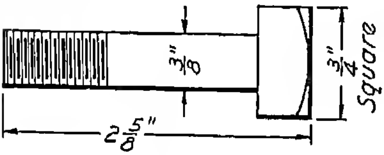
7. To find the distance around a cylindrical tank, a boy wraps a measuring tape around the outside, marking where the ends meet. He finds that the measurement is exactly 51 in. How many feet is this equal to?

8. A strip of metal used in making a safety guard on a disk grinder measures 28 in. long. What is the equivalent number of feet in this strip?

9. How many pine boards each 9 in. wide by 6 ft. long are needed to cover an opening in a floor that measures $4\frac{1}{2}$ ft. wide and 6 ft. long?

10. How many pieces like that in the drawing to the right can be cut from a bar of stock 3 ft. 9 in. long? Allow $\frac{1}{8}$-in. waste for cutting off each piece.



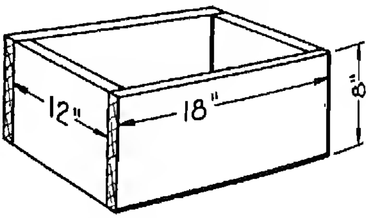


11. The bolt to the left is to be made from a bar of $\frac{3}{4}$-in. square stock $8\frac{1}{4}$ ft. long. If $\frac{1}{8}$ in. is to be allowed for waste due to cutting off each bolt, how many such bolts can be made from this piece of stock?

12. Pieces measuring $9\frac{3}{4}$ in. long are to be cut from a board 8 ft. 10 in. long. Neglecting the width of the saw cut, how many full-length pieces can be cut from this board?

13. To provide shims for use in the garage, a mechanic selects a strip of thin brass $3\frac{1}{2}$ ft. long and clips off 14 pieces each $2\frac{1}{4}$ in. long. As there is no waste in this cutting, how much material is used for these shims, and what is the length in inches of the piece that remains?

14. The sides and ends for 2 boxes like that in the sketch to the right are to be cut from a board 8 in. wide and 12 ft. long. Adding $\frac{1}{8}$ in. for each piece cut, what is the total length in inches of the lumber used? What is the length in inches of the piece that remains?



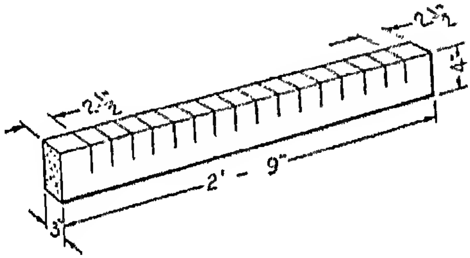
15. A partition being erected has 2 by 4-in. studding placed 16 in. on centers. The length of the partition is 17 ft. 6 in.

How many studs are needed for this, allowing one stud at each end of the partition?

16. Cellar stairs are to be installed in a house in which the top landing of the stairs is 5 ft. 3 in. above the cellar floor. If the riser is 7 in., how many risers are needed for these stairs?

17. An attic stairs is $8\frac{3}{4}$ ft. high and has 15 risers. Determine the size of the riser.

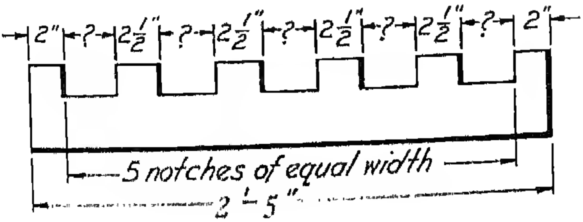
18. In the rack illustrated below 15 saw cuts are to be made as shown. Calculate the spacing of these cuts.



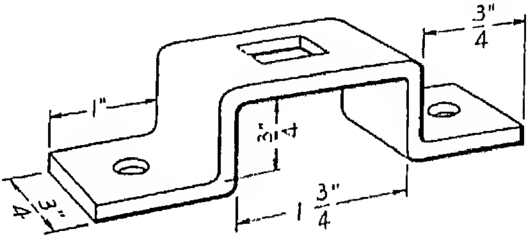
19. In laying out a flight of stairs to be erected in a small barn that has been converted into a garage, the carpenter finds that the distance from the garage floor to the attic floor is 7 ft. 6 in.

Using a $7\frac{1}{2}$-in. riser, how many such risers will be necessary for such a flight of stairs? If a 7-in. riser were used, what would be the length of the odd riser?

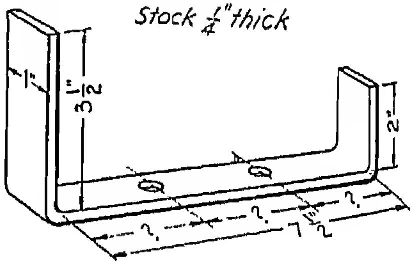
20. Determine the missing dimensions of the 5 equal notches in the following drawing.



21. How many braces as per the following dimensions can be made from a strip of flat $\frac{1}{8}$-in. stock 14 ft. 8 in. long? As these pieces are sheered off in a cutter there is no waste of material.

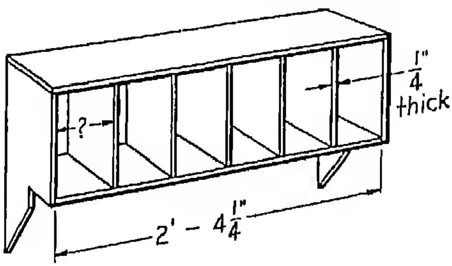


22. What is the length of material needed for 2 iron pieces bent up as per drawing below? Calculate the spacing of the two holes indicated. The centers of these holes lie the same distance from the uprights as from each other. How many such pieces may be cut from a strip that measures 5 ft. 10 in. long?

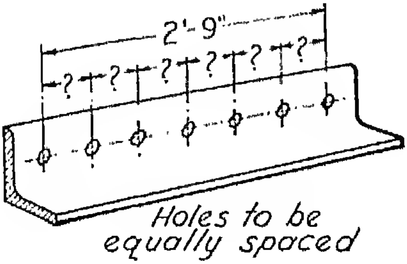


23. Square steel rods measuring $\frac{5}{16}$ in. on the side weigh $\frac{1}{3}$ lb. per linear foot. Determine the total length in feet and inches of a bundle of such rods weighing $23\frac{3}{4}$ lb.

24. Calculate the width of the openings in the set of "pigeon-holes" shown. The spaces are of equal width.

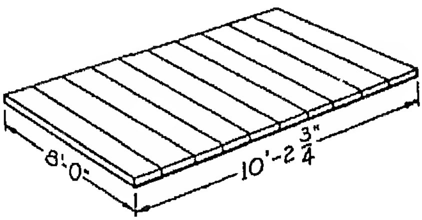


25. According to the following sketch of an angle plate, there are to be drilled seven holes spaced equally distant apart. Determine the length of the center distances as indicated by the question marks.



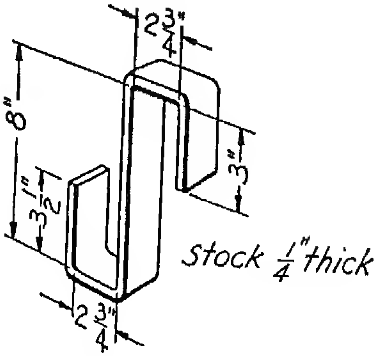
26. How many risers are needed in building a flight of stairs which connects two floors that are 10 ft. apart? A $7\frac{1}{2}$-in. riser is recommended. Make a sketch showing a layout of these risers.

27. How many boards each 10 in. wide are needed to build a wooden platform 8 ft. wide and 10 ft. $2\frac{3}{4}$ in. long? The boards are laid in the direction of the width of the platform as shown, and are placed $\frac{1}{4}$ in. apart.

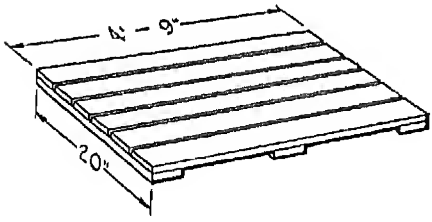


28. A ceiling 12 ft. long and 10 ft. wide is to be papered. The strips are to be put on so that they run in the direction of the 10-ft. width. If each strip is 18 in. wide, how many are used?

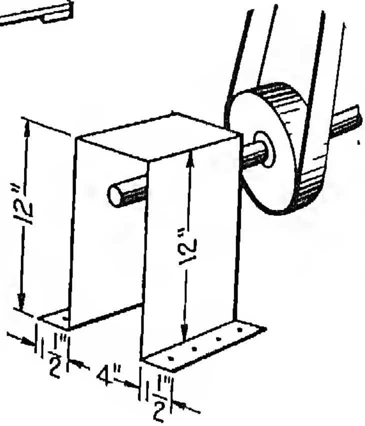
29. How many feet of wrought iron are needed to make 8 brackets according to the following dimensions?



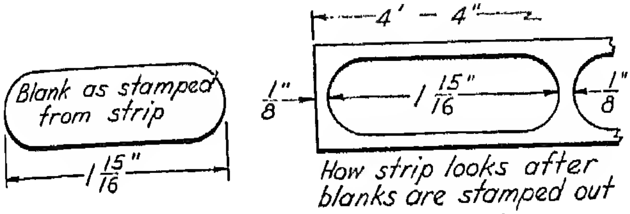
30. In constructing the special platform as shown in the following drawing, the strips are to be $2\frac{1}{2}$ in. wide. Three cross strips of this same thickness and width are used on the bottom. What is the total length of stripping used allowing 2 ft. for the cutting waste?



31. Six sheet-iron safety guards as shown to the right are required to cover shafting as illustrated. What is the length in feet and inches of the material in each guard? What is the total length of material required for the 6 guards?



32. How many strips of sheet metal, each 4 ft. 4 in. long are needed in order to blank out on a foot press 200 pieces according to the dimensions in the following drawing? The first piece is blanked out $\frac{1}{8}$ in. from the end, and each succeeding blank is to lie $\frac{1}{8}$ in. from the previous blank as indicated.*



*Answers to these problems will be found on page 158.

ANSWERS TO PROBLEMS

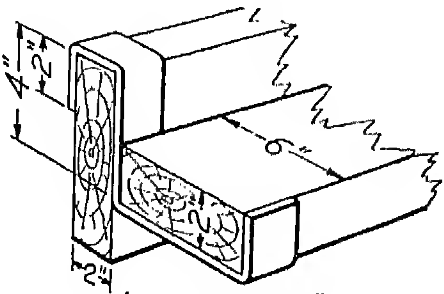
Pages 150 to 157.

1. 7,920 in.
2. 45 in.; 27 in.
3. $30.40.
4. 63 ft.; $4\frac{1}{2}$ ft.
5. $8\frac{5}{8}$ in.
6. $2\frac{1}{4}$ in.
7. $4\frac{1}{4}$ ft.
8. $2\frac{1}{3}$ ft.
9. 6 boards.
10. 8 pieces.
11. 36 bolts.
12. 10 pieces.
13. $10\frac{1}{2}$ in.
14. 121 in.; 23 in.
15. 14 studs.
16. 9 risers.
17. 7 in.
18. 2 in.
19. 12 risers; 6 in. odd riser.
20. 3 in.
21. 33 pieces.
22. $20\frac{1}{2}$ in.; $2\frac{1}{2}$ in.; 5 pieces.
23. 71 ft. 3 in.
24. $4\frac{1}{2}$ in.
25. $5\frac{1}{2}$ in.
26. 16 risers.
27. 12 boards.
28. 8 strips.
29. 13 ft. 8 in.
30. 35 ft. 6 in.
31. 2 ft. 7 in.; 15 ft. 6 in.
32. 8 strips.

Review Problems Involving Reduction of Measurements

1. Change 6 yd. 2 ft. 5 in. to inches. How many feet in $4\frac{1}{2}$ rods? Change 198 inches to yards. How many yards long is a carpet strip that measures $16\frac{1}{2}$ ft. long?

2. What is the length of a bar of iron stock needed to make 6 brackets like that illustrated in the following sketch?



Iron stock $\frac{1}{2}$" thick

3. A line of steam pipe which is $24\frac{1}{2}$ yd. long is to be supported by pipe hangers, the center lines of which lie $10\frac{1}{2}$ ft.

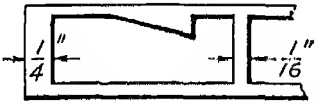
apart. Counting one hanger for each end of the line, how many hangers are needed for the entire length?

4. How many posts 9 ft. center distance apart, will be needed to enclose a piece of land, the perimeter of which measures 60 rods?

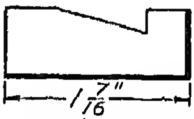
5. How many strips of wallpaper $\frac{1}{2}$ yd. wide will it take to paper the ceiling of a room measuring 10 ft. 6 in. wide and 19 ft. 6 in. long? These strips are to extend in the direction of the width of the ceiling.

6. A rack $9\frac{3}{4}$ ft. long is to have 15 hooks placed equally distant apart. The center lines of the hooks at each end are to lie $2\frac{1}{2}$ in. from the end. How far apart are the center lines of the other hooks? Draw a plan showing such an arrangement.

7. How many blanks of the following dimensions can be punched from a strip 4 ft. 8 in. long? A space of $\frac{1}{16}$ in. is allowed between each punching as shown. The first blank is started $\frac{1}{4}$ in. from the end of the strip as illustrated in the sketch below.



Showing how strip looks after blanks are punched

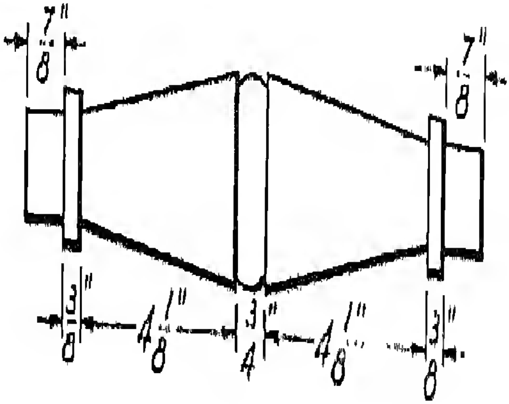


Blank as punched from strip

8. The distance between the first and second floor as shown on a set of house plans is 11 ft. 3 in. The building specifications state that the stairs to be erected between these floors are to have 18 risers. At this rate, what is the dimension of these risers?

9. What would be a suitable riser and tread in a stairs which extends from the floor to a balcony, a distance of $3\frac{1}{2}$ ft.? Draw a diagram showing the general dimensions.

10. To make 10 spindles as per specifications in the following drawing, how many linear feet of material are needed? No allowance is made for finishing up.



ADDITION OF UNITS OF LENGTH

Several measurements involving yards, feet, and inches may be added by reducing each measurement to the same denomination and then adding. This sum may then be changed to whatever unit is desired.

However there is a shorter and better method shown in the following example:

Example:

To replace a broken belt on a woodworking machine it is found that three pieces when fastened together will just make the required length of belt needed. These three pieces of belt measure 6 ft. 10 in., 8 ft. 11 in., and 7 ft. 9 in. What will be the total length of the belt made from these pieces?

Solution and Explanation:

To use the *first* method explained above, each length is reduced to its equivalent number of inches. After finding the total number of inches in the three pieces, this amount is in turn divided by 12 to change it to its equivalent number of feet and inches, as worked out below:

6 ft. 10 in. changed to inches equals 82 in.
8 ft. 11 in. changed to inches equals 107 in.
7 ft. 9 in. changed to inches equals 93 in.

The sum of 82 in., 107 in., and 93 in., equals 282 in.; 282 in. reduced to its equivalent number of feet equals 282 ÷ 12, which works out as,

$$\begin{array}{r} 12)282(23\tfrac{6}{12}, \text{ or } 23\tfrac{1}{2} \text{ ft.} \\ \underline{244\phantom{(23\tfrac{6}{12}, \text{ or } 23\tfrac{1}{2} \text{ ft.}}} \\ 42\phantom{(23\tfrac{6}{12}, \text{ or } 23\tfrac{1}{2} \text{ ft.}} \\ \underline{36\phantom{(23\tfrac{6}{12}, \text{ or } 23\tfrac{1}{2} \text{ ft.}}} \\ 6\phantom{(23\tfrac{6}{12}, \text{ or } 23\tfrac{1}{2} \text{ ft.}} \end{array}$$

That is, the sum of the three lengths of belting is $23\frac{1}{2}$ ft.

By the *shorter method* of calculation, the sum of these three lengths of belting would be found as explained as follows:

The quantities are arranged so that like units of length are in columns, feet under feet, and inches under inches, as shown. Each column is then added separately, the sum being placed below the line as shown.

6 ft.	10 in.
8 ft.	11 in.
7 ft.	9 in.
21 ft.	30 in.

The sum of these three measurements is therefore 21 ft. 30 in. But 30 in. is over one foot and should be changed to its equivalent number of feet and inches by dividing it by 12. This division gives $2\frac{1}{2}$ ft.

The 21 ft. and the $2\frac{1}{2}$ ft. added together equal $23\frac{1}{2}$ ft.

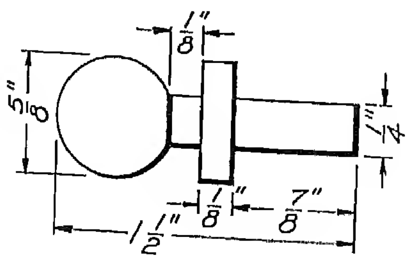
This measurement agrees with the one calculated by the first method, but is obtained by a much shorter process.

Problems Relating to the Addition of Units of Linear Measure

1. Add 9 ft. 7 in., 4 ft. $2\frac{1}{2}$ in., 14 in., and 2 ft. 8 in. Find the sum of 6 ft. 2 in., 4 ft. 10 in., 4 ft. $8\frac{1}{4}$ in., $9\frac{1}{2}$ in., and $17\frac{1}{4}$ in.

2. In checking over a stock of round steel bars, 6 pieces are found which measure 5 ft. 9 in., $2\frac{1}{2}$ ft., 6 ft. 4 in., 4 ft., 9 ft. 7 in., and 3 ft. 3 in. Give the combined length of these pieces.

3. To complete the job of making 110 pins like that in the following drawing, 3 pieces of stock are used which measure 4 ft. 10 in., 6 ft. 8 in., and 3 ft. 9 in. What is the total length of stock used on this job? How many inches were wasted?



4. To build a bookshelf for his room, a young man buys 3 boards each measuring in length, 8 ft. 9 in., 6 ft. 8 in., and 4 ft. 6 in. What is the total length of this material?

5. In order to construct a guard which would protect the machine operator from a moving belt, 3 strips of angle iron were necessary. These strips measured 3 ft. 10 in., 4 ft. 9 in., and 2 ft. 6 in. What is the total length of the angle iron used for this guard?

6. The distance around a three-sided plot of ground measures 18 yd. $2\frac{1}{2}$ ft. on one side, 24 yd. 1 ft. on another side, and 17 yd. 2 ft. on the third side. What is the total distance in feet?

7. In selecting the border for a small room a paper hanger takes the following measurements:

1 side measures	4 yd. 2 ft. 10 in.
1 side (deducting two windows) measures	3 yd. 0 ft. 5 in.
1 side (deducting vent opening) measures	4 yd. 1 ft. 0 in.
1 side (deducting a door) measures	3 yd. 1 ft. 9 in.

How many yards of border does he need for this job?

8. To extend a line of shafting which measures 18 ft. 10 in., a piece measuring 7 ft. $10\frac{1}{2}$ in. is joined to it by means of a flange coupling. What is the total length of the line after this addition?

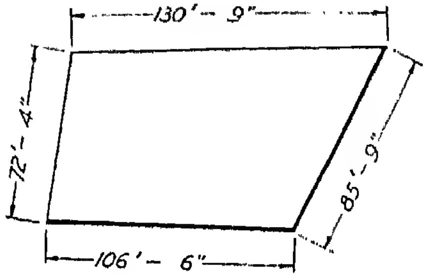
9. In measuring a room to determine the number of feet of picture molding needed, one side is found to need 14 ft. 2 in.; another 10 ft. 3 in.; another $6\frac{1}{2}$ ft.; another 9 ft. 1 in. Allowing 10% for waste, what is the total number of feet of molding necessary?

10. A steam pipe line is made up of four lengths of piping. One length measures 6 ft. 8 in.; another 5 ft. 11 in.; and another 8 ft. 0 in., while the fourth length measures 6 ft. 10 in. What is the total length of the "run"?

11. During a repair job on the wiring of a house, an electrician used from a 50-ft. coil of copper wire, three lengths of

wire. One length measured 16 ft. 9 in.; another 12 ft. 6 in.; and the third length 9 ft. 6 in. What is the combined length of these pieces?

12. What is the distance around the four-sided plot of land in the following drawing?



13. To blank out 85 pieces on a foot press 4 strips each measuring $2\frac{1}{2}$ ft., 3 ft. 8 in., 2 ft. 10 in., and 4 ft. 3 in. are used. What is the total length of the strips used?

14. A machine-shop inventory shows that there are three bars of steel on hand that measure 8 ft. 9 in., 11 ft. 8 in., and 9 ft. 7 in. each. If this steel weighs $3\frac{3}{8}$ lb. per foot length, what is the total weight of this material?

15. Determine the total length of the three timbers, as listed, when they are fastened end to end. These pieces measure 10 ft. 8 in., 11 ft. 10 in., and 8 ft. 9 in. each.

16. In checking up the amount of 1-in. strip brass in the shop stock room the stock-room boy finds that there are 6 coils of brass that measure as follows: 4 ft. 8 in., 5 ft. 4 in., 7 ft., 6 ft. 11 in., 3 ft. 7 in., 5 ft. 9 in. What is the combined length of these 6 strips?

17. Four pieces of single thickness belting 2 in. wide measure 3 ft. 7 in., 21 in., 5 ft. 9 in., and $7\frac{1}{2}$ ft. each. These are fastened to make a temporary belt. What is their combined length?

18. Two lengths of lead pipe were used on a plumbing job as follows. One piece measured 4 ft. 10 in. long, while the other piece measured 3 ft. 8 in. long. Calculate the weight of this pipe if it is listed at 2 lb. per foot of length.*

SUBTRACTING UNITS OF DIFFERENT DENOMINATIONS

To subtract measurements involving units of two denominations, the numbers are first arranged in columns the same as in addition. The process of subtracting is then carried out by first subtracting the numbers of the lower denominations. After this, the numbers of the next higher denomination are subtracted.

How this is done is shown in the following practical problems.

Example 1:

From a coil of lead pipe measuring 12 ft. 9 in. long, pieces amounting to 8 ft. 6 in. are used during one week's time. How many feet remained in this coil at the end of this period?

Solution and Explanation:

The number of feet remaining is equal to the difference between the original length and the amount that was used. This difference is determined by subtraction as follows:

The units are arranged as above explained and are subtracted separately as shown. The results are placed in their proper columns as in addition.

$$\begin{array}{r} 12 \text{ ft. } 9 \text{ in.} \\ 8 \text{ ft. } 6 \text{ in.} \\ \hline 4 \text{ ft. } 3 \text{ in.} \end{array} = \text{difference}$$

That is, the length of the lead pipe remaining in the coil is 4 ft. 3 in. However, this same result could have been obtained by reducing the dimensions to inches and then subtracting. By changing this difference back to feet and inches, the result would be equal to that obtained by the shorter method.

*Answers to these problems will be found on page 169.

Example 2:

Three lengths each measuring 5 ft. 9 in., 2 ft. 8 in., and 3 ft. 6 in. are cut from a piece of drill rod 14 ft. 6 in. long. How much of the drill rod remains?

Solution and Explanation:

The combined length of the pieces used is found by adding the separate lengths as previously explained. This equals:

$$\begin{array}{r r}
5 \text{ ft.} & 9 \text{ in.} \\
2 \text{ ft.} & 8 \text{ in.} \\
3 \text{ ft.} & 6 \text{ in.} \\
\hline
10 \text{ ft.} & 23 \text{ in.}
\end{array}$$

As 23 in. equals 1 ft. 11 in., then 10 ft. 23 in. becomes 11 ft. 11 in., which is the total amount cut from the full-length rod.

Since this amount is cut from a rod 14 ft. 6 in. long then there remains left an amount which is equal to the difference between the original length 14 ft. 6 in. and that which was cut off. In this subtraction the units are arranged as in the previous problem and the subtraction begins in the lower unit, inches. This works out as follows:

$$\begin{array}{r r}
13 & 18 \\
\cancel{14} \text{ ft.} & \cancel{6} \text{ in.} \\
11 \text{ ft.} & 11 \text{ in.} \\
\hline
2 \text{ ft.} & 7 \text{ in.}
\end{array}$$

As shown 11 in. cannot be subtracted from 6 in., so 1 unit is taken from the next higher denomination "feet," and added to the number of inches. This 1 unit, 1 foot, equals 12 in., so 12 in. is added to the 6 in. which changes that item to 18 in. This enables the subtraction of the lower units to take place. At the same time it reduces 14 ft. to 13 ft.

The result of this subtraction, as shown, is 2 ft. 7 in.

That is, the length of the drill rod that remains after cutting off the pieces listed equals 2 ft. 7 in.

Problems Involving Subtraction of Units of Different Denominations

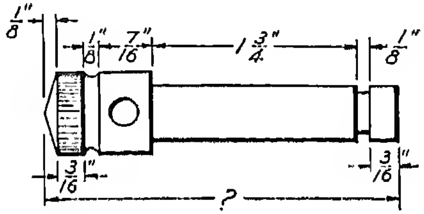
1. Subtract 9 ft. 10 in. from 14 ft. 2 in.

   Subtract 5 ft. $3\frac{1}{2}$ in. from 12 ft. 4 in.

   Subtract 11 ft. 9 in. from 18 ft. 5 in.

2. How many feet of window-screen wire remain in a 25-ft. roll after using pieces totaling 18 ft. 4 in. for repairs to house window screens?

3. Determine the number of feet of $\frac{1}{2}$-in. stock needed in making 26 pieces as per the following dimensions.



In cutting these off and finishing them to size there is a waste of $\frac{3}{16}$ in. on each piece. At this rate how many feet of stock should remain in a 9-ft. bar from which these 26 pieces are made?

4. To build a wooden shelf, 2 pieces of pine each 3 ft. 10 in. long are cut from a board that measures 12 ft. 6 in. long. Neglecting the width of the saw cut, what is the length of the piece that remains?

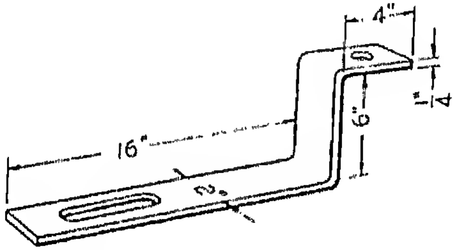
5. Two pieces of belting, one 5 ft. 9 in. long and the other 11 ft. 10 in. long are to be joined together with a third piece to make a belt that is to measure 22 ft. 4 in. long. What must be the length of the third piece needed to make up the required length?

6. A room which measures 12 ft. 9 in. wide by 18 ft. long, has 2 windows on one side extending from 18 in. above the baseboard to the ceiling. The total width occupied by these

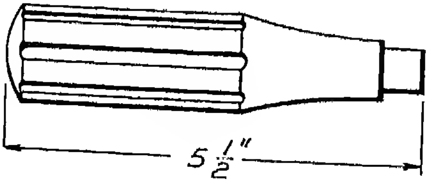
windows is 6 ft. 10 in. How many feet of picture molding are needed to go around this room?

7. In order to use a 10-in.-wide board for shelving, it is necessary to cut a piece 2 ft. 5 in. long off one end because of knotholes and cracks. The original length of the board was 14 ft. 3 in. What is the length of the piece that remains?

8. Four angular braces like the following are cut from a piece 12 ft. 4 in. long. What is the total amount of material used, allowing $\frac{1}{8}$ in. for cutting off each piece? How much remains?

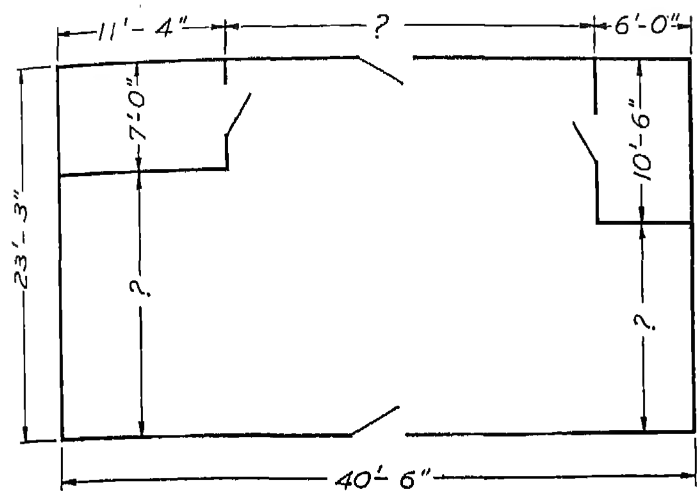


9. Stock for turning 12 screw-driver handles like those below is to be cut from a piece of maple 8 ft. 9 in. long. One inch is added to the length of each handle for finishing and cutting off in the lathe. What is the total amount in feet and inches used? What is the length in feet and inches of the piece that remains?



10. Determine the length of baseboard molding needed for a room that measures 10 ft. 9 in. wide by 12 ft. 6 in. long. In this room there are 2 doors, and a deduction of 3 ft. 4 in. is to be made for each of these.

11. On the following floor plan determine the length of the dimensions that are noted by the question marks.



12. A workbench measuring 18 ft. 0 in. long is to be shortened so as to measure 15 ft. 9 in. What should be the length of the portion that is to be cut off?*

ANSWERS TO PROBLEMS

Pages 162 to 165.

1. 17 ft. $7\frac{1}{2}$ in.; 17 ft. 11 in.
2. 31 ft. 5 in.
3. 15 ft. 3 in.; 1 ft. 6 in.
4. 19 ft. 11 in.
5. 11 ft. 1 in.
6. 182 ft. 6 in.
7. 16 yd.
8. 26 ft. $8\frac{1}{2}$ in.
9. 44 ft.
10. 27 ft. 5 in.
11. 38 ft. 9 in.
12. 395 ft. 4 in.
13. 13 ft. 3 in.
14. 108 lb.
15. 31 ft. 3 in.
16. 33 ft. 3 in.
17. 18 ft. 7 in.
18. 17 lb.

*Answers to these problems will be found on page 170.

Pages 167 to 169.

1. 4 ft. 4 in.; 7 ft. $\frac{1}{2}$ in.; 6 ft. 8 in.
2. 6 ft. 8 in.
3. 6 ft. $9\frac{1}{4}$ in.; 2 ft. $2\frac{3}{4}$ in.
4. 4 ft. 10 in.
5. 4 ft. 9 in.
6. 54 ft. 8 in.
7. 11 ft. 10 in.
8. 8 ft. $9\frac{1}{2}$ in.; 3 ft. $6\frac{1}{2}$ in.
9. 6 ft. 6 in.; 2 ft. 3 in.
10. 39 ft. 10 in.
11. 23 ft. 2 in.; 16 ft. 3 in.; 12 ft. 9 in.
12. 2 ft. 3 in.

Review Problems Involving Addition and Subtraction of Linea Measure

1. What is the sum of the following measurements:

2 yd. 2 ft., 3 yd. 7 in., 6 ft., 1 ft. 8 in.

2. To make up an order of mahogany spindles, 3 pieces o mahogany were used. One piece measured 3 ft. long, anothe 2 ft. 5 in. long, and the third piece 3 ft. 10 in. long. What i the combined length of these three pieces?

3. An ornamental iron bracket requires the following 4 piece for its construction:

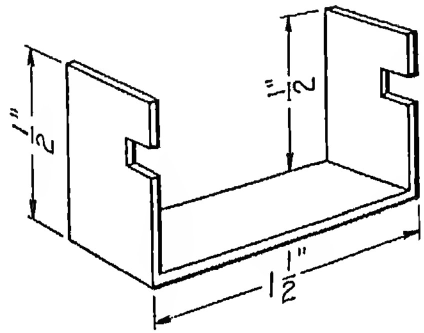
1 piece 2 ft. 6 in.
1 piece 2 ft. 8 in.
1 piece 3 ft. 5 in.
1 piece 1 ft. 11 in.

This stock weighs $1\frac{1}{2}$ lb. per foot of length. What is the approximate weight of this material?

4. In connection with his day's work a stock-room boy cuts from a steel bar 14 ft. 9 in. long, 3 pieces of stock measuring 2 ft. 9 in., 3 ft. 10 in., and 4 ft. 8 in. If this stock is listed as weighing 6 lb. per linear foot what should be the approximate weight of the piece that remains?

5. In constructing a small closet for home use a boy purchased 2 boards each 12 ft. long. After he has finished making the closet he found that he had one piece left that measured 1 ft. 10 in. long and another piece that measured 2 ft. 7 in. long. What is the length of lumber actually used in constructing this cabinet?

6. Two strips of thin copper are used in making 42 pieces according to the following drawing. One piece measured 5 ft. 10 in. long and the other 4 ft. 7 in. long. How much material was wasted in making these pieces?



7. The present line of shafting in a woodworking shop measures 11 ft. 8 in. long. This is to be extended, however, until the entire length is 19 ft. 4 in. How long a piece of shafting must be added to the present line to make up this difference?

8. Three pieces of leather belting, 30 ft. $6\frac{3}{4}$ in. long, 15 ft. $5\frac{3}{8}$ in. long, and 12 ft. $9\frac{13}{16}$ in. long respectively, have been cut from a roll of belting which originally contained 89 ft. $6\frac{1}{2}$ in. of belting. How much of the belting is left for future use?

9. From an original stock of 3 pieces of round tool steel, each 6 ft. in length, the following pieces were used: $6\frac{5}{8}$ in., $8\frac{1}{4}$ in., 2 ft. $2\frac{3}{16}$ in., $7\frac{7}{16}$ in. The dimensions given include the wastage due to the saw cut. How much of the original stock is left for future use?

MULTIPLYING UNITS OF TWO OR MORE DENOMINATIONS

Measurements involving units of two or more denominations may be multiplied by a short method somewhat resembling that in addition as explained on page 161. This method is illustrated in the following example.

Example 1:

What is the combined length of 7 strips of oak flooring each measuring 8 ft. 10 in. long?

Solution and Explanation:

If each strip measures 8 ft. 10 in. long, then 7 strips will measure 7 times that amount.

To multiply such measurements express the quantities as shown below and multiply each item separately. When this is done, change the item of the lowest denomination to its equivalent in the next higher denomination. This amount is then added to the unit of the higher denomination much the same as in the process of addition.

$$\begin{array}{r} 8 \text{ ft. } 10 \text{ in.} \\ \times 7 \\ \hline 56 \text{ ft. } 70 \text{ in.} \end{array}$$

Thus, 8 ft. 10 in. multiplied by 7 equals 56 ft. 70 in.

The amount 70 in., is then reduced to its equivalent in feet, which is 5 ft. 10 in. From this, the unit 5 ft. is added to the 56 ft. changing that amount to 56 ft. + 5 ft., or 61 ft. This changes 56 ft. 70 in. to its equivalent 61 ft. 10 in.

That is, the 7 strips above mentioned have a total length of 61 ft. 10 in.

This method of multiplying is used to advantage in calculating the amount of floor covering needed for a given surface.

To determine the amount of carpet, linoleum, or other floor covering needed to cover a given surface, the number of strips to be laid is first calculated. This number is multiplied by the length of each strip, the resulting product is then reduced to yards, giving the number of yards required.

Allowance should be made for the waste in matching the patterns.

This process of multiplying such units is applied in problems like the following.

Example 2:

A floor 18 ft. wide and 18 ft. 10 in. long is to be covered with linoleum. The linoleum strips which are one yard wide are to be laid in the direction of the length of the room. What is the total number of yards needed to cover the floor? No allowance is to be made for waste or matching.

Solution and Explanation:

Since the room is 18 ft. wide, then the number of strips needed to extend across the floor would be found by reducing 18 ft. to yards.

This becomes:

$$18 \div 3, \text{ or } 6$$

Since each strip is one yard wide, 6 strips are needed. 6 strips each 18 ft. 10 in. long makes the total amount needed.

$$\begin{array}{r} 18 \text{ ft. } 10 \text{ in.} \\ \times 6 \\ \hline 108 \text{ ft. } 60 \text{ in.} \end{array}$$

Changing 60 in. to feet equals 5 ft. This changes 108 ft. 60 in. to its equivalent of 113 ft. Since there are 3 ft. in 1 yd., then in 113 ft. there are:

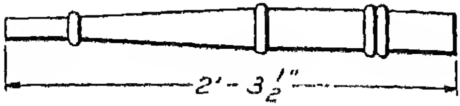
$$113 \div 3, \text{ or } 37\tfrac{2}{3}$$

That is, $37\frac{2}{3}$ linear yards will exactly cover the above floor, no allowance being made for waste or matching.

Problems Involving Multiplication of Units of Two or More Denominations

1. To fence in a small garden 6 lengths of wire fence are used, each measuring 15 yd. 2 ft. long. What is the total length of fence material required for this?

2. An order to a woodworking shop calls for 10 table legs like the following. Adding $1\frac{1}{4}$ in. to the length of each leg for waste, what is the total length of stock needed for this order?



3. How many linear yards of carpet are needed to cover a room 12 ft. wide by 20 ft. 3 in. long? This carpet is 1 yd. wide and is laid in the direction of the length of the room. No allowance is made for matching.

4. To finish a certain job on a machine there are used 7 bars of $1\frac{1}{4}$-in. round steel, each bar being 10 ft. 4 in. long. What is the total length of all this material?

5. A storm door 6 ft. 10 in. high is constructed by using 5 boards each 6 in. wide and 6 ft. 10 in. long. What is the total number of linear feet in these boards as used in making this door?

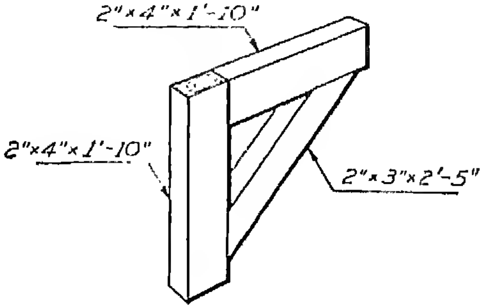
6. To build a steam-heating coil 5 lengths of pipe each measuring 8 ft. 9 in. long are used. What is the total length of piping used in making this heating coil?

7. What is the cost of the linoleum necessary to cover an office floor which measures 12 ft. wide by 19 ft. 6 in. long? This linoleum sells at $2.25 per yard and comes 36 in. wide. It is to be laid in the direction of the longest dimension.

8. How many linear yards of carpet 27 in. wide are needed to cover a floor that measures 13 ft. 6 in. wide by 16 ft. 6 in. long, the carpet being laid in the direction of the length of the room. If the price of this carpet is $1.60 per linear yard, what is the total cost of the carpet needed?

9. In constructing a brace like the one in the following sketch, there are used 2 pieces of 2 by 4 pine each measuring 1 ft. 10 in. long, and 1 piece of 2 by 3 pine measuring 2 ft.

5 in. long. What is the total length of each size needed to make 6 such braces?



10. How many linear yards of linoleum are needed to cover a room 18 ft. wide by 20 ft. 6 in. long? The linoleum used is 1 yd. wide and is to be laid in the direction of the length of the room. No allowance is made for matching or fitting.

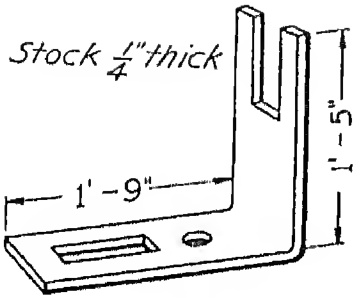
11. In constructing a board fence 120 boards each 6 ft. 5 in. long are needed. What is the total number of linear feet in this number of boards?

12. Three pieces of matting each 14 ft. 8 in. long are sewed together in one strip for a hall carpet. Deducting an allowance of 6 in. for sewing, what is the total length of this strip?

13. Fifty-four posts each 9 ft. 3 in. on center are used in building a fence enclosing a plot of ground. What is the distance around this plot?

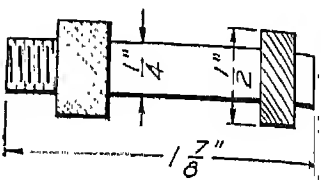
14. How many yards of carpet 27 in. wide are needed to cover a floor 9 ft. wide by 26 ft. 3 in. long. This carpet is to be laid in the direction of the length of the room. If the price of the carpet is $1.25 per linear yard, what is the total cost of the carpet needed for this room?

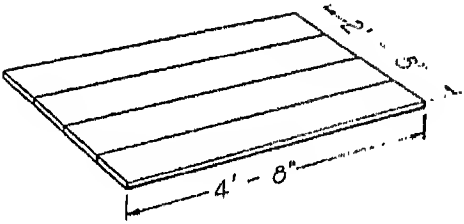
15. To construct a board fence, a carpenter estimated that he would use 86 boards each 4 ft. 9 in. long. How many linear feet does this equal?



16. Determine the total length of flat stock needed to construct 4 angular braces according to the drawing to the left.

17. To turn out 392 pieces like those in the sketch to the right, 6 pieces of stock each 10 ft. 8 in. long are used. What is the total length of material used? What is the percentage of wastage?





18. What is the combined length of 8-in. boards needed for the hatchway cover with measurements as illustrated to the left?

19. In order to make a certain picture frame, 5 ft. 10 in. of picture-frame molding are needed. How many feet of this molding will be required to construct 8 such frames?

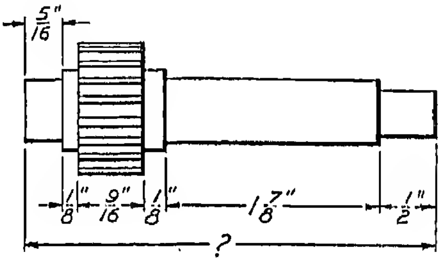
20. The stock-room boy in a machine shop finds in checking his materials that he has the following $\frac{1}{4}$-in. flat stock on hand:

3 pieces	1 in. wide each	6 ft. 8 in. long
5 pieces	$\frac{3}{4}$ in. wide each	8 ft. 5 in. long
12 pieces	$1\frac{1}{2}$ in. wide each	10 ft. 3 in. long

What is the total length of each size of stock?

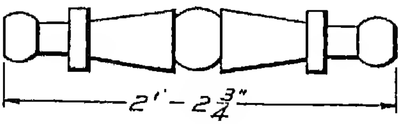
21. Four straps of band iron are used around a heavy wooden box containing machine parts labeled "For Export Shipment." Each strip measures 8 ft. 4 in. long. What is the total length of band iron used on this box?

22. To turn out an order of 110 brass-pinion shafts like the following, 3 pieces of brass stock each 11 ft. 9 in. long were used. What was the total length of stock used? If there was no spoilage or wastage due to cutting off each piece what was the amount of stock needed?



23. Thirty-six pieces of 4 by 4-in. oak each 2 ft. 8 in. long are needed for 9 tables. What is the total length in feet and inches represented by this number of pieces?

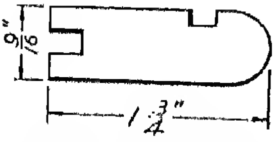
24. A cabinet-shop order calls for 6 pieces of mahogany turned as per the following dimensions.



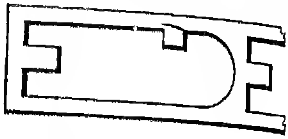
There is to be added to each piece $1\frac{1}{4}$ in. for finishing and cutting off. What is the total length of material required for this order?

25. In punching out an order of 112 blanks like the follow-

ing, 5 strips of sheet brass each 3 ft. 8 in. were used. Determin the total length of stripping used. How much was waste?



Blank as punched from strip



Waste strip after blanks are punched

26. The young man in the following picture used 10 strips of webbing each 2 ft. 3 in. long in reupholstering the chair as shown. How many yards of webbing is this equal to?

27. In preparation for the winters' storms a young man plans to put special weatherstrips on two doors of the house in which he lives.

With this in mind he obtains the following measurements: 2 top and 2 bottom strips each 2 ft. 10 in. long; and 4 side strips each 6 ft. 10 in. long. What is the total length of weather-strip required for these two doors?*

*Answers to these problems will be found on page 182.

DIVISION INVOLVING TWO OR MORE UNITS

The division of numbers involving two or more units is carried on by a short process in a manner similar to that used in multiplying linear units. How such calculations are worked out is illustrated in the following problem.

Example:

A space 27 ft. 9 in. long is to be marked off into 6 equal divisions. How long will each division be?

Solution and Explanation:

This problem might be solved by reducing 27 ft. 9 in. to inches and then dividing that amount by 6. The quotient could then be changed back to its equivalent in feet and inches. But the following process is shorter and convenient.

To lay off into 6 equal divisions the space which measures 27 ft. 9 in. long, is really to divide 27 ft. 9 in. by 6. This is expressed as (27 ft. 9 in.) ÷ 6, and works out as follows:

$$\begin{array}{r} \quad\;\; 3 \quad\quad\; 45 \\ 6\overline{)\cancel{27}\text{ ft.}\;\; \cancel{9}\text{ in.}} \\ \hline 4\text{ ft.}\;\; 7\tfrac{1}{2}\text{ in.} \end{array}$$

In this process of division 6 will go into 27 four times with 3 remaining. The number 27 is crossed out as noted and 3 is placed *above* it while the quotient 4 is placed *below* it. The remainder 3 is in feet, so before it can be carried over to the unit of the next lower denomination it must be changed to inches, and then *added* to the number of inches already expressed. Three feet equals 36 in. This, added to the 9 inches changes it to 45 in. This is the *new* dividend and it is placed *above* the 9 in. which is now crossed out.

Six will then go into 45, seven and one-half times. This gives the complete division as 4 ft. $7\frac{1}{2}$ in.

That is, each of the 6 divisions as marked off on the space 27 ft. 9 in. will be equal to 4 ft. $7\frac{1}{2}$ in.

The correctness of this answer may be determined by checking it with the method referred to at the beginning of this problem.

Problems Involving Division of Two or More Denominations

1. A line of shafting 23 ft. 3 in. long is to have 4 hangers equally spaced throughout its length, one being placed at each end of the line. How far apart will they be located?

2. To lay out a stenciled border on the ceiling of a room requires that the length of the ceiling, which is 18 ft. 4 in., be divided into 8 equal parts in order that the units of the design may be properly spaced. What is the length of each of the 8 divisions?

3. A shelf support 18 ft. 8 in. long is to be divided up into 14 equal spaces for hat hooks. How far apart will these hooks be?

4. A pipe line 41 ft. 9 in. long is made up of 7 lengths, one of which is 5 ft. The remaining six are all equal in length. What is the length of these 6 pipes?

5. If it takes 278 ft. 4 in. of wire fencing to enclose a square plot of ground, how much is used for each side of this plot?

6. A plank 10 ft. $4\frac{1}{2}$ in. long is to be cut up into 3 equal parts. Neglecting the width of the saw cut, how long should each piece be?

7. A strip of angle iron 10 ft. 9 in. long is cut into 6 equal pieces to be used as braces. How long will each be, no allowance being made for the width of the saw cut?

8. A rack 17 ft. 9 in. long has 12 hooks spaced equal distances apart. The hooks on either end, however, lie 2 in. from each end of the rack. Show by diagram how far apart the other hooks are placed.

9. Seven posts are needed for a wire fence 49 ft. 9 in. long. If these are to be equally spaced, how far apart will they be? Draw a diagram illustrating their positions, indicating the distance they are apart.

10. To provide material for a certain job, an apprentice is told to cut into 6 equal parts a bar of flat brass stock which

measures 13 ft. 9 in. long. What is the length in feet and inches of each piece as measured off before the cutting?

11. Eight equal lengths of steam pipe are joined together in making a pipe line 61 ft. 4 in. long. What is the length of each section?

12. A bar of iron 11 ft. 5 in. long is to be cut up into 5 equal lengths. The saw cut is $\frac{1}{16}$ in. wide. Make a drawing showing how such a bar would be measured off. What is the length of each piece as cut?

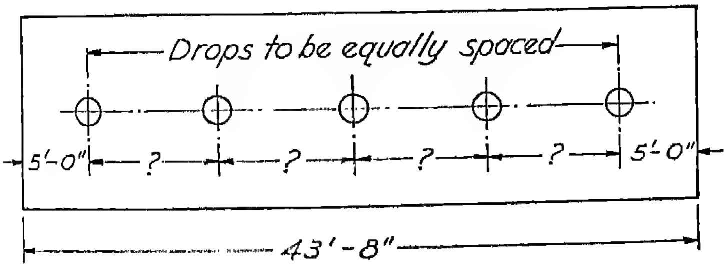
13. A sheet of galvanized iron measuring 6 ft. 8 in. long is to be cut into 5 equal pieces. How wide will each strip be?

14. Three metal bands, or hoops, are made by cutting a strip of $\frac{3}{4}$-in. band iron measuring 14 ft. 9 in. long into 3 equal lengths. Allowing $1\frac{1}{2}$ in. for making the lap joint, what is the diameter of each hoop?

15. A corridor 72 ft. 6 in. long has 6 droplights equally spaced. The end droplights, however, are 5 ft. from each end of the corridor. What should be the distance between the drops?

16. A pine board 15 ft. 4 in. long has a piece 8 in. long cut off one end because of knots and cracks. The remainder of the board is to be cut up into 8 equal lengths. Neglecting the width of the saw cut, what would be the length of each piece?

17. How far apart should the droplights be placed in the following diagram of a hall ceiling? Drops to be equally spaced.



18. A workbench 22 ft. 6 in. long is to have 4 metal bench legs spaced equal distances apart. The center lines of the end legs are to lie 1 ft. from each end. How far apart are the center lines of the other legs?*

ANSWERS TO PROBLEMS

Pages 173 to 178.

1. 94 yd.
2. 24 ft. 2 in.
3. 27 yd.
4. 72 ft. 4 in.
5. 34 ft. 2 in.
6. 43 ft. 9 in.
7. $58.50.
8. $52.80.
9. 22 ft. of 2 by 4;
   14 ft. 6 in. of 2 by 3.
10. 41 yd.
11. 770 ft.
12. 43 ft. 6 in.
13. 499 ft. 6 in.
14. 35 yd.; $43.75.
15. 408 ft. 6 in.
16. 12 ft. $8\frac{1}{2}$ in.
17. 64 ft.; 4.3%.
18. 18 ft. 8 in.
19. 46 ft. 8 in.
20. 20 ft.; 42 ft. 1 in.; 123 ft.
21. 33 ft. 4 in.
22. 35 ft. 3 in.; 32 ft. 1 in.
23. 96 ft.
24. 14 ft.
25. 18 ft. 4 in.; 2 ft. wasted.
26. $7\frac{1}{2}$ yd.
27. 38 ft. 8 in.

Pages 180 to 182.

1. 7 ft. 9 in.
2. 2 ft. $3\frac{1}{2}$ in.
3. 1 ft. 4 in.
4. 6 ft. $1\frac{1}{2}$ in.
5. 69 ft. 7 in.
6. 3 ft. $5\frac{1}{2}$ in.
7. 1 ft. $9\frac{1}{2}$ in.
8. 1 ft. 7 in.
9. 8 ft. $3\frac{1}{2}$ in.
10. 2 ft. $3\frac{1}{2}$ in.
11. 7 ft. 8 in.
12. 2 ft. $3\frac{1}{4}$ in.
13. 1 ft. 4 in.
14. 18.302 in. diameter.
15. 12 ft. 6 in.
16. 1 ft. 10 in.
17. 8 ft. 5 in.
18. 6 ft. 10 in.

Review Problems Involving Multiplication and Division of Linear Measurements

1. Six strips of linoleum each 20 ft. 9 in. long are required to cover an office floor. What is the total number of yards in these strips?

2. A maple board 14 ft. 10 in. long is to be used in making repairs to a workbench. The carpenter on this job finds he must

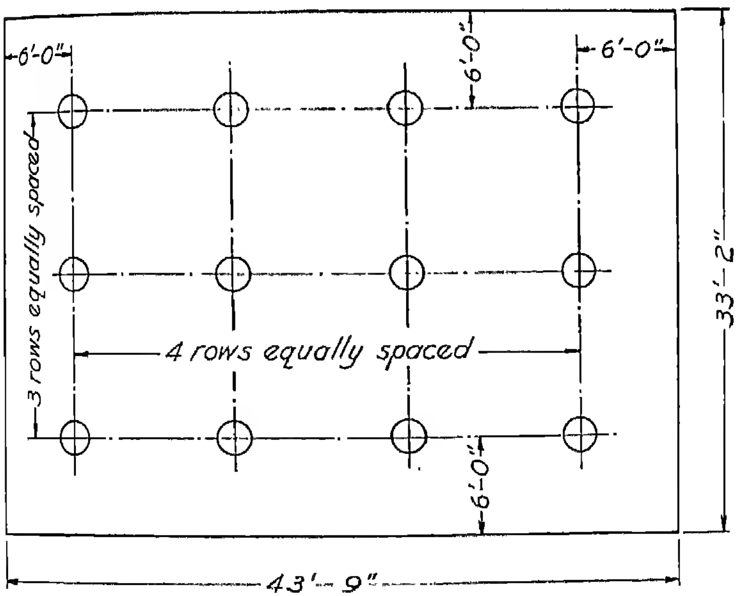
*Answers to these problems will be found on this page.

cut off from this board a piece 1 ft. 7 in. long because of cracks and knots. The part remaining is to be cut into 3 equal lengths. How far apart are the saw cuts? Draw a plan of this board showing the location of the sawing lines.

3. What is the length of a coil of wire which weighs 12 lb. if 18 ft. 4 in. of this wire weighs one pound?

4. A young man planning to put up shelving in his cellar, orders 3 boards each 10 in. wide and 12 ft. long; 2 pieces each 8 in. wide and 8 ft. 4 in. long; and 3 pieces each 6 in. wide and 9 ft. 6 in. long. Calculate the total number of linear feet of lumber used for this shelving.

5. Lay out on the following ceiling plan, the position of "droplights" according to the following specifications. There are to be three rows with 4 "drops" in each row. The outside "drops" are to be 6 ft. from the walls as shown.



6. Four lengths of steam pipe each 8 ft. 9 in. long are used in replacing a defective pipe line. What is the combined length of these pieces?

7. A copper tube 10 ft. 3 in. long is to be cut into 3 equal parts. Neglecting the width of the saw cut, how long would each piece be?

8. A boy who has the job of erecting a fence in his back yard finds that he has been allowed six fence posts. If the distance between the centers of the end posts is 41 ft. 8 in., what is the distance between the centers of the other posts?

BOARD MEASURE

Board feet, shop practice,
estimates, practical jobs.

Board Measure

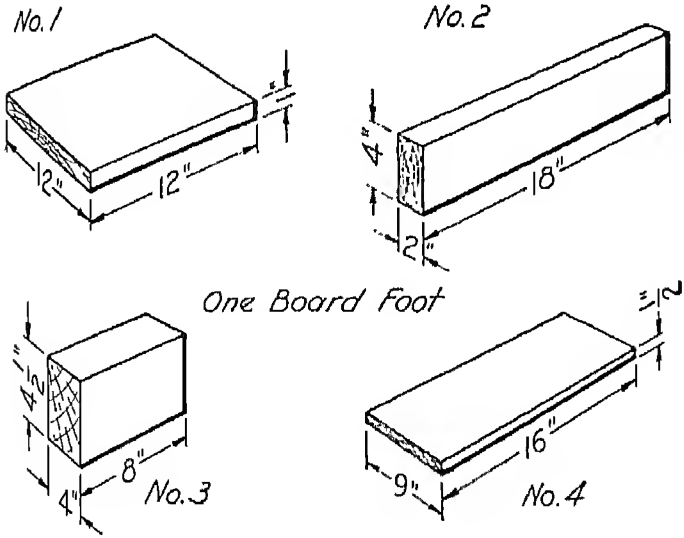
While units of linear measure are used in measuring *lengths* of lumber, the *amount*, or *quantity*, of this material is indicated by a particular unit called the *board foot.*

This unit is the measure of a board which is one foot long, one foot wide, and one inch thick. Such a board contains one board foot. This is abbreviated bd. ft., as in 36 bd. ft.

At times the expression "ft. B.M." is used instead of the expression "board measure," as 36 ft. B.M. Ft. B.M. is the abbreviation for feet board measure.

The number of board feet in a given piece of lumber is determined by multiplying the *length* in *feet*, by the *width* in *feet*, by the *thickness* in *inches*. If the thickness is less than 1 in., it is considered as equal to 1 in. in such calculation.

The following sketches may help in making clear the meaning of board measure.



In each of the previous pieces there is one board foot illustrated.

Board No. 1 is 1 ft. long, 1 ft. wide, and 1 in. thick. The product of these dimensions is 1. Therefore, there is 1 board foot in this piece of lumber.

In board No. 2, the length is 18 in., or $1\frac{1}{2}$ ft., and the width is 4 in., or $\frac{1}{3}$ ft. The thickness being 2 in., this amount is used in the following calculation as previously explained. The multiplication of the length by the width and by the thickness is worked out by cancellation as follows,

$$1\tfrac{1}{2} \times \tfrac{1}{3} \times 2 \text{ or } \frac{\cancel{3}}{\cancel{2}} \times \frac{1}{\cancel{3}} \times \frac{\cancel{2}}{1} = 1.$$

This indicates that in board No. 2 there is one board foot.

NOTE: Should the length or the width be given in inches, it is not necessary to change it directly into feet before using it in the calculation. Instead, such measurements may be expressed in fractional form by using 12 as a denominator. This will avoid the extra process of changing the measurements into inches.

Applying this rule to the above problem the length would be $\frac{18}{12}$ ft. and the width would be $\frac{4}{12}$ ft.

The solution of this problem would then become:

$$\frac{18}{12} \times \frac{4}{12} \times \frac{2}{1}, \text{ or } \frac{\overset{6}{\cancel{18}}}{\underset{3}{\cancel{12}}} \times \frac{4}{\underset{2}{\cancel{12}}} \times \frac{\cancel{2}}{1} = 1.$$

As seen, this gives the same result as the above.

In board No. 3 the length is 8 in., the width $4\frac{1}{2}$ in., and the thickness 4 in. The board-foot measure of this piece is calculated as suggested in the above note.

$$\frac{8}{12} \times \frac{4\frac{1}{2}}{12} \times \frac{4}{1}, \text{ or } \frac{\cancel{8}}{\underset{3}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{3}{\cancel{24}}} \times \frac{\cancel{4}}{1} = 1.$$

That is, there is 1 board foot in piece No. 3.

In piece No. 4, the length is 16 in., the width is 9 in., and the thickness is $\frac{1}{2}$ in. As previously explained, this thickness

being less than 1 in., it is considered as 1 in. in the calculation. The solution then becomes:

$$\frac{16}{12} \times \frac{9}{12} \times 1, \text{ or } \frac{\overset{4}{\cancel{16}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{3}{\cancel{12}}} \times 1 = 1.$$

In determining the board measure of lumber that is 1 in. or less in thickness, the figure 1 may be omitted from the calculation as its use does not effect the result. According to this the above solution would become:

$$\frac{\overset{4}{\cancel{16}}}{\underset{4}{\cancel{12}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{3}{\cancel{12}}} = 1.$$

This gives the same result as the above calculation.

This method of calculating board measure is used in such problems as the following.

Example 1:

How many board feet are there in a piece of lumber 14 ft. long, 9 in. wide, and $\frac{7}{8}$ in. thick?

Solution and Explanation:

As this piece is less than 1 in. thick the board-foot measure is determined as follows:

$$14 \times \frac{9}{12}, \text{ or } \frac{\overset{7}{\cancel{14}}}{1} \times \frac{\overset{3}{\cancel{9}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{12}}} = \frac{21}{2} \text{ or } 10\tfrac{1}{2}.$$

That is, there are $10\frac{1}{2}$ bd. ft. in the above piece of lumber.

Example 2:

Determine the number of board feet in 4 pieces of lumber each of which measures 30 in. × 6 in. × 2 in.

Solution and Explanation:

In this problem the thickness of the lumber is over 1 in., and the actual thickness, 2 in., is used in the calculation. The

length in inches, and the width in inches are changed to feet in the cancellation as shown.

$$\frac{30}{12} \times \frac{6}{12} \times 2, \text{ or } \frac{\overset{5}{\cancel{30}}}{\underset{6}{\cancel{12}}} \times \frac{\cancel{6}}{\underset{2}{\cancel{12}}} \times \frac{\cancel{2}}{1} = \frac{5}{2} \text{ or } 2\tfrac{1}{2}.$$

That is, in one of these boards there are $2\frac{1}{2}$ bd. ft. of lumber. In 4 boards there are $4 \times 2\frac{1}{2}$, or 10 bd. ft.

From a study of this problem it may be seen that the number of board feet in the 4 pieces of lumber might have been determined in one operation, by multiplying the length by the width, by the thickness, by the total number of pieces. The solution according to this method works out as:

$$\frac{\overset{10}{\cancel{30}}}{\underset{\cancel{3}}{\cancel{12}}} \times \frac{\cancel{6}}{\underset{\cancel{2}}{\cancel{12}}} \times \frac{\cancel{2}}{1} \times \frac{\cancel{4}}{1} = 10 \text{ board feet.}$$

This illustrates how the board measure of a given number of pieces of lumber of the same dimensions may be determined.

LUMBER TERMS

Lumber in quantities is bought and sold by the thousand board feet, the letter "M" being used to indicate thousand board feet. As for example: Sugar pine sells at \$95 per M. This means that sugar-pine lumber sells at \$95 per thousand board feet.

Occasionally certain stock sizes of lumber may be sold by the linear foot. Small amounts of lumber may also be sold by the linear foot, or by the single board foot.

Lumber may be purchased either in "rough stock" or "dressed stock." Rough stock is lumber that is in the rough condition as it leaves the saw. Dressed stock is lumber that has been *planed*, or *surfaced*, on one or more sides as directed. This latter stock is used for finished work, for trim, or for cabinetwork.

When lumber is ordered dressed on one side only, it is specified as D.1.S. When dressed on two sides, it is specified as

D.2.S. If it is to be dressed on one side and one edge it becomes D.1.S.1.E. If the 2 sides and 2 edges are to be dressed, the expression used is D.4.S.

Sometimes the word *surfaced* is used instead of the word *dressed* and the letter D becomes changed to the letter S. In such case S.2.S. would be used instead of D.2.S., and S.4.S. instead of D.4.S. However, to avoid confusion the letter D will be used to refer to either surfaced or dressed lumber in the calculations which follow.

It should be remembered that when lumber is to be dressed on two sides, the thickness is reduced by from $\frac{3}{16}$ in. to $\frac{1}{4}$ in. Because of this when a board is ordered dressed, its final thickness is not as much as its original or nominal thickness. For example, a $1\frac{1}{2}$-in. oak board D.2.S. will measure approximately $1\frac{5}{16}$ in. A 1-in. chestnut board ordered D.2.S. will measure about $\frac{13}{16}$ in. A 2 by 4-in. piece of birch will measure $1\frac{3}{4}$ by $3\frac{3}{4}$ in. when ordered dressed on all sides.

If it is desired that the given dimensions be those to which the lumber should measure after dressing, it should be so specified. This will necessitate "dressing down" a piece of slightly larger dimensions as above explained. This fact might well be remembered in ordering lumber.

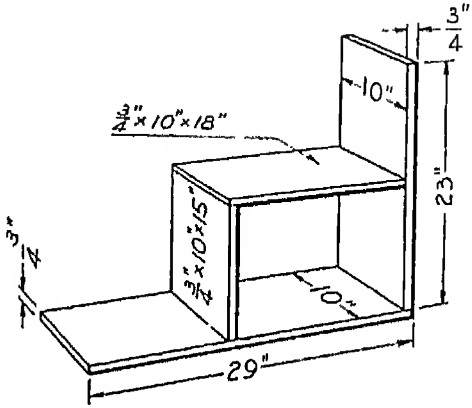
Problems Involving Calculations in Board Feet

1. How many board feet are there in each linear foot of a 1-in. board which measures 3 in. wide; 4 in. wide; 9 in. wide; and 10 in. wide?

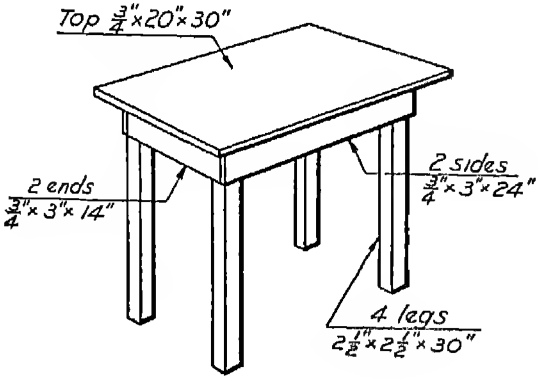
2. Calculate the cost of the following bill of material:

18 pieces red oak	12 ft.	×	9 in.	×	$\frac{3}{4}$ in.	@	\$135 per M
5 pieces red oak	10 ft.	×	4 in.	×	2 in.	@	170 per M
5 pieces red oak	10 ft.	×	6 in.	×	3 in.	@	185 per M
12 pieces red oak	14 ft.	×	12 in.	×	$1\frac{1}{2}$ in.	@	155 per M
3 pieces red oak	8 ft.	×	4 in.	×	$1\frac{1}{2}$ in.	@	155 per M
7 pieces red oak	12 ft.	×	8 in.	×	$\frac{3}{4}$ in.	@	135 per M
9 pieces red oak	12 ft.	×	6 in.	×	2 in.	@	170 per M
6 pieces red oak	10 ft.	×	8 in.	×	2 in.	@	170 per M

3. At 12¢ per board foot what is the cost of the lumber used in making the piece illustrated in the following drawing? An addition of 20% is to be made for waste.



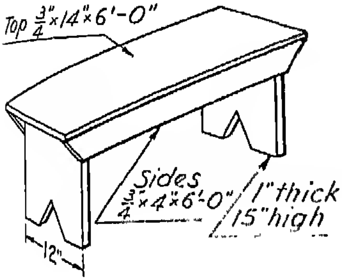
4. How many board feet of oak are there in 24 of the small tables as shown in the following drawing? If this lumber costs \$96 per M when finished to the specified thickness what would be the cost of this material, no allowance being made for waste?



5. Calculate the number of board feet of each kind of material in this list, also the total cost of the bill of lumber.

128 pieces $1\frac{1}{2}$ in. × 6 in. × 12 ft. spruce
30 pieces 2 in. × 4 in. × 10 ft. sugar pine

Spruce sells at \$70 per thousand, and sugar pine sells at \$87 per thousand.



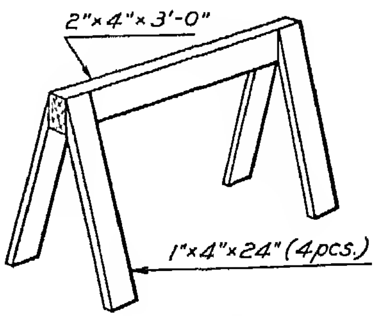
6. Find the number of board feet needed in building 6 small benches as shown in the drawing to the left. No allowance is to be made for waste. If this material costs \$90 per M when finished to sizes given, what is the total cost of the lumber used in these benches?

7. A young man having some repairing to do at home presents the following list of material to the lumber dealer. The boy is told that when this lumber is dressed as specified it will cost him 12¢ a board foot. What does the bill amount to?

4 boards $\frac{3}{4}$ in. × 9 in. each 9 ft. long
6 boards $1\frac{1}{2}$ in. × 10 in. each 6 ft. long
6 boards $\frac{3}{4}$ in. × 12 in. each 8 ft. long

All boards to be first-grade sugar pine.

8. How many board feet of lumber are needed to build 12 carpenter's horses according to specifications in the sketch to the right?

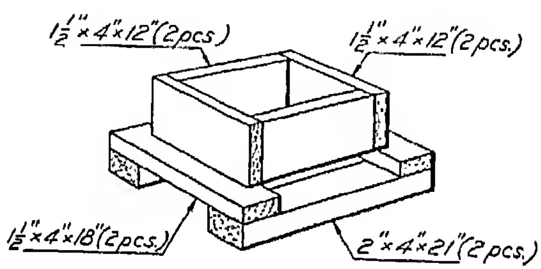


9. To construct a wooden sidewalk 6 ft. wide and 50 ft. long, boards 1 in. thick and 10 in. wide are to be used. The boards for this walk are to be cut 6 ft. long and laid in the direction of the width of the

sidewalk. In cutting these to size, 5% is allowed for waste. Running lengthwise under these boards for the entire length of the walk there are to be three rows of 4 by 4-in. "stringers," each 10 ft. long.

What is the total cost of this lumber at $60 per thousand board feet?

10. Make out the bill of material for 12 concrete forms as illustrated. At $60 per M, what is the cost of this material?



11. How many linear feet of a board $1\frac{1}{2}$ in. thick, 9 in. wide, will it take to make up 54 bd. ft.?

12. How many linear feet of 2 by 4 will it take to make 32 bd. ft.?

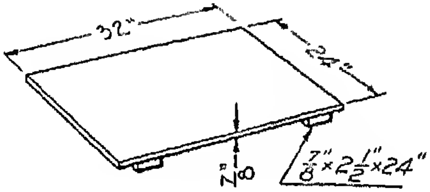
13. Calculate the number of board feet in the following bill of material:

10 planks 14 ft. × 6 in. × 2 in.
18 planks 12 ft. × 8 in. × 2 in.
6 planks 12 ft. × 6 in. × 2 in.

At $57 per M what is the cost of this material?

14. Determine the number of board feet in a piece of lumber that measures 12 ft. long, 10 in. wide, and 8 in. thick.

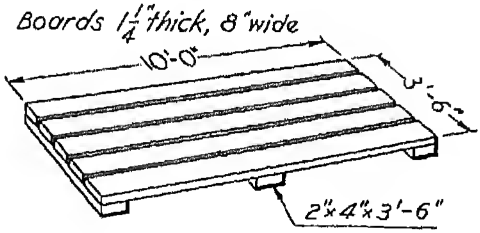
15. A shop order calls for 24 drawing boards as per the following dimensions. What is the total number of board feet in these drawing boards as finished?



16. How many board feet of $1\frac{1}{4}$-in. lumber will be required in making a wooden platform 18 ft. wide by 24 ft. long? These boards are to be nailed to stringers already in position.

17. Calculate the board feet in 3 pieces of oak $2\frac{3}{4}$ in. thick, 8 in. wide, and 12 ft. long.

18. Determine the number of board feet of fir needed to build 3 platforms according to the following dimensions. No allowance is to be made for waste.

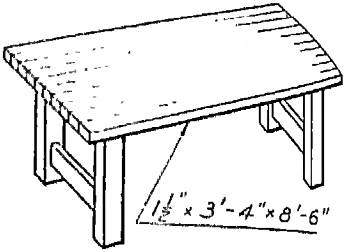


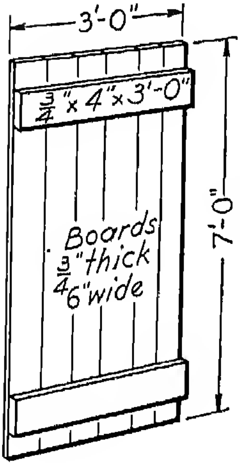
19. What is the total amount of the following bill of material:

12 pcs. chestnut	8 ft. ×	$2\frac{1}{2}$ in. ×	$1\frac{1}{4}$ in.	@ 16¢ per board foot
12 pcs. chestnut	6 ft. ×	4 in. ×	4 in.	@ 15¢ per board foot
18 pcs. oak	12 ft. ×	10 in. ×	$\frac{7}{8}$ in.	@ 9¢ per board foot

20. How many board feet of birch are there in the finished bench top in the drawing as shown below?

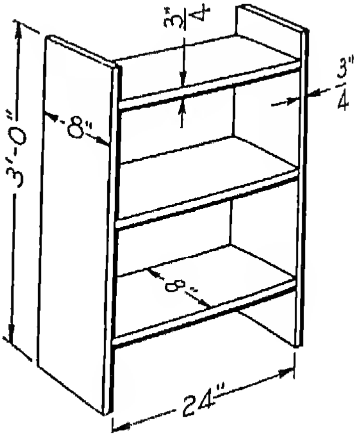
21. A house repair job calls for 6 pieces of lumber 2 in. × 4 in. × 10 ft. and 8 pieces each 1 in. × 8 in. × 12 ft. At 8¢ per board foot what is the cost of this material?





22. At 12¢ per board foot when dressed on all sides and cut to the dimensions given what is the cost of the material needed to construct the storm door in the sketch to the left?

23. In building the set of shelves as shown in the drawing to the right, how many board feet of lumber are used, adding 25% of lumber for waste?



24. How many linear feet of board 1 in. thick 6 in. wide will it take to make 68 bd. ft.?

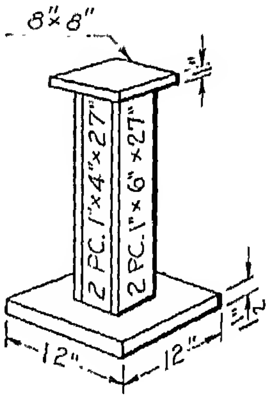
25. Calculate the total board feet in the following list:

2 pieces	fir	4 in.	× 6 in.	×	$8\frac{1}{2}$ ft.
2 pieces	fir	2 in.	× 8 in.	×	12 ft.
5 pieces	pine	$1\frac{1}{2}$ in.	× 8 in.	×	10 ft.
2 pieces	pine	$\frac{7}{8}$ in.	× 9 in.	×	12 ft.

26. How many board feet in 15 pieces of 2 by 4 each 16 ft. long?

27. Calculate the board feet in a timber 4 in. thick, 6 in. wide, and $14\frac{1}{2}$ ft. long.

28. Determine the number of board feet of lumber in 24 pedestals as per the dimensions in the drawing to the right.



29. A repair job on a house porch requires the following lumber:

3 pcs. 2 in. × 8 in. × 10 ft.
3 pcs. 2 in. × 4 in. × 10 ft.
2 pcs. 2 in. × 6 in. × 10 ft.

What will be the cost of this lumber at $50 per thousand board feet?*

ANSWERS TO PROBLEMS

Pages 191 to 197.

1. $\frac{1}{4}$ bd. ft.; $\frac{1}{3}$ bd. ft.; $\frac{3}{4}$ bd. ft.; $\frac{5}{8}$ bd. ft.
2. $121.86.
3. 85¢.
4. 263 bd. ft.; $25.25.
5. 1152 bd. ft.; 200 bd. ft.; $98.04.
6. 81 bd. ft.; $7.29.
7. $14.40.
8. 56 bd. ft.
9. $30.90.

*Answers to these problems will be found on pages 197, 198.

10. \$4.20.
11. 48 ft.
12. 48 ft.
13. 500 bd. ft.; \$28.50.
14. 80 bd. ft.
15. 148 bd. ft.
16. 540 bd. ft.
17. 66 bd. ft.
18. 146 bd. ft.
19. \$34.60.
20. $42\frac{1}{2}$ bd. ft.
21. \$8.32.
22. \$2.76.
23. 10 bd. ft.
24. 136 ft.
25. 134 bd. ft.
26. 160 bd. ft.
27. 29 bd. ft.
28. 130 bd. ft.
29. \$4.

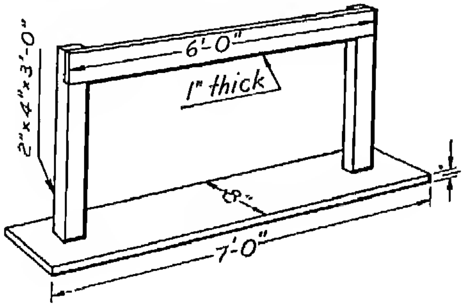
Review Problems Involving Calculations in Board Measure

1. Calculate the total number of board feet in the following list of lumber:

10 pieces fir	12 ft.	× 8 in.	× 2	in.
20 pieces fir	12 ft.	× 4 in.	× 2	in.
12 pieces pine	12 ft.	× 9 in.	× $1\frac{1}{2}$	in.
8 pieces chestnut	14 ft.	× 9 in.	× 1	in.

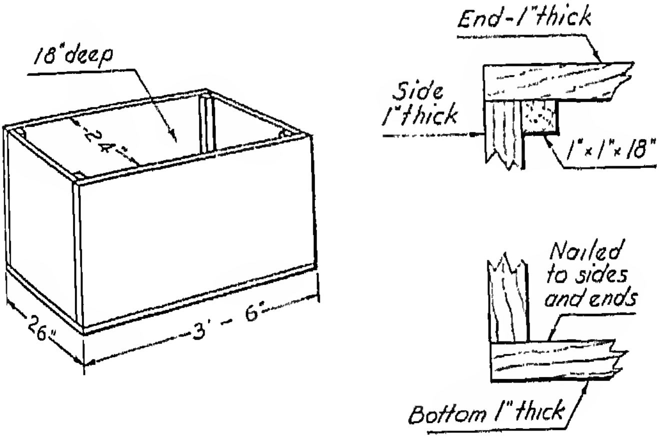
2. How many board feet in a piece of fir timber that measures 6 in. × 8 in. × 10 ft.?

3. Determine the number of board feet of pine needed for six display racks as finished according to the following dimensions. An allowance of 20 per cent is to be made for waste.
The top board is 6 in. wide.



4. To erect a wire fence around a garden 36 pieces of lumber are needed each measuring 5 ft. long, 4 in. wide, and 2 in. thick, also 70 pieces measuring 10 ft. long, 3 in. wide, and 2 in. thick. If this material costs $40 per M, calculate the cost of these pieces.

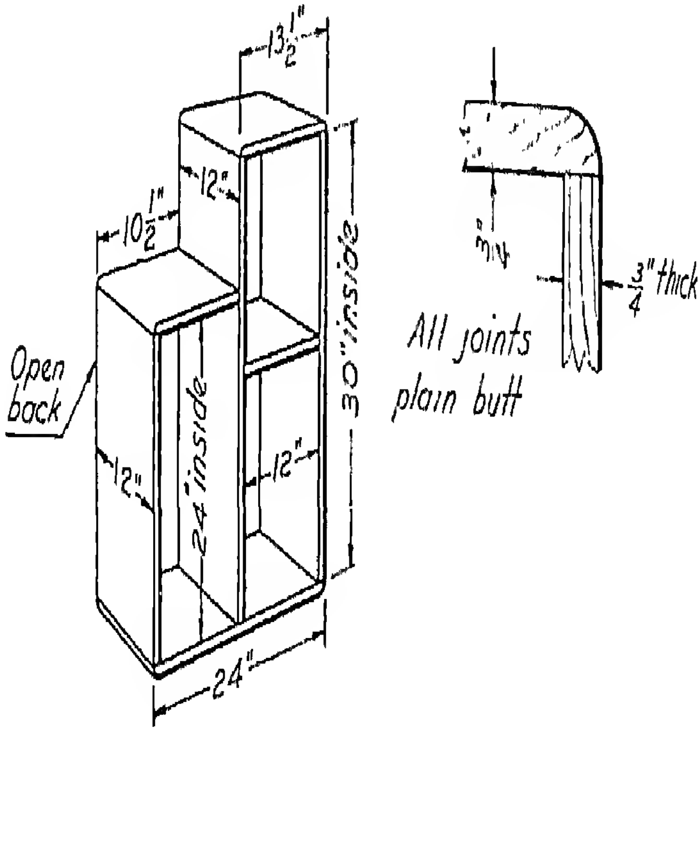
5. Calculate the number of board feet of lumber in twelve packing boxes as per the following dimensions. The boxes are to be open at the top as shown. Determine the total cost of the material in these boxes when the lumber sells at $40 per thousand board feet.



6. How many board feet of 1 by 8-in. sheathing 12 ft. long will be required for a partition that measures 12 ft. high and 24 ft. long? An allowance of $12\frac{1}{2}\%$ is made for waste.

7. It is estimated that 24 pieces of fir each 12 ft. long, 9 in. wide, and $\frac{7}{8}$ in. thick, together with 16 pieces of fir 12 ft. long, 4 in. wide, and 2 in. thick are needed to erect a partition between two rooms. If this lumber costs $50 per thousand board feet what is the cost of these pieces?

8. Determine the cost of $\frac{3}{4}$-in. whitewood needed for the modernistic bookshelf and rack in the following drawing. This lumber costs 15¢ per board foot, when dressed to the sizes given.



SQUARE MEASURE

Surfaces, areas, weights, estimates, trade calculations.

Square Measure

Areas are calculated by means of units of *square measure* in much the same manner that lengths and distances are calculated by units of linear measure. However, in calculating areas, the *two* dimensions representing length and width are *multiplied* together. Their product gives area.

If the dimensions are in inches their product is *square inches*, and the area is in *square* inches, abbreviated sq. in. If the dimensions are in feet, their product is in *square feet*, abbreviated sq. ft., and the area is in *square* feet. If in yards their product is *square yards*, abbreviated sq. yd., and the area is in *square* yards; and so on.

This is illustrated in the following example.

Example:

What is the area in square inches of a table top that measures 28 in. long and 20 in. wide?

Solution and Explanation:

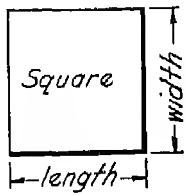
As explained, the area of this table top in square inches is equal to the product of the two dimensions that represent length and width.

In this particular case 28 in. is the length and 20 in. is the width.

The product of these two numbers is 28 × 20, or 560.

Therefore, the area of this table top is 560 square inches.

Most of the calculations on areas deal with the following shapes. How the areas of these shapes are calculated is explained and illustrated as follows.

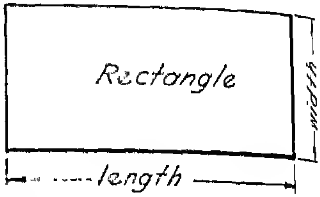


THE SQUARE

The square is a plane figure having four equal sides and four square or right-angle corners.

Referring to the figure on page 203 the area of a square is equal to the *length* multiplied by the *width*. Since the length equals the width one might state that the area of a square is equal to the product of two sides of the square.

THE RECTANGLE



A *rectangle*, sometimes called an oblong, is a plane figure, also having four sides and four square or right-angle corners. It differs from a square in that only the two opposite sides are equal.

By referring to the figure the area of a rectangle is expressed as the product of the length and the width. Sometimes the word *height* or *thickness* is used for either length or width. The method of calculating the area, however, remains the same.

The following example shows how this rule is applied.

Example:

Determine the number of square feet of plasterboard needed to cover a ceiling that measures 15 ft. wide and 18 ft. long.

Solution and Explanation:

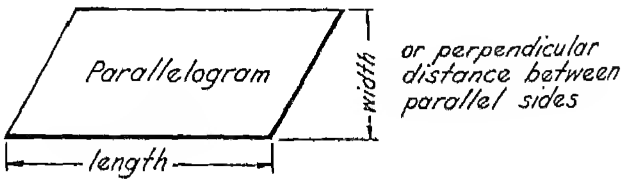
This ceiling is rectangular in shape and its area is found by multiplying the length by the width.

This equals, 18×15, or 270.

That is, 270 sq. ft. of plasterboard are needed to cover the above ceiling.

THE PARALLELOGRAM

The *parallelogram* is a plane figure whose *opposite* sides are *parallel* and *equal*, but whose corners are *not square corners.*

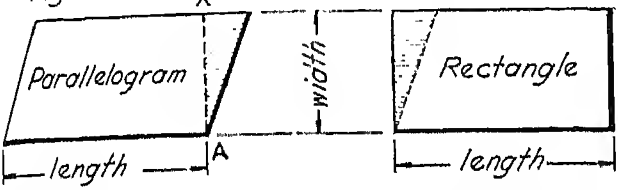


The area of a parallelogram is found by multiplying the length of one side by the width, or the perpendicular distance between it and the opposite side.

It will be noticed that this is the same rule that was used in finding the area of a rectangle. That this same rule applies to both the rectangle and the parallelogram may be readily seen by drawing a parallelogram like the one in the accompanying sketch, Figure 1, and erecting a perpendicular at the corner marked A.

After this is done, cut along the perpendicular AX erected at A and move the triangular piece that has been cut off over to the left side, occupying the position as shown in Figure 2.

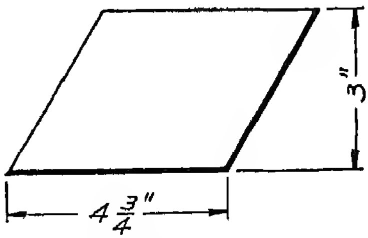
Fig 1 Fig. 2



The figure so made is a rectangle whose area is identical with that of the parallelogram. Accordingly, the area of a parallelogram is calculated by the same rule that is used in calculating the area of a rectangle.

This is illustrated in the following problem.

Example:



Fourteen pieces of sheet iron, as per the drawing to the right, are cut from a strip of metal 3 in. wide. What is the total area of these pieces?

Solution and Explanation:

This piece is a parallelogram in shape. The area of one piece is found by multiplying the length by the width. This equals:

$4\frac{3}{4} \times 3$ or $\frac{57}{4}$, which is $14\frac{1}{4}$ square inches.

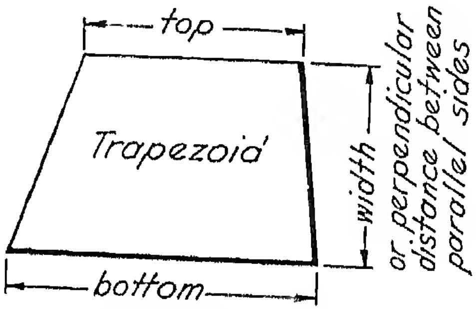
In 14 pieces, the area is 14 times the area of one piece, or

$14 \times 14\frac{1}{4}$, which equals $199\frac{1}{2}$ square inches.

That is, in the 14 pieces of metal mentioned there are 199 square inches.

THE TRAPEZOID

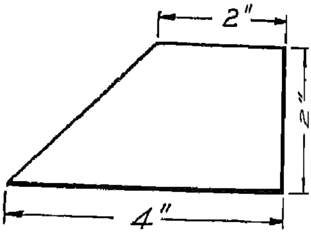
The *trapezoid* is a plane figure having four sides, two o which are *parallel* but *not equal*. The area of a trapezoid i equal to one half the sum of the parallel sides multiplied b the width, or the *perpendicular* distance between them.



Because the lengths of the opposite sides are unequal they might be referred to as "top" and "bottom" as shown in the figure.

How this rule is applied in a practical case is shown in the following example.

Example:



Determine the weight of 50 metal blanks made as per the drawing to the right. This metal weighs .02 lb. per square inch.

Solution and Explanation:

The area of this trapezoid equals:

$\frac{1}{2}(2 + 4) \times 2$; or, $\frac{1}{2} \times 6 \times 2$ which equals 6 square inches

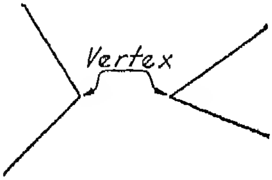
The area of 50 pieces is:

$$50 \times 6, \text{ or } 300 \text{ square inches}$$

If this metal weighs .02 lb. per square inch, then 300 square inches weigh: $300 \times .02$, or 6 lb.

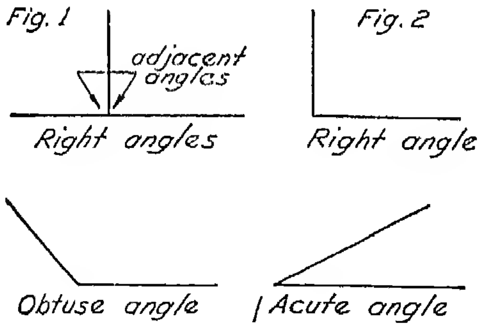
That is, 50 pieces of this metal according to the drawing on page 206 weigh 6 lb.

ANGLES



An *angle* is the opening between two straight lines which meet at a point called the *vertex*. The amount of the opening determines the size of the angle.

When two straight lines meet at a point, and form equal *adjacent* angles, as shown in Figure 1 below, these angles are said to be *right angles*. The lines forming a right angle are perpendicular to each other and form a square corner, as in Figures 1 and 2.

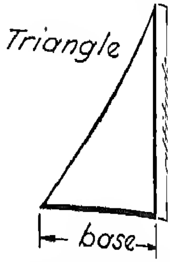


An angle *less than* a right angle is called an *acute angle*.

An angle *greater than* a right angle is called an *obtuse angle*.

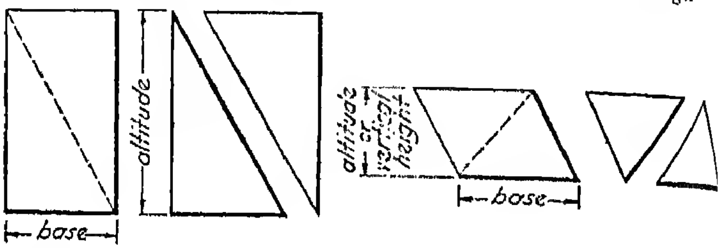
THE TRIANGLE

A *triangle* is a plane figure having 3 sides and 3 angles.



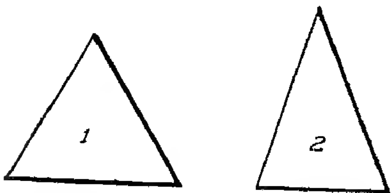
Although they may be of various shapes as illustrated below, the area of a triangle equals one half the product of the *base* by the *altitude*, or the *vertical height*, which is the same as saying one half the base times the height.

To prove that this rule is true, cut out of paper a rectangle also a parallelogram, as shown in the following drawings.



After drawing the diagonals as indicated, cut along the lines, cutting each piece into two equal triangular parts a shown. Each one of these triangles equals one half the whol area of the figure of which it is a part. Since the area of th rectangle, or the parallelogram, equals the base multiplie by the vertical height, then the area of each triangle woul accordingly equal one half the base multiplied by vertic height.

VARIOUS SHAPES OF TRIANGLES



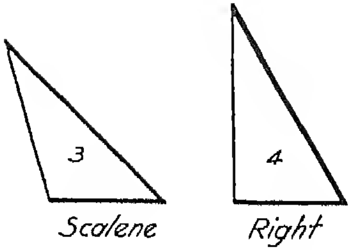
Equilateral Isosceles

1. An *equilateral triangle* i one in which all angles an all sides are equal.

2. An *isosceles triangle* i one in which two sides an two angles are equal.

3. A *scalene triangle* is one in which none of the sides or angles are equal.

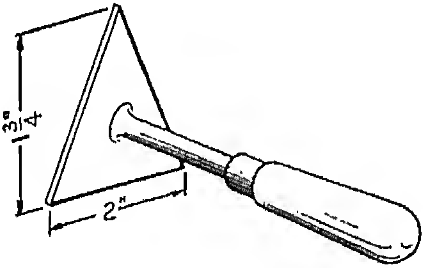
4. A *right triangle* is one which has one right angle, or one square corner.



The area of each of these triangles as previously explained is equal to one half the product of the base and the altitude. The following is a practical application of this rule.

Example:

How many square inches of sheet steel are needed to make 12 scraper blades according to the dimensions in the following drawing, assuming there is no loss due to cutting to size?



Solution and Explanation:

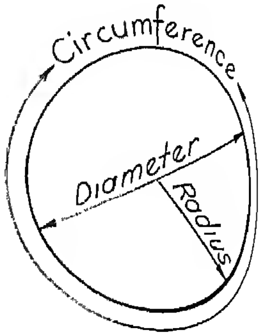
This blade is triangular in shape. It measures $1\frac{3}{4}$ in. high and has a base that is 2 in. long.

According to the above rule the area of this triangle is:

$$\frac{1}{2} \times 2 \times 1\frac{3}{4}, \text{ or } 1\frac{3}{4} \text{ square inches.}$$

The area of 12 such triangles is $12 \times 1\frac{3}{4}$, or 21 square inches. That is, 21 square inches of sheet steel are necessary to make 12 scraper blades according to the given dimensions.

THE CIRCLE



The area of a circle is equal to 3.1416 × radius × radius; or to 0.7854 × diameter × diameter. This also approximately equals, 0.08 × circumference × circumference.

When a number is multiplied by itself, it is said to be squared, or raised to the second power. This is usually indicated by placing a small 2 just above and to the right of the number. For example, 3 × 3 may be expressed as 3^2. This is read as three square or the square of three.

In the same manner radius × radius becomes radius^2; diameter × diameter, diameter^2; circumference × circumference, circumference^2.

Radius may be abbreviated as *rad.*, *R.*, or *r*.

Diameter may be abbreviated as *diam.*, *D.*, or *d*.

Circumference may be abbreviated as *circum.*, *cir.*, or *C*.

The rules for the area of a circle are usually expressed in abbreviated form as:

$$\begin{aligned} \text{Area of a circle equals, } & 3.1416 \times r^2, \text{ or,} \\ & 0.7854 \times d^2, \text{ or,} \\ & 0.08 \times cir^2. \end{aligned}$$

The above rules are applied in such problems as the following.

Example 1:

What is the cross-section area of a shaft which measures 2.625 in. in diameter?

Solution and Explanation:

According to the above rule the circular area of this shaft is: $0.7854 \times d^2$; or $0.7854 \times 2.625 \times 2.625$. Carrying the answer of this multiplication to the second decimal place, the result is 5.41.

That is, the area of this shaft equals 5.41 sq. in.

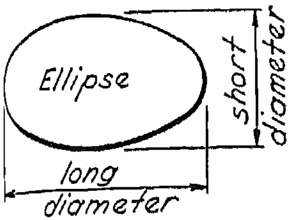
Example 2:

In order to determine the area of a cylindrical tank, a tape is stretched around its circumference. This is found to measure $62\frac{1}{2}$ in. What is the circular area of this tank?

Solution and Explanation:

It will be sufficiently accurate in this problem to carry the answer to the second decimal place. Following the above rule, the calculation then becomes:

$0.08 \times cir^2$, or $0.08 \times 62\frac{1}{2} \times 62\frac{1}{2}$, which is approximately 312.50. That is, the circular area of the above tank is approximately $312\frac{1}{2}$ sq. in.



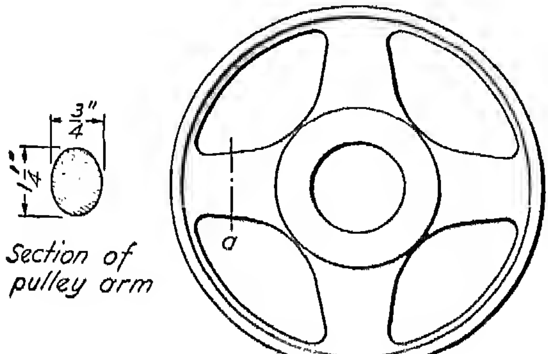
THE ELLIPSE

The area of an ellipse is found by multiplying the short diameter by the long diameter, and this product by 0.7854.

The following example shows how this rule is applied to a practical problem.

Example:

The arms of the pulley as shown below are elliptical in cross section, measuring $\frac{3}{4} \times 1\frac{1}{4}$ in. at point a. What is the area of the cross section at this point?



Solution and Explanation:

The area of the elliptical section according to the above rule is found by multiplying $\frac{3}{4} \times 1\frac{1}{4}$, and that product by .7854. This equals $\frac{3}{4} \times \frac{5}{4} \times 0.7854$, which is approximately .7363, or about $\frac{3}{4}$ sq. in.

A term frequently used in the calculation of areas is "perimeter." This is the distance *around* a figure, and is equal to the total length of *all* the sides added together. In the case of a square, a rectangle, or a parallelogram this equals the sum of the four sides. In a triangle this would equal the sum of the three sides. In like manner the circumference of a circle, or of an ellipse, is sometimes called the perimeter.

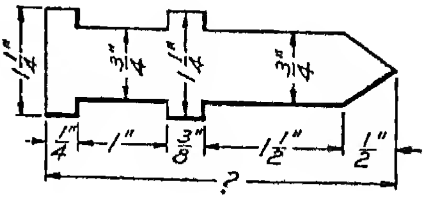
IRREGULAR FLAT SURFACES

Areas of irregular flat surfaces, or surfaces other than those just considered, may be calculated by dividing them up into regular shapes resembling those already illustrated. The areas of these shapes are calculated separately, and then added together to determine the total area.

The following problem illustrates how such surfaces are calculated.

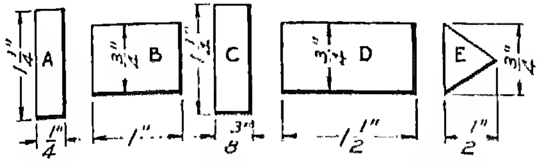
Example:

What is the area of a blank made according to the following drawing?



Solution and Explanation:

The area of this blank may readily be determined by dividing it up as shown in the illustration on page 213, and calculating each area separately. The sum on the separate areas *A*, *B*, *C*, *D*, and *E* will give the total area required.



A, B, C, and D are rectangles, and E is a triangle.

The area of each of these figures is calculated according to rules previously explained.

The area of $A = 1\frac{1}{4} \times \frac{1}{4} = \frac{5}{4} \times \frac{1}{4} = \frac{5}{16}$ sq. in.
The area of $B = \frac{3}{4} \times 1 = \frac{3}{4}$ sq. in.
The area of $C = 1\frac{1}{4} \times \frac{3}{8} = \frac{5}{4} \times \frac{3}{8} = \frac{15}{32}$ sq. in.
The area of $D = \frac{3}{4} \times 1\frac{1}{2} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$, or $1\frac{1}{8}$ sq. in.
The area of $E = \frac{1}{2} \times \frac{3}{4} \times \frac{1}{2} = \frac{3}{16}$ sq. in.

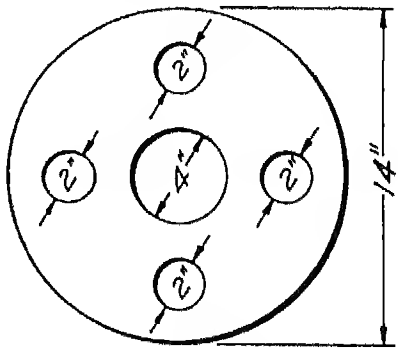
The sum of these separate areas, which in turn constitutes the total area of the blank, equals $\frac{5}{16} + \frac{3}{4} + \frac{15}{32} + 1\frac{1}{8} + \frac{3}{16}$. Reducing these fractions to similar fractions and adding them together the sum equals $2\frac{15}{32}$.

That is, the area of the above blank is $2\frac{15}{32}$ sq. in.

Problems Involving Calculations of Areas of Flat Surfaces

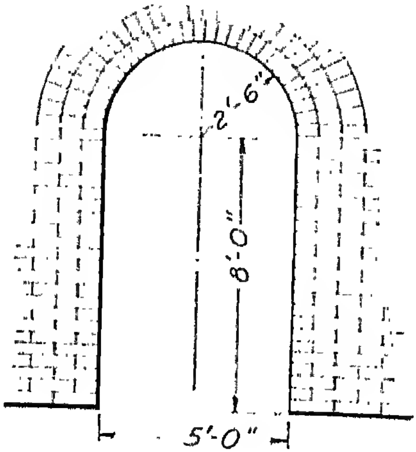
1. How many square feet of surface in a concrete sidewalk that measures $5\frac{1}{2}$ ft. wide and 258 ft. long?

2. By cutting holes in the 14-in. cast-iron disk as illustrated, what is the total area reduction? What is the per cent reduction of area?

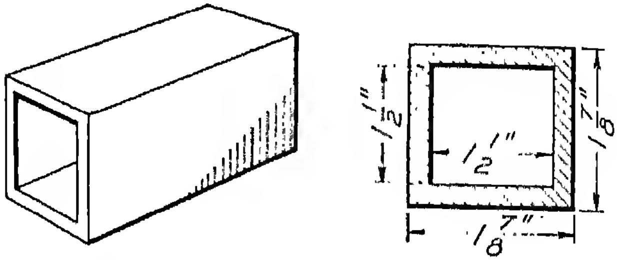


3. Determine the area of the floor space in a gymnasium that measures $62\frac{1}{2}$ ft. wide and 98 ft. long.

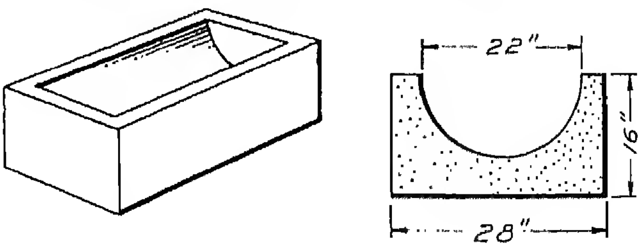
4. What is the area of the archway in the following sketch?



5. Determine the area of metal in the cross section of the cast-iron sleeve as shown below.

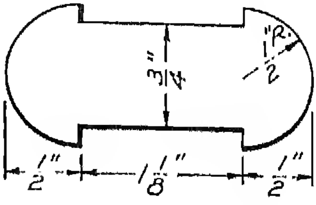


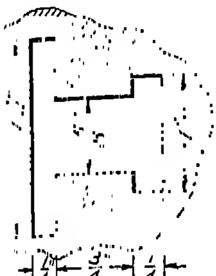
6. Calculate the cross-section area through the concrete trough as illustrated in the following drawing.



7. At 30¢ per square foot, including labor and materials, what is the cost of a concrete sidewalk that measures 5 ft. wide by 32 ft. long?

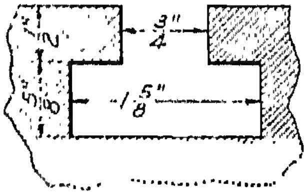
8. Determine the area of the sheet-steel punching shown in the sketch to the right. If 10 sq. in. of this material weigh ½ lb., what will be the weight of 1200 punchings?

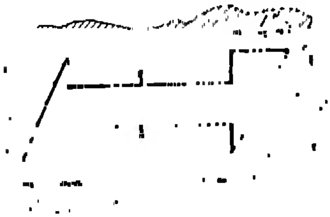




9. Calculate the area of the L-shaped hole as cut in the piece of metal shown in the sketch to the left.

10. What is the area through the T slot in the drawing to the right?



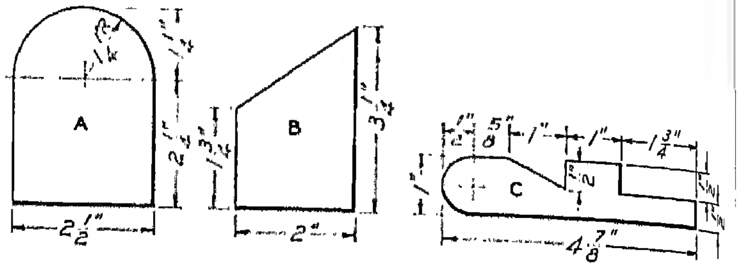


11. Determine the area of the cut-out portion illustrated in the detailed sketch as shown to the left.

12. A pulley arm that is elliptical in cross section, measures 2½ in. across the long diameter and 1¾ in. across the short diameter. What is the area of this section?

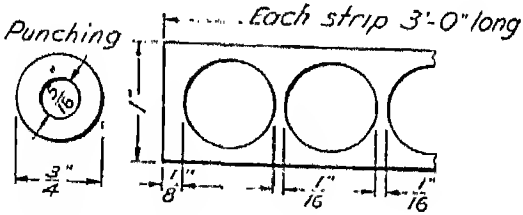
13. Calculate the cross-sectional area of the metal in a hollow shafting measuring 2½ in. outside diameter and 1 in. inside diameter. Draw such a section, placing on it the given dimensions.

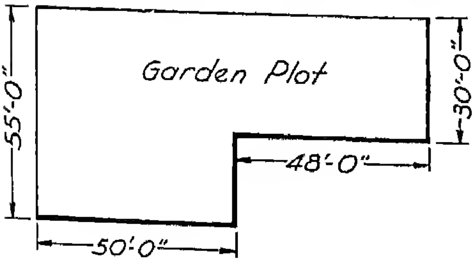
14. Determine the areas of the following shapes, carrying the results to the third decimal place.



15. Copper wire is listed as being able to withstand, without breaking, a pulling strain of 30,000 lb., for each square inch of cross-section area. At this rate how much pulling strain will a $\frac{1}{4}$-in. diameter copper wire be able to withstand before breaking?

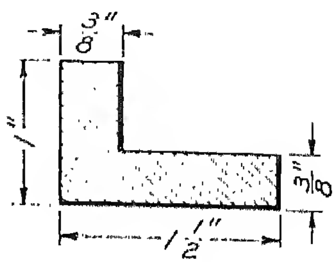
16. Calculate the weight of 1,000 washers of the following size. These are punched from strips of 18-gauge sheet iron 3 ft. long which weighs 2 lb. per square foot area. The first "punching" in each strip lies $\frac{1}{8}$ in. from the end, and there is $\frac{1}{16}$ in. between the following blanks as shown. How many full strips are needed?





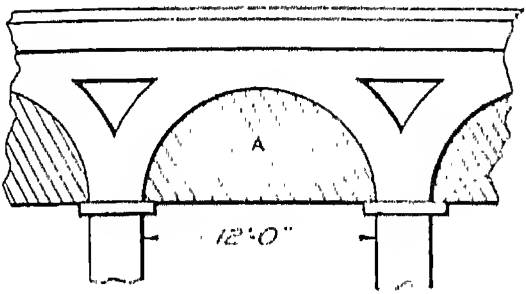
17. How many square feet in the garden plot illustrated to the left.

18. Find the cross-section area of the angle iron to the right.



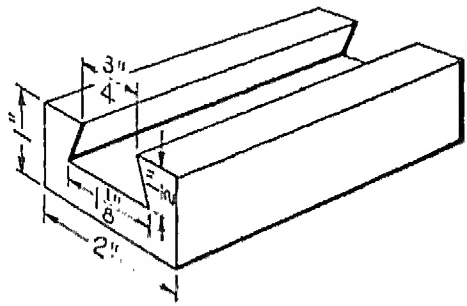
19. After cutting a 6-in. circular piece from a sheet of fiber that is 10 in. square, what per cent of the material is left?

20. On a certain job of painting the interior of a large hall, there are to be decorated 10 semicircular portions like those shown by A in the following sketch. What is their total area?



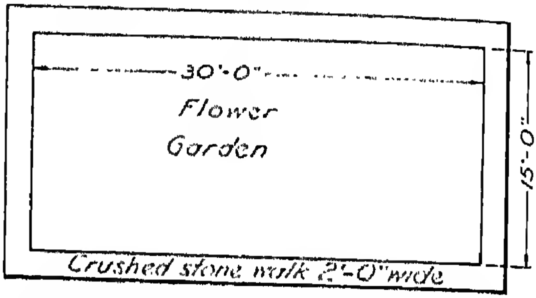
21. An open rectangular tank that measures 8 ft. long and 3 ft. wide inside, is 2 ft. deep. How many square feet of copper sheet are needed to line this tank?

22. Calculate the cross-section area of the material in the "dovetail" slotted block as shown in the following drawing.



23. A power hack saw such as is used for cutting steel, is advertised as being "able to cut through steel at the rate of $\frac{1}{2}$ square inch per minute." At this rate how long will it take to cut through a piece of steel that measures $3\frac{1}{2}$ in. in diameter?

24. Calculate the number of bags of fine crushed stone needed for covering the top of the dirt sidewalk in the following layout. One bag of this stone is advertised as being sufficient to cover 25 sq. ft. of surface.



25. The steam gauge on a steam engine indicates that the pressure of the steam is 112 lb., per square inch. If the diameter of the piston in the engine is $9\frac{1}{2}$ in., what is the total pressure of the steam on the engine piston?*

COMMON UNITS OF SQUARE MEASURE

The units of square measure common to shopwork, and practical work in general, are contained in the following table. This table is used principally in transforming units of one denomination to equivalent units of another denomination.

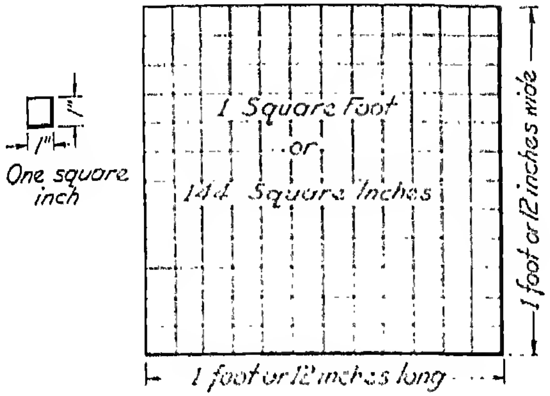
144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.) or 1,296 sq. in.
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.) or $272\frac{1}{4}$ sq. ft.
160 square rods	= 1 acre (A) or 4,840 sq. yd. or 43,560 sq. ft.
640 acres	= 1 square mile (sq. mi.)

*Answers to these problems will be found on page 228.

A square inch is the measurement of a surface whose length is 1 in. and whose width is 1 in. In the same way a square foot is the measurement of a square surface whose length is 1 ft. and whose width is 1 ft.

In each case the area denotes the number of square units of measure.

From the figure below it is seen that such area is equal to the product of the numbers representing the length and the width of the surface.



Explanation:

The unit, one square foot, represents a surface which measures 1 ft. long and 1 ft. wide. Since 1 ft. contains 12 in., the length equals 12 in. and the width equals 12 in. The product of these two dimensions is 12 × 12, or 144.

By inspecting this figure it will be seen that 1 sq. ft. contains 144 sq. in.

According to this calculation a surface which measures $\frac{1}{2}$ ft. wide and 2 ft. long also contains 1 sq. ft. area. The product of the 2 ft., or 24 in., and the $\frac{1}{2}$ ft., or 6 in., equals 1 sq. ft., or 144 sq. in. This may be proven by dividing up the surface according to the dimensions given and dividing it up like the square above.

The larger units, as rod, acre, and mile, are used only in referring to large areas, principally areas of land.

CHANGING FROM ONE DENOMINATION TO ANOTHER

In square measure, the reduction from higher to lower denominations is done much the same as in linear measure.

That is, to reduce a measurement of higher denomination to its equivalent in a lower denomination, the figure representing the higher denomination is multiplied by the number of units of the lower denomination it takes to make one of the higher denomination.

For example, to reduce square feet to square inches, the area in square feet is multiplied by 144, as it takes 144 square inches to make one square foot. How this is done is illustrated in the following problem.

Example 1:

How many square inches of material are there in a piece of sheet copper which contains $2\frac{1}{4}$ sq. ft.?

Solution and Explanation:

Since there are 144 square inches in one square foot, then in $2\frac{1}{4}$ square feet there are:

$2\frac{1}{4} \times 144$ or 324 square inches, which represents the number of square inches in the above sheet of copper.

To reduce to its equivalent number of square feet a surface which is measured in square yards, the number representing the square yards is multiplied by 9, as there are 9 square feet in one square yard.

An application of this rule is seen in the following problem.

Example 2:

How many square feet of surface are there in a sign, which is made of canvas, measuring $1\frac{1}{4}$ yd. wide and 2 yd. long?

Solution and Explanation:

The area of this sign equals $2 \times 1\frac{1}{4}$, or $2\frac{1}{2}$ square yards.

Since there are 9 square feet in one square yard, then in $2\frac{1}{2}$ square yards there are:

$$2\frac{1}{2} \times 9, \text{ or } 22\frac{1}{2} \text{ square feet.}$$

That is, there are $22\frac{1}{2}$ square feet in this canvas sign.

As in linear measure it often becomes necessary to change lower units to equivalent higher units, so it is in square measure.

This is accomplished by dividing the number representing the smaller unit by the number of these smaller units it takes to make one of the larger units to which the smaller one is to be changed.

This is illustrated in the following problem.

Example 3:

A piece of land rectangular in shape measures 534 ft. long and 210 ft. wide. How many acres are there in it?

Solution and Explanation:

The total area of this land in square feet is:

$$534 \times 210, \text{ or } 112{,}140 \text{ square feet.}$$

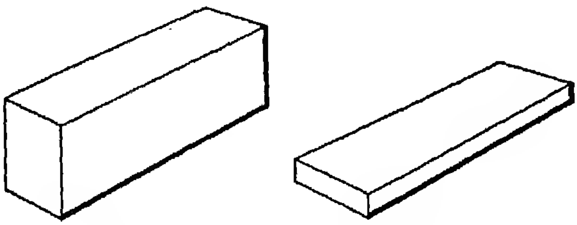
Since there are 43,560 square feet in one acre, then in 112,140 square feet there are as many acres as 43,560 is contained in 112,140, which equals:

$$112{,}140 \div 43{,}560, \text{ or } 2.57 \text{ acres (approximately)}$$

That is, the above piece of land contains approximately 2.57 acres.

SURFACE AREAS OF SOLIDS

Rectangular Solids



Rectangular Shapes

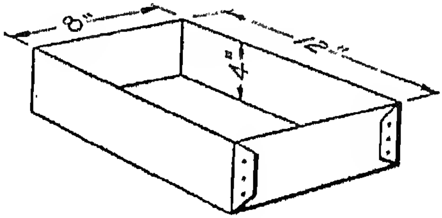
Surface areas of *rectangular* shaped solids may readily be determined by adding together the total areas of all six faces. These being rectangular in shape, they may be calculated the same as rectangles, as previously explained.

Calculations involving the areas of such solids are frequently necessary in determining areas and weights of sheet metal and other materials required in making boxes, containers, and similar items.

How these calculations are worked out is illustrated in the following practical problem.

Example:

Determine the weight of sheet iron required in making 9 work pans according to the following drawing. This metal is listed as weighing $1\frac{1}{4}$ lb. to the square foot.



Solution and Explanation:

Inspection shows that this pan may be made out of one sheet of metal with but little waste. The width of this sheet is equal to the width of the bottom, plus the width of the two sides, that is $8 + 4 + 4 = 16$ in.

The length of the sheet is equal to the length of the bottom, plus the width of the two sides; $12 + 4 + 4 = 20$ in.

The area of the required sheet then is $16 \times 20 = 320$ sq. in., which will be the amount required for one pan.

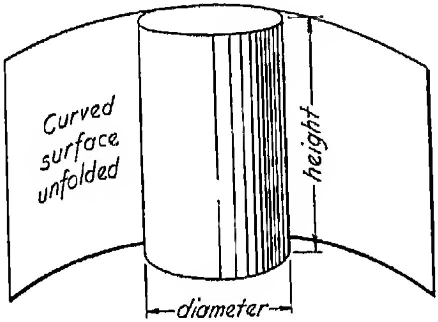
For 9 such pans this becomes 320×9, or 2880 square inches.

Reducing 2880 to square feet there results $2880 \div 144$, or 20 square feet.

If this metal weighs $1\frac{1}{4}$ lb. to the square foot then the weight of metal required for the 9 pans becomes $20 \times 1\frac{1}{4}$, or 25 lb.

Cylinder

The area of a *cylindrical* surface like that illustrated below, is equal to the area of the curved side as it appears when unfolded.



The area of this curved surface is equal to the circumference of the circular base multiplied by the height of the cylinder. That is, it is equal to the diameter × 3.1416 × height of cylinder.

To prove that this is correct, construct a cylinder out of heavy paper and measure the area of the cylindrical part when unfolded. Then compare this with the calculated area.

This rule is helpful in calculating amounts of metal to be used in constructing tanks or other cylindrical items, in determining radiation surfaces of pipes, areas of surfaces to be painted, and such.

The following problem shows an application of this rule.

Example:

Determine the number of square feet of sheet metal needed in building a tank 2 ft. in diameter and 5 ft. high, allowing 5% for lapping and waste.

This tank is open at one circular end.

Solution and Explanation:

The total area of the material in this tank equals the area of the circular bottom plus the area of the cylindrical portion.

Area of bottom = 2 × 2 × 0.7854 = 3.1416 square feet.

Area of cylindrical portion = 2 × 3.1416 × 5 = 31.416 square feet. Therefore, the total area equals:

3.1416 + 31.416 = 34.5576 square feet.

Adding 5% to this area for lapping and waste, this amount is increased by 34.5576 × .05, or 1.7278 square feet.

This latter amount added to the above total area equals: 34.5576 + 1.7278, or 36.2854 square feet.

That is, the total number of square feet of sheet metal needed to build one of the above tanks is 36.2854 square feet, or approximately $36\frac{1}{4}$ square feet.

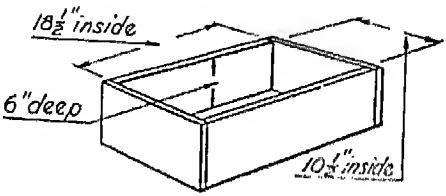
Problems Involving Reduction in Square Measure

1. A floor measuring 18 ft. by 24 ft. is to be covered with linoleum that measures 1 yd. wide. How many square yards of this material are needed? Lay out a plan showing the arrangement of the strips.

2. A ceiling measuring 12 ft. by 16 ft. is to be covered with beaverboard. The size of the beaverboard that is to be used measures 48 in. wide and is 8 ft. long. What is the cost of the beaverboard needed if it sells at $1.60 per sheet?

3. If 1 gallon of a special ready-mixed paint will cover approximately 30 sq. yd. two coats, how many gallons will be needed to paint two coats on both sides of a board fence which measures 6 ft. high and 135 ft. long?

4. How many square feet of thin sheet copper are needed in lining the box shown in the sketch?

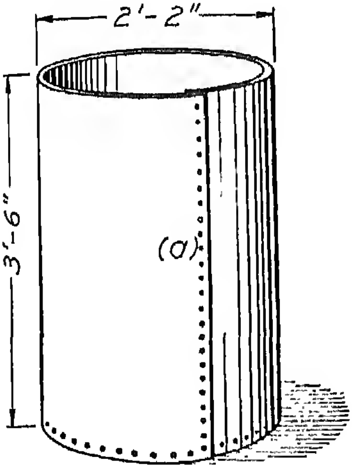


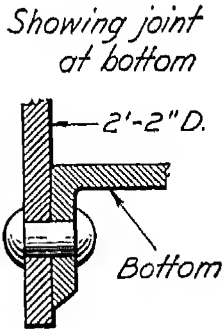
5. How many acres in a rectangular tract of land which measures 242 ft. long and 270 ft. wide?

6. A room in a factory building is being arranged for the temporary storage of bags of cement. Because of the construction of the floor it can be loaded to only 1250 lb., per square yard.

If the measurements of the floor are $18\frac{1}{2}$ ft. by 36 ft., what is the total number of pounds that it can safely carry?

7. How many square feet of sheet metal are in a cylindrical tank made according to the following specifications? The tank is to be open at the top as shown. There is an allowance of 2 in. for the seam at (*a*).

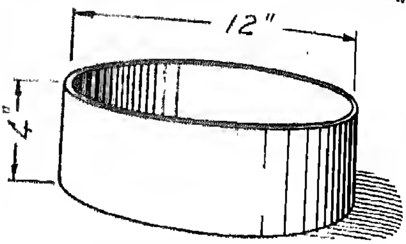




The diameter of the bottom, which sets up into the tank, is extended 1 in. on each side for riveting as shown in the detailed sketch.

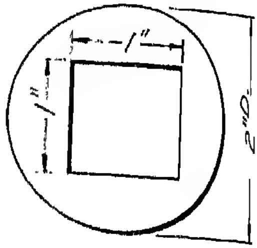
8. A hot-water heating coil is made up of copper tubing which measures $\frac{3}{4}$ in. in diameter outside. The coil is in four sections each 38 in. long. How many square feet outside heating surface are there in this coil?

9. Sheet copper weighing 0.82 lb. to the square foot costs 32¢ per pound. At this rate what is the cost of the copper used in making 16 drip pans as illustrated in the drawing to the right, allowing 10% for lapping and waste.



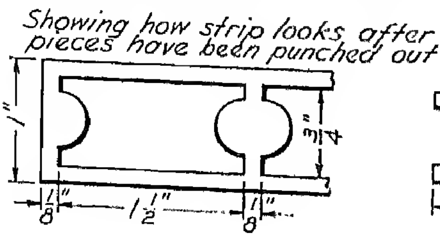
10. How many square yards in a roll of wallpaper that measures 18 in. wide and 24 ft. long?

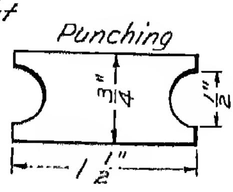
11. Two hundred and twenty wrought-iron blanks 2 in. in diameter as shown at the right are to be made on a special order. The material out of which these blanks are to be made weighs 3.3 lb., per square foot. What will be the total weight of these 220 blanks?



12. From a rectangular piece of sheet iron 36 in. wide and 42 in. long by $\frac{1}{8}$ in. thick, a circular piece $2\frac{1}{2}$ ft. in diameter is to be cut. If this metal weighs 5 lb., per square foot area, what will be the weight of the material left over, after the circular piece is cut out.

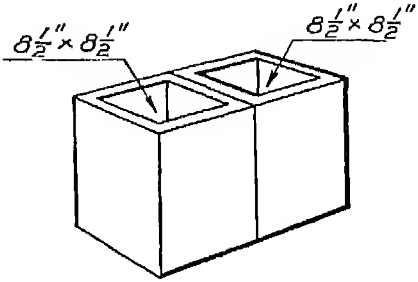
13. How many pounds of metal scrap remain after blanking out 200 of the pieces as shown below from a strip of copper 1 in. wide? This strip is made up in roll form and weighs 0.54



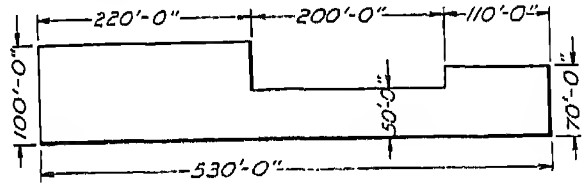


lb., per square foot. The first punching begins $\frac{1}{8}$ in. from the end of the strip and an allowance of $\frac{1}{8}$ in. is made between punchings as shown. For convenience the strip is clipped off 1 in. from the last punching.

14. What is the area in square feet of the flue openings in the sketch to the right?

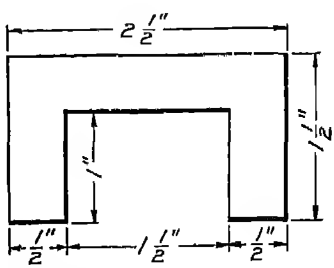


15. A piece of land having the dimensions below is advertised for sale as containing 1 acre. What does it contain?



16. It is advertised that one quart of a certain grade of varnish will cover 9 sq. yd. two coats when applied to hardwood floors. At this rate how many quarts are needed to varnish a floor two coats that measures 21 ft. × 15 ft.?

17. The sheet iron out of which the lamination to the right was made for a small transformer weighs .5 lb., per square foot. From this determine the weight of 720 such pieces.



*Answers to these problems will be found on page 228.

ANSWERS TO PROBLEMS

Pages 213 to 218.

1. 1419 sq. ft.
2. 25.13 sq. in.; 16.3%.
3. 0125 sq. ft.
4. 49.82 sq. ft.
5. $1\frac{17}{64}$ sq. in.
6. 257.93 sq. in.
7. $48.
8. 1.629 sq. in.; 97.75 lb.
9. $1\frac{5}{32}$ sq. in.
10. $1\frac{27}{64}$ sq. in.
11. 1 sq. in.
12. 3.44 sq. in.
13. 4.12 sq. in.
14. (*a*) 8.079 sq. in.; (*b*) 5 sq. in.; (*c*) 3.643 sq. in.
15. 147.3 lb.
16. 5.07 lb.; 23 strips.
17. 4190 sq. ft.
18. $\frac{31}{64}$ sq. in.
19. 71.7%.
20. 505.488 sq. ft.
21. 68 sq. ft.
22. $1\frac{11}{32}$ sq. in.
23. 19.24 min.
24. 8 bags.
25. 7938.78 lb.

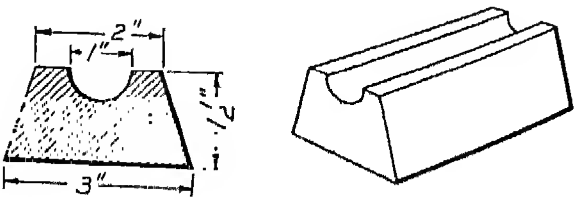
Pages 224 to 227.

1. 48 sq. yd.
2. $9.60.
3. 6 gal.
4. 3.77 sq. ft.
5. $1\frac{1}{2}$ acres.
6. 92,500 lb.
7. 28.68 sq. ft.
8. 2.49 sq. ft.
9. $8.46.
10. 4 sq. yd.
11. 10.8 lb.
12. 27.96 lb.
13. .526 lb.
14. 1 sq. ft.
15. .91 acre.
16. $3\frac{9}{16}$ or 4 qt.
17. $5\frac{5}{8}$ lb.

Review Problems Involving Reduction of Areas

1. A strip of rubber matting used in a gymnasium locker room measures 27 in. wide and 24 ft. long. How many square yards in this strip?

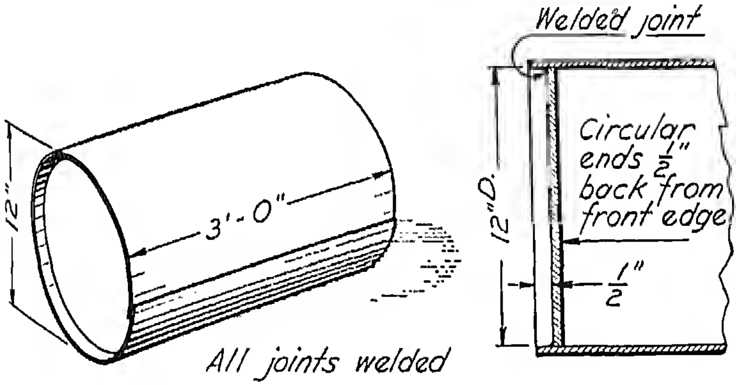
2. Calculate the cross-section area of the casting in the following sketch.



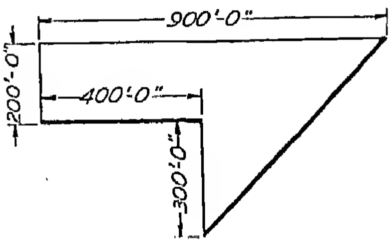
3. A concrete sidewalk 5 ft. wide is to be constructed around the outside of a circular plot of ground that measures 60 ft. in diameter. Upon how many square yards of sidewalk surface should the estimate for labor cost be based?

4. Sheet tin is required in lining a wooden tank having inside dimensions of 18 in. deep × 24 in. wide × 24 in. long. How many square feet of metal does this equal?

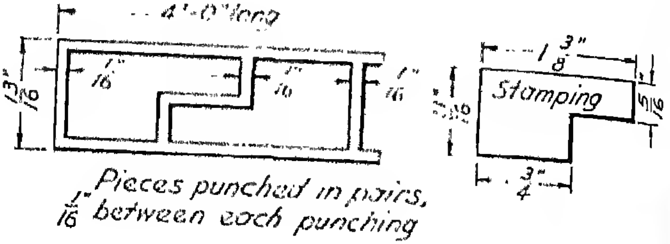
5. In the following drawing of a steel tank it is noted that all joints are to be welded. Calculate the weight of sheet steel in this tank allowing $1\frac{1}{4}$ in. for the lap joint on the cylinder. The circular ends are cut to fit the diameter given. This sheet steel is listed as weighing $1\frac{1}{3}$ lb. per square foot.



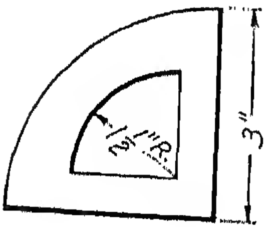
6. Determine the number of acres in the following irregular plot of land.

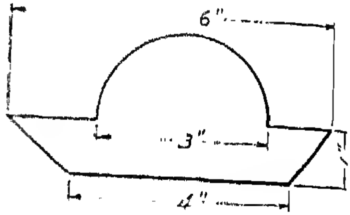


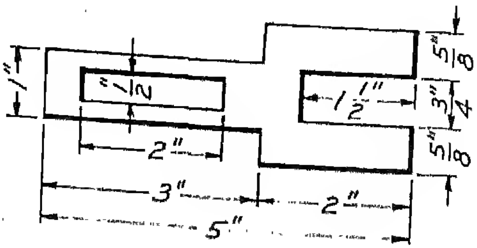
7. How many strips of brass each 4 ft. long are needed for 420 pieces as per the following drawing? If this stock weighs .72 lb. per square foot, what is the weight of these 420 pieces?



8. Calculate the areas in square inches of the following flat shapes.







Calculations Involving Square Measure as Applied to Various Trades

Calculations involving square measure are quite common in various branches of the building trades, such as estimating areas, materials, and costs. This is especially so in lathing, plastering, brickwork, shingling, painting, and paperhanging. Square measure is used also in the printing trades, in estimating composition and paper.

While these applications are rather extensive, only a few of the more elementary conditions will be considered here, so that the student may see how square measure may be used to advantage in solving such practical problems.

LATHING

Lathing work is estimated according to the number of square yards of surface to be covered. In figuring costs, or materials, in this branch of work, it is the rule in many localities to deduct from the total area under consideration one half the area of such openings as windows, doors, and the like, that lie in the surface being calculated. This deduction is made because of the extra time and labor required in performing the more difficult task of "finishing up" to an opening. Sometimes a certain percentage of the total area to be plastered is deducted for such openings, instead of calculating these openings separately.

In such cases the amount to be deducted should be known before the correct surface estimate can be given.

Both metal and wooden laths are used in building construction. The metal lath may be obtained in various sizes, a common size being 24 by 96 in. The standard size wooden lath comes in bundles of 100, each lath being $\frac{1}{4}$ in. thick, $1\frac{1}{2}$ in. wide, and 48 in. long.

A bundle of wooden laths, $1\frac{1}{2} \times 48$ in. will cover approximately 7 sq. yd.

To calculate the number of bundles of laths needed for a given job, the area in square yards to be covered is divided by 7. A fractional part of a bundle is considered as a whole bundle in such calculations. How this is applied is shown.

Example:

At 75 cents per bundle what is the cost of laths needed to cover the side walls and ceiling of a room that measures 18 ft. long, 16 ft. wide, and 10 ft. high? A deduction of 45 sq. ft. is allowed for doors and windows.

Solution and Explanation:

The first step in the solution of this type of problem is to find the total area of the side walls and ceiling. After this is determined, 45 sq. ft. are deducted for the openings as mentioned.

The remaining number of square feet is then reduced to square yards. By dividing this number of square yards by 7, the number of bundles may be determined.

Area of all side walls = perimeter of room × height.

Perimeter of room = 18 + 16 + 18 + 16, or 68 ft.

The square foot area of the side walls = 68 × 10, or 680 sq. ft.

The square foot area of the ceiling = 16 × 18, or 288 sq. ft.

The combined area of side walls and ceiling becomes 680 + 288 or 968 sq. ft. Deducting 45 sq. ft. from this amount, there results:

$$968 - 45, \text{ or } 923 \text{ sq. ft.}$$

This, reduced to square yards is, $923 \div 9 = 102\frac{5}{9}$ sq. yd.

Since one bundle of 100 laths will cover 7 sq. yd., then in order to cover $102\frac{5}{9}$ sq. yd. it will take as many bundles as 7 is contained in $102\frac{5}{9}$, which equals $102\frac{5}{9} \div 7$, or 14.65.

As previously explained a portion of a bundle is to be considered as a whole bundle in estimating. Therefore, it will take 15 bundles for the side walls and ceiling of the room.

At 75¢ per bundle, 15 bundles will cost 15 × .75, or $11.25.

Hence, it will cost $11.25 for the laths necessary for the above job.

PLASTERING

Plastering also is estimated by the square yard. As in lathing, it is usually the custom to deduct one half the area of doors, windows, and such openings as are in the surfaces to be

plastered, in order to obtain the correct area upon which the estimate is to be based.

When solving problems in plastering, the total area in square feet is first determined. From this is subtracted one half the area of the openings. The remainder reduced to square yards equals the amount of the surface on which the plastering estimate is to be made.

The following is typical of such problems.

Example:

A ceiling measuring 22 ft. by 32 ft., and having a 4-ft. circular opening in the center for a skylight, is to be plastered. At \$1.50 per square yard what is the cost of plastering this ceiling after deducting one half the area of the circular opening?

Solution and Explanation:

This problem is worked out in a manner similar to the one on lathing. The total area of the ceiling is first determined. From this one half the area of the circular opening is deducted. The area in square feet remaining represents the correct amount on which the estimate is to be made.

Area of ceiling $= 22 \times 32$, or 704 sq. ft.

Area of circular opening $= .7854 \times 4 \times 4$, or 12.57, which is approximately $12\frac{1}{2}$ sq. ft.

Deducting one half of this circular area, $6\frac{1}{4}$ sq. ft., from the total area of the ceiling there results,

$$704 - 6\tfrac{1}{4}, \text{ or } 697\tfrac{3}{4} \text{ sq. ft.}$$

This is the square foot area on which the estimate is to be made.

Changed to square yards this equals,

$$697\tfrac{3}{4} \div 9 = 77.53, \text{ or } 78 \text{ sq. yd.}$$

At \$1.50 per square yard the cost of plastering this area equals:

$$78 \times \$1.50, \text{ or } \$117.00.$$

That is, the cost of plastering this ceiling is \$117.00.

BRICKWORK

The standard size of the common brick is $8 \times 3\frac{3}{4} \times 2\frac{1}{4}$ in. It is estimated that in ordinary brickwork there are $6\frac{1}{2}$ of these bricks to each square foot of surface. While this number varies slightly according to the width of the joint between the bricks, $6\frac{1}{2}$ will be used in the calculations which follow.

If a brick wall were made 1 ft. thick it would require 3 rows of these bricks. This would mean that in such a wall there would be $3 \times 6\frac{1}{2}$, or $19\frac{1}{2}$ bricks for each square foot of wall surface.

A wall 4 rows thick would be 16 in. thick, and would contain $4 \times 6\frac{1}{2}$ or 26 bricks to each square foot of wall surface.

According to the above, the number of bricks in a given wall 1 ft. thick would be determined by multiplying each square foot area (length $\times$ height) of that wall by $19\frac{1}{2}$. If the wall is 16 in. thick the square foot area is multiplied by 26.

The following problem shows an application of this rule.

Example:

Determine the number of bricks needed to construct a wall 1 ft. thick, 6 ft. high, and 120 ft. long.

Solution and Explanation:

The first step in the solution of this problem is to determine the area of this wall in square feet. This is then multiplied by $19\frac{1}{2}$ giving as a result the number of bricks needed.

$$\text{Area of wall} = 120 \times 6 = 720 \text{ sq. ft.}$$

There being $19\frac{1}{2}$ bricks to each square foot of this wall space, then in 720 sq. ft. there would be $720 \times 19\frac{1}{2}$, or 14,040 bricks.

That is 14,040 bricks would be needed to construct the above wall.

Should this wall have been 16 in. thick, then it would have required 720×26, or 18,720 bricks, for according to the above rule there would be 26 bricks in a 16-in. wall for each square foot of wall surface.

Problems Involving Calculations in Lathing, Plastering, and Brickwork

1. How many square yards are to be estimated on in plastering two sides of a partition wall 12 ft. high and 24 ft. long. This partition contains two openings each of which measures 4 ft. by $6\frac{3}{4}$ ft. An allowance of 50% is to be made for these openings.

2. At 75¢ per bundle what will laths cost to cover the sides of a room 16 ft. wide, 20 ft. long, and 10 ft. high? There is to be a deduction of 54 sq. ft. for doors and windows.

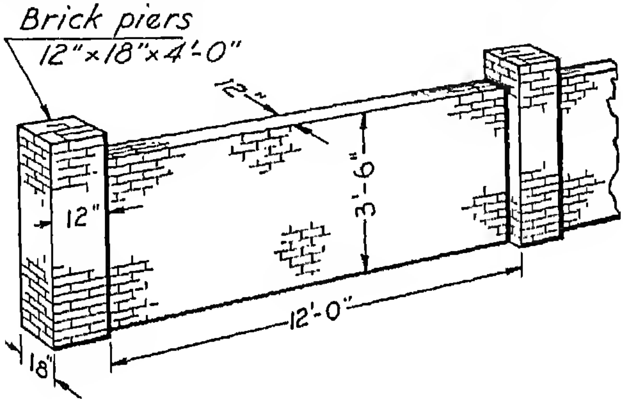
3. A brick wall 1 ft. thick, 4 ft. high, and 180 ft. long is to be constructed using the standard size brick. There are two openings in the wall, one measuring 20 sq. ft. and the other 28 sq. ft. How many bricks are needed for this wall, full deduction being made for the openings mentioned?

4. Determine the number of square yards to be plastered in a room measuring 16 ft. 6 in. wide, 18 ft. long, and 10 ft. high, including side walls and ceiling. No deduction is to be made for openings.

5. The side walls and ceiling of a hall measuring 13 ft. high, 16 ft. wide, and 45 ft. long are to be lathed and plastered.

A deduction of 80 sq. ft. is to be made for openings. How many bundles of laths are needed for this job? Allowing the full amount for deductions, how many square yards of surface are to be plastered?

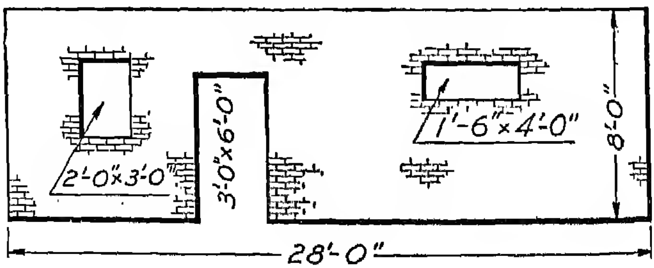
6. A brick fence is to be built as shown in the following sketch. It has 10 sections each 12 ft. long, $3\frac{1}{2}$ ft. high, and 1 ft. thick; also 11 brick piers each 18 in. wide by 12 in. deep, and 4 ft. high. Calculate the number of bricks needed.



7. How many bundles of laths will be required to replace a ceiling that measures 21 ft. by 27 ft. How many square yards of plastering will there be in this ceiling?

8. How many square yards should be estimated on in plastering a ceiling that measures $13\frac{1}{2}$ ft. wide and 26 ft. long?

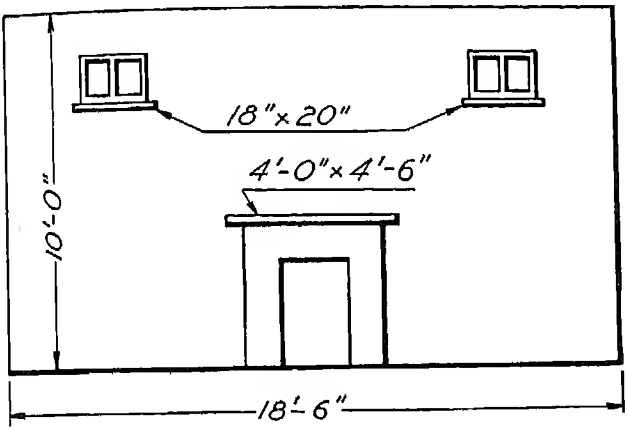
9. How many bricks are needed to build the following section of a 12-in. wall? Full deduction is to be made for openings.



10. How many square yards of plastering should be estimated on in plastering a hall ceiling measuring 9 ft. wide and 26 ft. long?

11. How many bundles of laths are needed to cover both sides of a partition $10\frac{1}{2}$ ft. high and 18 ft. long? A deduction of 22 sq. ft. is to be made for openings.

12. At $1.50 per square yard, determine the cost of plastering the wall in the following sketch, making full deductions for openings.*



SHINGLING

While wooden shingles are still used considerably, the asphalt shingle, the asbestos shingle, the composition shingle, and the strip shingle, have been used to a great extent during recent years.

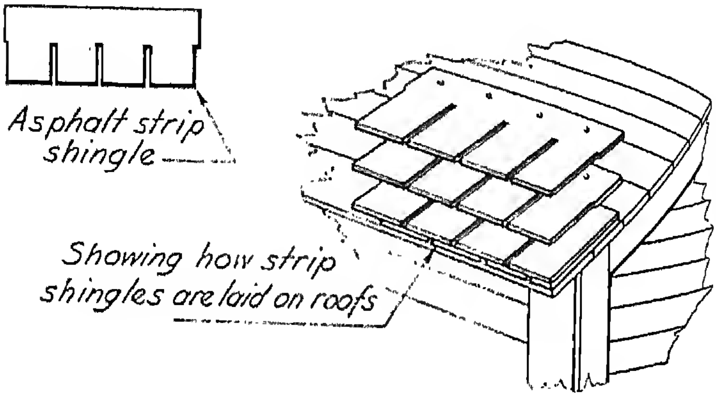
Wooden shingles are put up in bundles of 250 each, and it takes approximately 4 such bundles or 1,000 shingles to cover 100 square feet, or "1 square," as it is often called.

The asphalt *strip shingle* consists of 3, sometimes 4 shingles to the strip. Although there are several brands and sizes of such strips, a common size is that which measures $12\frac{1}{2}$ in.

*Answers to these problems will be found on page 260.

wide and 40 in. long. When "*laid on* 4 inches *to the weather*," this size requires 90 such strips to cover 100 square feet.

Another common size measures 10 × 36 in. and is put up 50 to the bundle. Each bundle of this size covers 50 square feet and 2 such bundles are needed to cover 100 square feet of roof surface.

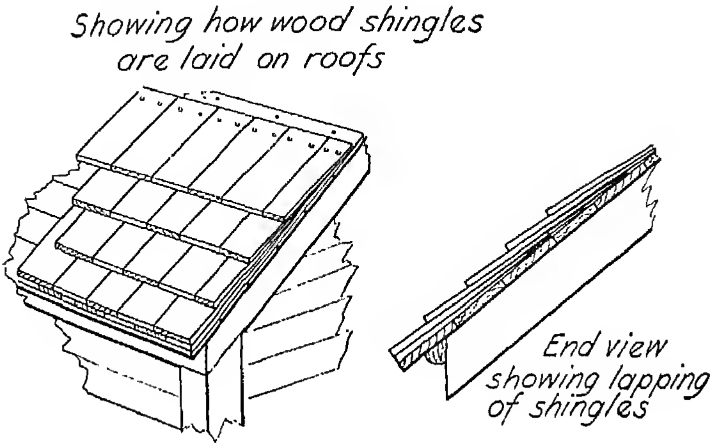


The *single type* of asphalt shingle is sold in various sizes, one of which measures 9 × 12½ in. These are put up in bundles of 95 each, and 4 such bundles, or 380 shingles, are needed to cover one square of 100 square feet, when laid 4 in. to the weather.

While there are other kinds of shingles in use, only calculations relating to those kinds and sizes above mentioned will be considered here, as it is felt that this will give sufficient information as to how square measure is applied in such work.

These shingles are laid on in *courses* as illustrated, so that the ends are exposed to the weather. In laying wooden shingles the end exposed is the heavy, or the thick, end. The lower shingle projects out 4 inches, sometimes 5 inches, from the one above it, and as the expression goes the "shingles are laid 4 in. to the weather."

The method of lapping wooden shingles in this manner may be seen in the following illustration.



To calculate the number of bundles of wood shingles needed to cover a given surface, the number of squares, or the number of 100 sq. ft. to be covered is first calculated. This is then multiplied by 4, as it takes approximately 4 bundles to cover 1 square, or 100 square feet. If the calculation results in a fractional part of a bundle, this is considered as a whole bundle in estimating.

If the asphalt strip shingles are to be used the number of squares is multiplied by the number of strips it takes to cover one square.

The asbestos shingle is made up in both the single type and the strip type. Their sizes vary somewhat, like the asphalt shingle. One common size asbestos shingle measures 8 × 16 in., and it takes 260 of them to cover 100 square feet. This special size is packed 13 shingles to the bundle making 20 bundles to 100 square feet. A common size asbestos strip shingle is one that measures 10 × 36 in. These strips come in bundles of 50 each, and 100 such strips are required to cover 100 square feet.

If the asbestos shingle is to be used the number of squ is multiplied by 20. This gives the number of bundles need In this calculation as in previous calculations a fractional of a bundle is considered a whole bundle.

If the asbestos strip shingle is to be used, the number strips needed equals the number of square feet to be cover This divided by 2 gives the equivalent number of bundl

These various types of shingles, or slight modifications them, are also used as "siding" in covering the sides of hous garages, barns, and such constructions.

Among the asbestos types of shingles, a special size referred to as the "siding shingle" seems to have become popular in certain sections of late. One common size as made by a well-known manufacturer, measures 12 × 24 in. and is put up 19 to the bundle, requiring 3 such bundles to cover 100 square feet.

The method of estimating quantities of shingles needed for siding is similar to that followed in estimations on roofing shingles. Some builders, however, deduct a standard amount of 15 square feet for each window and 20 square feet for each door instead of calculating out these areas. It is customary in using this kind of shingle to allow a small percentage for breakage and waste.

Calculations involving the use of these rules are illustrated in the following problems.

Example 1:

How many bundles of wood shingles are needed to cover a roof that measures 18 by 24 ft. The shingles are to be laid 4 in. to the weather.

Solution and Explanation:

The number of squares to be covered should first be determined. This is then multiplied by 4, giving the number of bundles required.

> $18 \times 24 = 432$, the square foot area to be covered.
> $432 \div 100 = 4.32$, or the number of *squares* to be covered.
> $4.32 \times 4 = 17.28$, which is the calculated number of bundles necessary.

Since a fractional part of a bundle is considered as equal to a whole bundle in estimating, then the required number of bundles, instead of being 17.28, becomes 18.

Example 2:

Determine the number of $12\frac{1}{4} \times 40$-in. asphalt strip shingles, laid 4 in. to the weather, that are needed to cover a shed roof measuring 15 ft. by 20 ft.

Solution and Explanation:

Following the general plan of the above problem,

> $15 \times 20 = 300$, the square foot area of the roof.
> $300 \div 100 = 3$, or the number of squares to be covered.
> $90 \times 3 = 270$, the number of strips needed to cover the roof.

That is, it takes 270 of the $12\frac{1}{4} \times 40$-in. size asphalt shingle strips to cover the above roof.

Example 3:

If it were decided to use a single asphalt shingle $9 \times 12\frac{1}{4}$ in. instead of the strip shingle in covering the roof in the above

example how many such shingles would be needed? How many bundles would this equal?

Solution and Explanation:

The area of this roof equals:

$$15 \times 20, \text{ or } 300 \text{ sq. ft.}$$

Reduced to squares this becomes, $\frac{300}{100}$, or 3 squares to be covered.

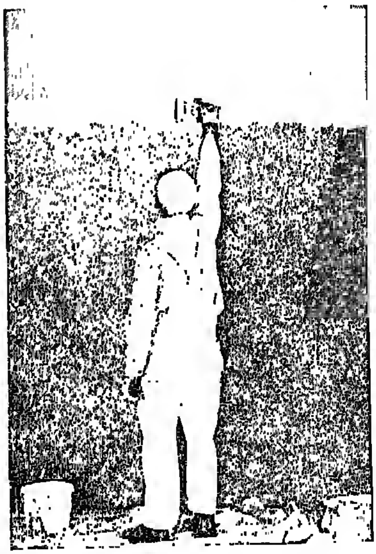
Since 380 of this type shingle are required to cover one square, then a total of 380×3, or 1140 shingles are needed to cover this roof.

Reduced to bundles each having a coverage of 95 sq. ft., this becomes: $1140 \div 95$ or 12.

That is, 1140 shingles are needed to cover the above roof. Since they come 95 shingles to the bundle, it will be necessary to purchase 12 bundles.

PAINTING

Although some painters estimate labor costs by the square yard, the more generally followed plan is that of estimating by the 100 square feet of surface to be painted. As a rule, no deductions are made for openings when estimating such costs, unless such allowances are definitely stated.



While various estimates, ranging from 400 sq. ft. to 800 sq. ft., are given as to the amount of surface that might be covered by a gallon of paint, it will be sufficiently accurate in the problems that follow to assume that one gallon will cover 500 sq. ft. of surface one coat.

On this basis, it will also be assumed that one gallon of paint used for both a first and a second coat would cover 250 sq. ft.

For unpainted, rough, porous, or new surfaces, the coverage in square feet would be slightly less than this amount depending upon the condition of the surface.

These same figures may also be used for the amount of surface that can be covered by a gallon of varnish, although, as in the case of paints, various estimates of coverage may be given in commercial advertisements.

The following case shows how these rules are used in practice.

Example:

How many gallons of paint are needed for two coats of paint on the side walls and ceiling of a room that measures 27 ft. long, 20 ft. wide, and 11 ft. high? A deduction of 100 sq. ft. is to be made for openings. At \$3 per gallon, what is the cost of this paint?

Solution and Explanation:

In such problems the number of gallons of paint required is determined by dividing the square foot area to be painted by 250, as two coats are required.

> Area of side walls = perimeter × height.
> Perimeter = 27 + 20 + 27 + 20, or 94 ft.
> Area of side walls = 94 × 11, or 1034 sq. ft.
> Area of ceiling = 27 × 20, or 540 sq. ft.
> Total area of side walls and ceiling = 1034 + 540, or 1574 sq. ft.

Deducting 100 sq. ft. for the openings, the estimated area to be painted is, 1574 − 100, or 1474 sq. ft.

The number of gallons needed to cover this surface two coats is, 1474 ÷ 250, which is 5.88, or 6 gallons.

At \$3 per gallon the cost of this is 6 × \$3, or \$18.

PAPERHANGING

Square measure is also used in estimating the amount of wallpaper required for a given job. A generally accepted

method of calculating such amounts is to determine the square foot area to be covered and, after making the allowable deductions, divide the remaining square foot area by the square foot area of a roll of paper.

In such estimates, a fractional part of a roll is considered as a whole roll.

It is a common rule to deduct one half the area of openings above a minimum limit, from the total area to be covered in estimating the number of rolls required. A percentage of the net area is sometimes added to make up for losses due to matching patterns and to waste.

Domestic wallpaper comes in single rolls 18 in. wide by 24 ft. long, and in double rolls which are 48 ft. long and 18 in wide. Imported wallpaper, also some domestic paper, may vary slightly from these dimensions, but the general method of estimating the number of rolls required is the same.

The single roll of wallpaper which measures 18 in., or $1\frac{1}{2}$ ft wide and is 24 ft. long contains $24 \times 1\frac{1}{2}$, or 36 sq. ft.

The double roll which is $1\frac{1}{2}$ ft. wide and 48 ft. long contains

$$48 \times 1\tfrac{1}{2}, \text{ or } 72 \text{ sq. ft.}$$

In determining the number of rolls of wallpaper required for a given room the procedure is as follows:

1. Determine the perimeter of the room.

2. Multiply the perimeter by the height.

3. Subtract from the above area the square foot area allowed for openings.

4. If any special additions are to be made for waste or matching patterns, they should be added to the above net result at this point.

5. The resulting square foot area is then reduced to rolls by dividing it by 36, if single rolls are to be used; or 72, if double rolls are to be used, unless other than the standard size roll is used.

6. If the ceiling is to be papered, its area is found by multiplying the length of the room by its width. To this square foot area is added allowances for waste and matching. The number of standard single rolls needed would be found by dividing the square foot area by 36. The number of standard double rolls would be found by dividing the ceiling area by 72.

These rules are applied in such problems as the following.

Example:

Determine the number of single rolls of wallpaper needed for side walls and ceiling of a bedroom that measures 18 ft. long by 15 ft. wide by $9\frac{1}{2}$ ft. high. In this room are 2 doors that measure 3×7 ft. and 3 windows that measure 3×5 ft. Full deduction is to be made for these openings. There is also to be an addition of 10% for waste in cutting and matching.

Solution and Explanation:

Perimeter of room $= 18 + 15 + 18 + 15$, or 66 ft.
Height of room $= 9\frac{1}{2}$ ft.
Square foot area of walls $= 66 \times 9\frac{1}{2}$, or 627 sq. ft.
Area to be deducted equals:

$$\begin{aligned} \text{Doors} &= 2 \times 7 \times 3 = 42 \text{ sq. ft.} \\ \text{Windows} &= 3 \times 5 \times 3 = \underline{45} \text{ sq. ft.} \\ \text{Total} &= 87 \text{ sq. ft.} \end{aligned}$$

Side wall surface to be covered equals:

$$627 - 87 = 540 \text{ sq. ft.}$$

To the above there is to be added 10% for cutting and matching patterns. This equals 10% of 540, or 54 sq. ft.

The total square foot area on which the side-wall paper is to be estimated thus equals $540 + 54$ or 594 sq. ft.

To determine the number of single rolls needed for the side walls, 594 is divided by 36. This equals:

$$594 \div 36 \text{ or } 16\tfrac{1}{2} \text{ rolls}$$

Since a fractional part of a roll is to be considered as a full roll in estimating, then the required number of rolls is 17.

Because a different pattern is used for the ceiling paper, as a rule, the requirements for the ceiling are figured separately.

Area of ceiling = 18 × 15, or 270 sq. ft.

Adding 10% for waste and matching, the total area is increased by 10% of 270, or 27 sq. ft.

The total area on which the ceiling paper is to be estimated then becomes 270 + 27, which equals 297 sq. ft.

To find the number of single rolls needed for this area, 297 is divided by 36. This equals:

$$297 \div 36 = 8\tfrac{1}{4}, \text{ or } 9 \text{ rolls.}$$

From these calculations it is seen that 17 single rolls are needed for the side walls and 9 single rolls are needed for the ceiling.

Problems in Shingling, Painting, and Paperhanging

1. How many gallons of paint are needed for two coats of paint on the side walls of a room that measures 14 ft. wide, 23 ft. long, and 10 ft. high? No allowance is to be made for openings.

2. Three ceilings in a house are to be painted two coats each. One ceiling is 12 by 18 ft., another 10 by 15 ft., and the third 8 by 15 ft. All ceilings are to be the same color. How many gallons of paint are needed for this job?

3. Determine the number of strips of asphalt shingles measuring $12\frac{1}{2} \times 40$ in. that are needed for a flat sloping roof that measures 15 by 30 ft., where the strips are laid 4 in. to the weather.

4. A hall ceiling 12 ft. wide and 35 ft. long is to be papered with double-roll wallpaper. No allowance is to be made for matching or waste. What is the cost of this paper at 32¢ per roll?

5. A barn, which is to be painted one coat, has a front and back each measuring 40 × 30 ft. The two sides each measure

65 × 30 ft. How many gallons of paint will be needed for this job?

6. The side walls and ceiling of a room 15 by 18 by 9 ft. high are to be papered. There is to be a deduction of 36 sq. ft. for side-wall openings, but no allowance for waste. How many single rolls will be required for the ceiling? How many for the side walls?

7. How many bundles of wood shingles laid 4 in. to the weather are needed to cover a common pitched roof, each side of which measures 16 by 30 ft.?

8. If one gallon of varnish will cover 500 sq. ft. of surface one coat, how many gallons will be needed to varnish a hall floor one coat that measures 40 by 72 ft.?

9. The side walls of a dining room 20 ft. wide by 20 ft. long and 10 ft. high are to be papered with an imported paper that measures 2 ft. wide and 36 ft. long. No deduction is to be made for openings, but an addition of 10% is to be made for waste and matching. Calculate the number of rolls required.

10. Determine the number of gallons of paint that are needed to cover a circular platform 2 coats that measures 25 ft. in diameter.

11. The side walls and ceiling of a hall 10 ft. wide by 28 ft. long and 10 ft. high are to be papered with double-roll wall-paper. How many rolls are required for this job if full deduction is to be made for four side-wall openings each measuring 4 by 7 ft.?

12. Ten cylindrical columns each 15 in. in diameter and 11 ft. long are to be painted two coats. Upon how many square feet should the labor estimate be made? How many gallons of paint are needed for this job?

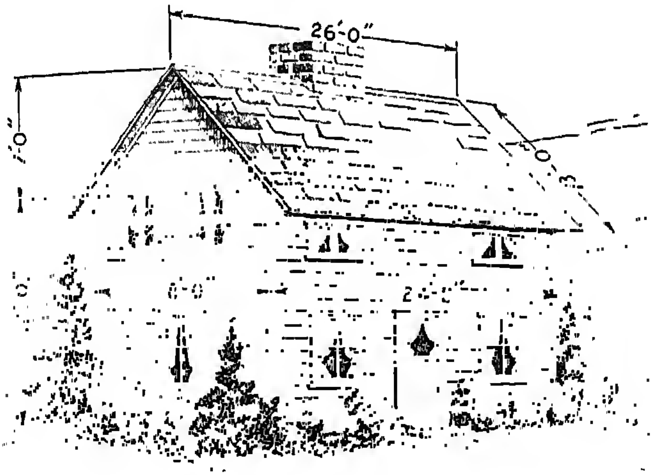
13. The main roof of a house is to be reshingled with asphalt strip shingles 10 × 36 in. laid 4 in. to the weather. Each of

the 2 sloping sides of the roof measure 15 × 30 ft. How many bundles of these shingles are needed?

14. How many gallons of paint will be needed to paint a strip 6 ft. wide two coats, around the outer edge of a floor which has a total width of 52 ft. and a total length of 85 ft.?

15. Determine the number of single rolls of wallpaper that are needed to cover the side walls of a room 9 ft. high, 16 ft. wide, and 22 ft. long. No allowance is to be made for openings or matching.

16. What is the area upon which the labor estimate should be calculated in applying 2 coats of paint to both sides of a board fence 6 ft. high and 165 ft. long? Because of the posts and stringers upon which the boards are nailed an addition of 10% is to be allowed. How many gallons of paint will be needed for this work?



17. The clapboards on the four sides of the cottage in the sketch are to be replaced with asbestos siding shingles which require three bundles to the 100 sq. ft. In calculating the square foot area to be covered, 150 sq. ft. are to be deducted

for windows and 40 sq. ft. are to be deducted for doors. How many bundles of siding shingles are needed for this job allowing 5% for wastage?

18. If it were decided to reshingle the roof of this cottage with a grade of asbestos shingle that requires 20 bundles to the 100 sq. ft. of roof surface, how many bundles would be needed?

19. Twenty square wooden pillars each measuring 15 in. by 15 in. by 10 ft. are to be painted 2 coats of white paint. How many gallons of this paint will be needed for the job?

20. The roof of a garage is to be reshingled with wood shingles laid 4 in. to the weather. This roof is of the common gable type and each side measures 12 by 25 ft. How many bundles of wood shingles will be needed for this?

21. Varnish that is advertised as covering 250 sq. ft. of surface two coats to the gallon is to be used in varnishing two coats on a floor that measures 40 ft. wide and 62 ft. long. How many gallons of this varnish are needed?

22. Determine the number of asphalt shingle strips of the size $12\frac{1}{2}$ by 40 in. that are needed to cover a gable-type roof each side of which measures 15 by 30 ft.

23. The wooden shingles on a flat sloping roof that measures 38 ft. long and 10 ft. wide are to be replaced with asbestos shingle strips. If two bundles of these shingles will cover 100 sq. ft., how many bundles will be required to cover this particular roof after adding 5% to the calculated area for spoilage?

24. At the close of its basketball season a school basketball team agrees to varnish its basketball court which measures 45 ft. wide and 70 ft. long. It is decided to apply two coats of varnish, and the team agrees to purchase a recommended grade of varnish that will cover 200 sq. ft. two coats to the gallon. If this varnish costs $3.50 per gallon, what is the total cost of the varnish needed for this job?*

*Answers to these problems will be found on pages 260 and 261.

PRINTING: TYPE MEASURE AND PAPER MEASURE

Another important use that is made of square measure is that of calculating the number of words to the printed page, also the number of pages needed for a piece of printed work.

One of the most important changes is our new system of lining. A glance at the specimen sheets issued within the past few years will show that the constant demand for something of this kind has led to ever recurrent efforts to solve the problem; but these attempts have been sporadic and inconsistent to this date, the failure to take into consideration every requirement causing the results to be very unsatisfactory. All our type is cast on the perfect new "Standard" Line system, including all Romans and Italics, Antiques and Gothics, and all other job

8-POINT TYPE

23 WORDS PER SQ. INCH (APPROX.)

One of the most important changes is our new system of lining. A glance at the specimen pages issued during the past few years will show that a constant demand for something of this kind has led to ever recurrent attempts to solve this weighty problem; but these attempts have been sporadic and inconsistent, failure to take into consideration all the requirements having rendered the results very unsatisfactory. All our type is cast on the new

10-POINT TYPE

16 WORDS PER SQ. INCH (APPROX.)

One of the most important changes is our new system of lining. A glance at specimen sheets issued during the past few years will show a constant demand for something of this kind has led to ever recurrent efforts to solve the problem; but these attempts have been sporadic and inconsistent, the

12-POINT TYPE

11 WORDS PER SQ. INCH (APPROX.)

Calculations relating to this are carried out by first estimating the number of words to the square inch. This, of course, varies according to the size of the type used, and to the manner in which it is spaced.

Some sizes have an average of 11 words to the square inch, other sizes range up to 23 words, and even higher.

Knowing the number of square inches of printed matter on a given page, the total number of words to that page is found by multiplying the area of the page in square inches by the number of words to the square inch.

In turn, the number of pages required for a given piece of printed matter may be determined by dividing its total number of words by the number of words to the printed page.

This rule is used in such practical problems as the following.

Example:

How many pages of 4 × 7-in. printed matter will be needed to set up an article containing 24,000 words, there being on an average 20 words to the square inch when the matter is set up?

Solution and Explanation:

The number of square inches of type matter on one page is 4 × 7 in. or 28 sq. in. Allowing 20 words to the square inch, one page would then contain 28 × 20, or 560 words.

With 560 words to the page, it will take as many pages to print the 24,000 words as 560 is contained in 24,000, which equals $42\frac{6}{7}$, or 43 pages.

PAPER MEASURE

It is the custom in the printing trade to express the size and weight of paper as follows: 25 × 38 in. — 80 lb.

This means that the size of this particular sheet, which is usually referred to as "stock," measures 25 × 38 in., and that the weight of one ream of it is 80 lb. This weight is known as "substance," or "basic weight" by printers. Should the weight be referred to in other quantities than the ream it is so specified.

A ream of paper when sold in bulk is considered as 500 sheets. When sold at retail in quire packages, however, a ream equals 12 quires there being 24 sheets to the quire. This would make 488 sheets to the ream if sold in this manner.

It is often necessary in printing to determine the weight of another ream of paper of the same kind, or same basic weight, but in a different size. This is done by multiplying the *weight* of the known paper by the *size* of the paper whose weight is to be determined. The resulting product is then divided by the size of the known paper. The quotient is the weight of a ream of the paper desired.

In calculating such weights of paper, the *nearest* pound is always used, and the fractional weight under one-half pound is discarded.

The following is a practical application of this rule.

Example 1:

If a ream of paper 17 × 22 in. weighs 24 lb., what is the weight of a ream of the same paper measuring 22 × 34 in.?

Solution and Explanation:

As previously explained the weight of the known paper, 24 lb., is multiplied by the square inch area (22 × 34) of the "unknown" paper. This product is then divided by (17 × 22) the area of the known paper.

By cancellation this works out as follows:

$$\frac{\cancel{22} \times \overset{2}{\cancel{34}} \times 24}{\cancel{17} \times \cancel{22}} = 48$$

That is, the weight of a ream 22 × 34 in. on the same basis as the 17 × 22 in. — 24 paper equals 48 lb.

When it is desired to find how many sheets of a certain size can be cut from a given "stock size" of paper, the process also becomes one of cancellation. That is, the product of the dimensions of the stock-size sheet are divided by the product of the dimensions of the required size. This gives the number of smaller sheets that may be cut from one of the larger sheets.

Multiplying this number by the number of stock sheets to be cut up, will give the total number of smaller sheets that may be obtained as a result of this cutting operation.

How this rule is applied is shown in the following problem.

Example 2:

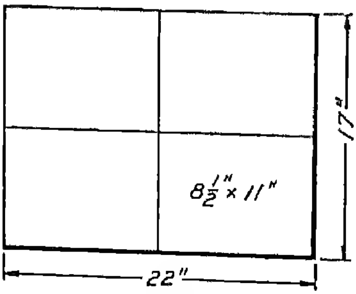
How many letterheads, $8\frac{1}{2} \times 11$ in. can be cut from one ream of paper (500 sheets) measuring 17×22 in.?

Solution and Explanation:

This problem is worked out as in ordinary cancellation, as explained, by first dividing the size of the larger sheet by the size of the letterhead. The result of this division gives the number of letterheads that can be cut from one full sheet. This quotient is then multiplied by 500, the number of large sheets in one ream.

$$\frac{\overset{2}{\cancel{17}} \times \overset{2}{\cancel{22}}}{\cancel{8\frac{1}{2}} \times \cancel{11}} = 4,$$ which equals the number of smaller sheets in one large size sheet.

$500 \times 4 = 2000$, equals the total number of letterheads that can be cut from one ream.



By laying out a plan showing how the large sheet is divided up, it is seen that the paper cuts exactly, and there is no waste.

In this method of calculating the amount of stock necessary for a job, the cancellation of the numerators and the denominators may not always come out evenly, however.

In such cases where there is this uneven cancellation, there is always some waste in cutting the sheets. This can best be explained by a few practical examples. In each case, however, it is well to make a layout of the sheet so as to show how the division takes place. This will serve to check the

calculation and give a clearer idea of how such cutting takes place.

Example 3:

How many sheets $6\frac{3}{4} \times 8\frac{1}{2}$ in. can be cut from 50 sheets of 17×28-in. paper?

Solution and Explanation:

In such a problem the first step is to find out how many of the smaller size sheets can be cut from one sheet of the large stock with as little waste as possible. This would be determined by dividing the area of the larger sheet by the area of the smaller sheet to be cut from it.

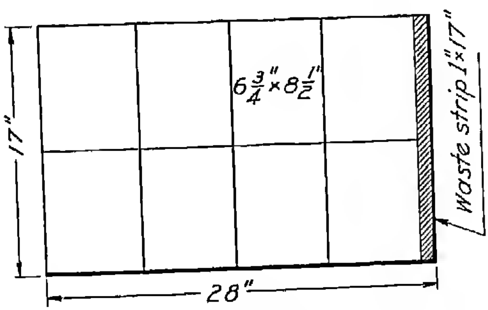
Expressed in cancellation this becomes:

$$\frac{17 \times 28}{6\frac{3}{4} \times 8\frac{1}{2}} \text{ or } \frac{\overset{2}{\cancel{17}} \times \overset{4}{\cancel{28}}}{\cancel{6\frac{3}{4}} \times \cancel{8\frac{1}{2}}} = 8$$

In this calculation $8\frac{1}{2}$ will exactly divide 17. But $6\frac{3}{4}$ will not exactly go into 28, the nearest amount being 4 with a slight remainder.

The product of these quotients equals 8. This indicates that not more than 8 of the smaller size sheets can be cut from one large sheet of stock.

Laid out to scale, a large sheet would be divided as follows:



By laying out a plan of a large sheet cut up into this number of smaller size sheets, it is seen that such a sheet is cut exactly in half in the short dimension, 17 in., and into 4 parts in the

long dimension 28 in. There results from this cutting a strip of waste 1 in. wide and 17 in. long. That, as might have already been seen, is due to the unequal division of $6\frac{3}{4}$ into 28, the remainder indicating waste.

If 8 sheets are cut from one large sheet of stock, then from 50 sheets there can be cut 50×8, or 400 sheets.

Example 4:

If a catalogue page measures 6×9 in. and the cover stock used is 20×25, how many sheets are needed to cover 1,000 of these catalogues?

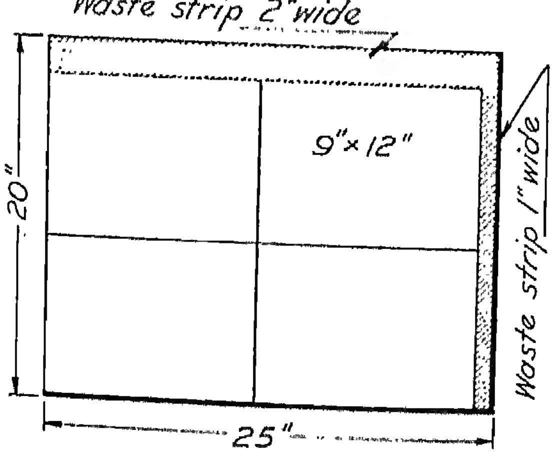
Solution and Explanation:

Since the pages are 6×9 in., one complete cover, front and back, would equal twice that area, and may be expressed as 12×9 in.

The number of such covers that can be cut from one sheet of stock measuring 20×25 would be determined by cancellation as follows:

$$\frac{20 \times 25}{12 \times 9} \qquad \frac{\overset{2}{\cancel{20}} \times \overset{2}{\cancel{25}}}{\cancel{12} \times \cancel{9}} = 4$$

That is, 4 covers, including front and back, can be cut from one sheet of stock measuring 20×25. By referring to the



above calculations, however, it will be seen that the division of the numbers is not exact. As already explained, this indicates waste. Laying out these covers it is seen that there is a small waste on two sides of the sheet.

It is to be understood, however, that in such layouts, the paper is to cut in the most economical way possible.

If there is a doubt as to a choice of two divisions a layout of each plan will aid in deciding which is to be used.

To get 1,000 covers will require as many sheets as 4 is contained in 1,000, which equals 250.

That is, 250 sheets are needed to cover 1,000 catalogues that measure 6 × 9 in.

Problems Relating to Type Measure and Paper

1. How many cards 3 × 5 in. can be cut from 100 sheets of card stock measuring $20\frac{1}{2}$ × $24\frac{3}{4}$ in.?

2. To print a certain job requires the use of a paper that measures 32 × 44 in. Calculate the weight of this paper on the basis of 25 × 38 in. — 70.

3. How many pages of 4 × 6-in. printed matter will it take for a 14,200-word article which averages 20 words to the square inch when set up in type? If the above article were later reprinted on a 5 × 8-in. page averaging 14 words to the square inch, how many pages would it cover?

4. How many sheets of 22 × 28-in. stock are needed for 1,400 cards cut $3\frac{3}{4}$ × $5\frac{1}{4}$ in.?

5. At 14 cents per pound what is the cost of 5 reams of paper 19 × 24 in. on the basis of 17 × 22 in. — 16?

6. An 8-page pamphlet set in 12-point type averages 11 words to the square inch. The size of the printed matter on each page is 3 × $5\frac{1}{2}$ in. This pamphlet is to be reprinted on a page measuring $2\frac{1}{2}$ × 4 in. How many pages will be required for the reprinting?

7. What is the equivalent weight of a ream of flat writing paper 16 × 21 in. on the basis of 22 × 34 in. — 28?

8. How many sheets of post-card material $28\frac{1}{2} \times 45$ in. will be needed to print 4,500 cards which measure 3 in. wide and $4\frac{1}{2}$ in. long?

9. At \$3 per ream determine the cost of 21×32 in. stock needed for 20,000 sheets cut 8×10 in.

10. Upon the basis of 17×22 in. — 28 lb., what is the weight of 42 reams of paper measuring 22×34 in.?

11. What is the approximate number of words in an article that requires 12 full pages of 6×8-in. printed matter? In this article there are on an average 16 words to the square inch. If this is later to be reprinted on a page that measures 3×6 in. using the same size type, how many pages are required for the reprint?

12. At \$2.20 per ream of 500 sheets, what is the cost of the material needed to print 4,000 letterheads $8\frac{1}{2} \times 11$ in.? The standard size of letter-paper sheet to be used for this job is 17×22 in.

13. How many sheets of stock 17×28 in. are needed to print 1,000 billheads measuring $8\frac{1}{2} \times 7$ in.?

14. An advertisement slip that measures $5 \times 6\frac{1}{4}$ in. is to be cut from stock measuring 20×25 in. How many of these sheets can be cut from 3 reams?

15. Ten-point type runs 16 words to the square inch. At this rate how many pages of type matter measuring $3\frac{1}{2} \times 6$ in. will be required to print an article of 6,600 words?

16. How many filing cards 4×5 in. can be cut from 340 sheets of Bristol board which measures $20\frac{1}{2} \times 24\frac{3}{4}$ in.?

17. Stock measuring 20×28 in. is used in making labels measuring 2×4 in. How many sheets are needed to print 35,000 labels?

18. How many pounds will a ream of stock 33×46 in. weigh on the basis of 25×38 in. — 40?

19. How many pieces, $6\frac{1}{4} \times 10$ in. can be cut from $1\frac{1}{2}$ reams of $25\frac{1}{2} \times 30\frac{1}{2}$-in. stock?

20. Determine the number of sheets of 25×38-in. stock needed to print 12,000 record forms which measure $4\frac{5}{8} \times 6\frac{1}{4}$ in.

21. How many cards $4 \times 5\frac{1}{2}$ in. can be cut from cardboard stock that measures $22\frac{1}{2} \times 28\frac{1}{2}$ in.? How many sheets are needed to make 4,200 cards?

22. Determine the number of pages of type matter, 3×5 in., that are needed to print a 6,200-word article. It is estimated that 18 words are to be used to the square inch.

23. A printing job that formerly required 3 reams of stock, 25×38 in. — 30 lb., is to be run on stock of the same base weight but measuring 32×44 in. What is the weight of this stock?

24. A brief historical sketch, containing 3,360 words which takes 6 full pages of 4×7-in. printed matter, is to be reprinted in pamphlet form on a page measuring $2\frac{1}{2} \times 4$ in. Using this same size type how many pages will be required for the reprint?

25. On the basis of 17×22 in. — 20 lb., what is the weight of 5 reams of stock measuring 28×38 in.?

26. A pamphlet requiring 4 sheets of paper that are cut $8 \times 6\frac{1}{4}$ in. is to be printed on a 25×38-in. — 60-lb. stock. How many sheets of this stock are needed to print 900 copies?

27. Determine the weight of 25 reams of book stock 36×48 in. on the basis of 25×38 in. — 50 lb. per ream.

28. An order for 8,000 handbills, 9×12 in., is to be printed on stock 24×36 in. — 30. At $4\frac{1}{2}$ cents a pound what is the cost of the stock needed for this job?

29. On the basis of 25×38 in. — 40 lb. what is the cost per ream of paper measuring 28×42 in. at 8¢ per pound?

30. Determine the number of sheets of stock, measuring

28×44 in., that are needed to print 1500 copies of a folder which requires three pieces of stock cut to the size of $8\frac{1}{2} \times 9$ in.

31. A pamphlet cover cut $8\frac{1}{4} \times 9\frac{3}{4}$ in. is to be made from stock that measures 20×26 in. How many sheets of this stock will be needed for 3,000 covers?

32. On the basis of 25×38 in. — 35 lb. what is the corresponding weight of a ream of book stock measuring 28×34 in.?

33. A ream of paper 22×34 in. weighs 80 lb. At this rate what would be the weight of the same kind of paper measuring $28\frac{1}{2} \times 45$ in.?

34. How many cards measuring 7×9 in. may be cut from 1,000 sheets of card stock each sheet of which measures $28\frac{1}{2} \times 45$ in.?

35. Blotter stock measuring 19×24 in., is to be used in making blotters that are 3 in. wide and 6 in. long. How many stock sheets are needed for an order of 3,000 such blotters?*

ANSWERS TO PROBLEMS

Pages 235 to 237.

1. 58 sq. yd.
2. $8.25.
3. 13,104 bricks.
4. 110 sq. yd.
5. 36 bundles; $247\frac{1}{3}$ sq. yd.
6. 9,477 bricks.
7. 9 bundles; 63 sq. yd.
8. 39 sq. yd.
9. 3,783 bricks.
10. 26 sq. yd.
11. 6 bundles.
12. $27.

Pages 247 to 250.

1. 3 gal.
2. 2 gal.
3. 405 strips.
4. 6 rolls; $1.92.
5. 13 gal.
6. 16 rolls side walls; 8 rolls ceiling.
7. 39 bundles.
8. 6 gal.
9. 13 rolls.
10. 2 gal.
11. 9 rolls side walls; 4 rolls ceiling.
12. 431.97 sq. ft.; 2 gal.
13. 18 bundles.
14. 6 gal.

*Answers to these problems will be found on page 261.

15. 19 rolls.
16. 2,178 sq. ft.; 9 gal.
17. 30 bundles.
18. 136 bundles.
19. 4 gal.
20. 24 bundles.
21. 10 gal.
22. 810 strips.
23. 8 bundles.
24. $56.

Pages 257 to 260.

1. 3,200.
2. 104 lb.
3. 30 pages; 26 pages.
4. 50 sheets.
5. $14.
6. 14 pages.
7. 13 lb.
8. 50 sheets.
9. $15.
10. 2,352 lb.
11. 9,216 words; 32 pages.
12. $4.40.
13. 125 sheets.
14. 24,000 sheets.
15. 20 pages.
16. 8,160 cards.
17. 500 sheets.
18. 64 lb.
19. 9,000 pieces.
20. 375 sheets.
21. 28 cards; 150 sheets.
22. 23 pages.
23. 132 lb.
24. 17 pages.
25. 285 lb.
26. 200 sheets.
27. 2,275 lb.
28. $2.70.
29. $4.
30. 300 sheets.
31. 500 sheets.
32. 35 lb.
33. 137 lb.
34. 20,000 sheets.
35. 125 sheets.

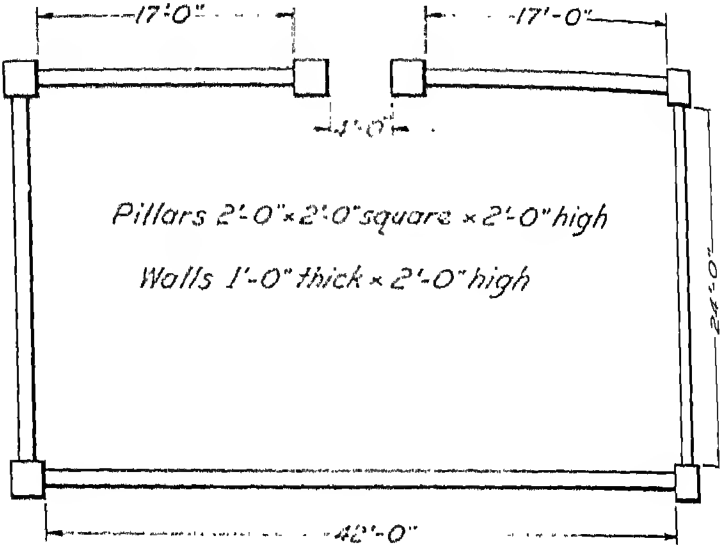
Review Problems Involving Applications of Square Measure to Trades

1. A common pitched roof measuring 30 ft. long with an 18-ft. slope on each side is to be reshingled with strip asphalt shingles. If sold in bundles of 25-sq.-ft. coverage how many bundles will be needed for this job?

2. A partition wall 9 ft. high and 15 ft. long is to be lathed and plastered on both sides. After deducting full allowance for one door that measures 3 ft. by 7 ft., how many bundles of lathes are required for this job?

Upon how many square yards should the plastering estimate be based allowing 50% deduction for the door opening?

3. A plain brick wall with pillars as noted is to be erected around a garden plot according to the following layout. What is the total number of bricks needed for this construction?



4. How many gallons of varnish will be needed in giving two coats to the three following floors? One floor measures 12 × 18 ft.; another 12 × 15 ft.; another 8 × 12 ft.

5. A lean-to flat roof 21 ft. long and 16 ft. wide is to be reshingled with wooden shingles laid 4 in. to the weather. At $1.35 per bundle what is the cost of the shingles needed for this job?

6. A leaky roof ruined the plaster on the ceiling of a room that measures $12\frac{1}{2}$ ft. wide by 18 ft. long. At $1.50 per square yard what would be the cost of replastering this ceiling?

7. The side walls of a bedroom that measures 12 ft. wide, 15 ft. long, and 9 ft. high are to be repapered. In this room there are 2 doors each measuring 3 by 7 ft., and 2 windows

each 3 by 5 ft. Allowing 50% for these openings, what will be the cost of the paper needed at 20¢ per single roll?

8. A brick partition wall in a cellar measures 14 ft. long by 7 ft. high and is 1 ft. thick. How many bricks are required in building this wall?

9. How many sheets 6 × 7 in. can be cut from a ream of paper that measures 38 × 50 in.? Show by sketch how each large sheet is to be cut.

10. The ceiling of a hall that measures 9 ft. wide and 23 ft. long is to be papered with a double-roll ceiling paper that costs 42¢ per roll. What is the cost of the paper required for this job?

11. A 16-page leaflet that has the printed matter measuring 5 × 8 in. to the page, is to be reprinted on a page size that measures 4 × 6 in. If there are approximately 16 words to the square inch, how many pages of the 4 × 6-in. size would this reprinted article require?

12. Determine the equivalent weight of a ream of book stock measuring 42 × 56 in. on the basis of 25 × 38 in. — 40 lb.

13. The cylindrical portion of a wooden tank 12 ft. in diameter and 10 ft. high is to be given two coats of red paint. How many gallons of paint will be required for this job if this paint covers only 200 sq. ft. 2 coats to the gallon?

14. How many sheets of stationery $8\frac{1}{2}$ × 11 in. can be cut from 2 reams of paper that measures 22 × 34 in.?

CUBIC MEASURE

Volumes, weights, capacities, estimates, industrial calculations, trade applications.

Cubic Measure

In the study of measurements so far, there was first considered the units of linear measure, which dealt with *one* dimension only. This was followed by square measure dealing with *two* dimensions, length and width, giving area.

Cubic measure deals with an additional dimension, *thickness*, and uses the *three* dimensions, length, width, and thickness, in determining volumes, capacities, weights, and the like.

Such calculations occur frequently in everyday practice and a working knowledge of cubic measure should prove very helpful.

HOW CUBIC MEASURE IS DETERMINED

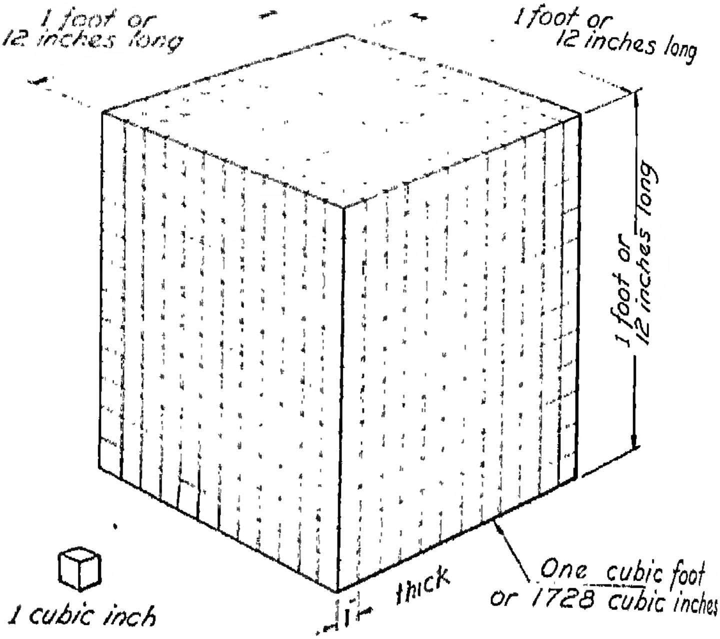
In the study of *square measure* it was explained that a square foot was the area of a surface 12 in. long and 12 in. wide, and that the product of these two dimensions was 144 square inches.

By referring to the following drawing it will be seen that if this 144 square inches were extended 12 inches in a *third* direction adding *thickness* to it, this amount would be increased, or multiplied, 12 times.

This gives a solid 12 inches long, 12 inches wide, and 12 inches thick, and contains 1,728 cubic inches which is the product of 12 × 12 × 12. This is the *cubic inch measure* of a solid which is 1 ft. wide, 1 ft. high, and 1 ft. thick, or 1 cubic foot.

From this explanation and from a study of the illustration which follows, a clear understanding may be had of what is meant by a cubic foot and cubic measure in general.

The solid pictured on page 268 contains twelve sections of cubes 1 in. on the edge, with 144 cubes in each section, or a total of 1,728 one-inch cubes.



It being 1 ft. long, 1 ft. wide, and 1 ft. thick, it is called one cubic foot.

To show the relation of the standard units of cubic measure to each other, and to aid in changing units of one denomination to equivalent units of another denomination, the following table is given.

1,728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)

To change from units of one denomination to equivalent units of another denomination, the process in cubic measure is similar to that used in square measure and linear measure, as shown in the following examples.

Example 1:

In excavating for a cellar, 12 truckloads of dirt were carted away. If each truckload contained $2\frac{1}{2}$ cu. yd., what was the equivalent number of cubic feet of dirt removed?

Solution and Explanation:

In problems of this kind it is good practice to first determine the number of cubic feet in one truckload. There being 27 cu. ft. in one cubic yard the number of cubic feet in $2\frac{1}{2}$ cu. yd. is found by multiplying 27 by $2\frac{1}{2}$. This works out as:

$$27 \times 2\frac{1}{2} \text{ or } 67\frac{1}{2} \text{ cu. ft.}$$

The equivalent number of cubic feet in 12 truckloads is found by multiplying $67\frac{1}{2}$ by 12.

$$67\frac{1}{2} \times 12 = 810$$

That is, 810 cu. ft. of dirt were carted away.

This problem illustrates that in order to change units of a *higher* denomination *to* units of a *lower* denomination, the number representing the higher denomination is multiplied by the number of the smaller units it takes to make one unit of the higher denomination.

Example 2:

How many gallons of oil will a can hold that has a capacity of 1,920 cu. in.? One cubic foot equals $7\frac{1}{2}$ gal. approximately.

Solution and Explanation:

In problems like this the cubic-inch measurement of the can should be reduced to cubic feet as the first step. There being 1,728 cubic inches in one cubic foot the capacity of the can in cubic feet is determined by dividing 1,920 by 1,728.

This works out as follows: $1920 \div 1728 = 1\frac{1}{9}$

As there are approximately $7\frac{1}{2}$ gal. to the cubic foot, the number of gallons in $1\frac{1}{9}$ cu. ft. would be:

$$1\tfrac{1}{9} \times 7\tfrac{1}{2}, \text{ or } \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{5}{\cancel{15}}}{\cancel{2}} = \frac{25}{3}, \text{ or } 8\tfrac{1}{3}$$

That is, in a can having a capacity of 1,920 cu. in. there are approximately $8\frac{1}{3}$ gal.

This problem illustrates that in order to change units of a *lower* denomination to units of a *higher* denomination, the number representing the lower denomination is divided by the number of these units it takes to make one unit of the higher denomination.

WEIGHTS OF MATERIALS

For reference in solving problems that involve weights of materials there are listed herewith the more commonly used materials and their approximate weights per unit of cubic measure.

Aluminum	weighs	.09	lb. per cu. in.
Brass	weighs	.30	lb. per cu. in.
Bronze	weighs	.32	lb. per cu. in.
Cast iron	weighs	.26	lb. per cu. in.
Copper	weighs	.32	lb. per cu. in.
Lead	weighs	.41	lb. per cu. in.
Steel	weighs	.28	lb. per cu. in.
Birch	weighs	40.	lb. per cu. ft.
Cedar	weighs	38.	lb. per cu. ft.
Chestnut	weighs	37	lb. per cu. ft.
Concrete	weighs	150.	lb. per cu. ft.
Earth (dirt)	weighs	100.	lb. per cu. ft.
Fir	weighs	32.	lb. per cu. ft.
Hemlock	weighs	25.	lb. per cu. ft.
Oak	weighs	50.	lb. per cu. ft.
Sand	weighs	100.	lb. per cu. ft.
Spruce	weighs	28.	lb. per cu. ft.
Water (fresh)	weighs	62.425	lb. per cu. ft.
(The weight of water is often given as $62\frac{1}{2}$ lb. per cu. ft.)			
Water (salt)	weighs	64.	lb. per cu. ft.
White pine	weighs	30.	lb. per cu. ft.
Yellow pine	weighs	40.	lb. per cu. ft.

To obtain a fair degree of accuracy in calculations that involve weights or volumes the answers should be carried to the second decimal place, unless otherwise specified.

Problems Involving Changing Units of One Denomination to Units of Another Denomination

1. Change: $17\frac{1}{2}$ cu. yd. to cubic feet; $\frac{7}{8}$ cu. yd. to cubic inches; 2,430 cu. in. to cubic yards; 1,850 cu. in. to cubic feet; 5.4 cu. ft. to cubic inches.

2. In excavating for a foundation of an automobile service station 16 truckloads of earth (dirt) were removed. How many cubic feet is this equal to, if each truckload contains 2 cu. yd.?

3. To empty a water tank, 54 bucketfuls of water were removed. How many cubic feet is this equal to if the bucket used contained 960 cu. in.?

4. A solid iron pedestal contains $2\frac{1}{2}$ cu. ft. of metal and weighs 1,116 lb. How much does a cubic inch of the metal weigh?

5. If a cubic foot of steel weighs 490 lb., what is the weight of a piece of this metal containing $42\frac{1}{2}$ cu. in.?

6. At the rate of $22\frac{1}{2}$ gal. per minute how long will it take to fill a rectangular tank which has a capacity of $4\frac{1}{2}$ cu. yd.? In this calculation assume there are $7\frac{1}{2}$ gal. to the cubic foot.

7. A solid metal ball measuring $45\frac{1}{2}$ cu. in. weighs $12\frac{1}{4}$ lb. Compute the weight of 1 cu. ft. of this material.

8. Calculate the weight of a cubic inch of oak in a timber that contains $2\frac{1}{2}$ cu. ft. and weighs 125 lb.

9. In excavating for a driveway and a garage, 72 wheelbarrow loads of dirt were removed. Each wheelbarrow load averaged 90 lb. If a cubic foot of dirt weighs 100 lb., how many cartloads of 1 cu. yd. each would this equal?

10. A checkup shows that lubricating oil flows from a faucet of an oil tank at such a rate that it fills a can containing 1,152 cu. in. in $1\frac{1}{2}$ min. If there are approximately $7\frac{1}{2}$ gal. to the cubic foot, how long will it take at this rate, to draw 50 gal. from this tank?*

VOLUMES OF REGULAR SOLIDS

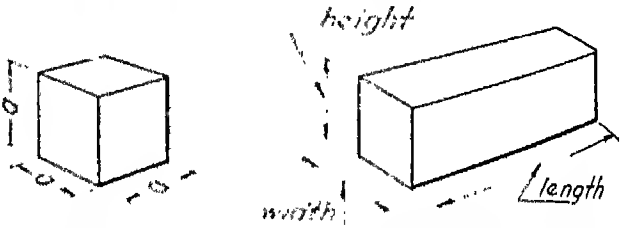
The cubical measure of regular solids, as for example, a cube, a sphere, or a rectangular prism, or a solid resembling any of these, is found by multiplying together the three dimensions

*Answers to these problems will be found on page 294.

expressing length, width, and thickness, or their equivalents. If these dimensions are in inches, the product is cubic inches. If they are in feet, the product is in cubic feet. If they are in yards the product is in cubic yards.

The method used in calculating the cubic measure of the more common *regular* figures is explained and illustrated below.

1. Cubes, Rectangular Prisms, or Forms Resembling Them



Cube

Rectangular prism

In the cube, all edges and all faces, or sides, are equal, and its volume is equal to $a \times a \times a$, or a^3.

NOTE: When a number, or a quantity, is multiplied by itself in this manner, it is said to be "cubed," or "raised to the third power." This multiplication is more briefly expressed by placing the small figure ^{3} to the right and slightly above the number, or quantity, to be cubed. In this case, $a \times a \times a$ would become a^3. The volume of the cube would then be said to equal a^3, where "a" equals the length of one of the edges.

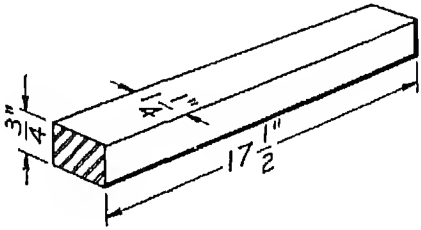
In a *rectangular* solid, or a *rectangular* prism, as it is sometimes called, the faces, or sides, are rectangular in shape and only the *opposite* sides and *opposite* edges are equal.

The volume of such a solid is found by multiplying together the length, width, and thickness. This is also the equivalent of multiplying the area of the base by the vertical height.

An application of these rules is seen in the following examples:

Example 1:

Calculate the weight of the steel bar in the following sketch, when this material weighs .28 lb. per cubic inch.



Solution and Explanation:

The weight of this piece is found by first determining its volume in cubic inches, and then multiplying that amount by .28.

The volume of the bar equals the product of the three dimensions expressing length, width, and thickness. This works out as follows:

$$17\tfrac{1}{2} \times 1\tfrac{1}{4} \times \tfrac{3}{4}, \text{ or } \tfrac{35}{2} \times \tfrac{5}{4} \times \tfrac{3}{4}, \text{ which equals:}$$

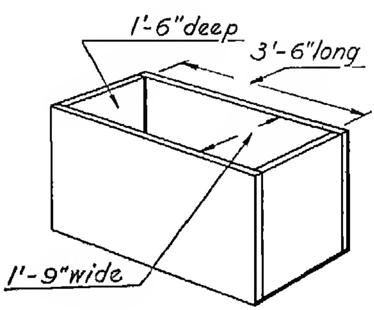
$$\tfrac{525}{32}, \text{ or } 16\tfrac{13}{32} \text{ cu. in.}$$

$16\frac{13}{32}$ cu. in. of steel weighing .28 lb. per each cubic inch equals:

$$16\tfrac{13}{32} \times .28, \text{ or } 4.59 \text{ lb., or approximately } 4.6 \text{ lb.}$$

Example 2:

Determine the cubic foot capacity of a box whose inside measurements are as indicated in the sketch.



Solution and Explanation:

Length of box inside = $3\frac{1}{2}$ ft.
Width of box inside = $1\frac{3}{4}$ ft.
Depth of box inside = $1\frac{1}{2}$ ft.

Since the capacity in cubic feet is found by multiplying together the length, width, and depth, the capacity of this box is,

$$3\tfrac{1}{2} \times 1\tfrac{3}{4} \times 1\tfrac{1}{2}, \text{ or } \tfrac{7}{2} \times \tfrac{7}{4} \times \tfrac{3}{2}, \text{ which equals:}$$

$$\tfrac{147}{16}, \text{ or } 9\tfrac{3}{16} \text{ cu. ft. capacity.}$$

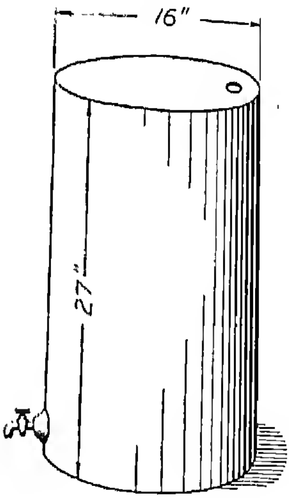
2. Cylinders, or Solids Resembling Cylinders

The cubic measurement of a cylinder is found by multiplying the area of the circular end by the length, or height, of the cylinder.

A practical application of this rule is as follows:

Example:

How many cubic feet of liquid in the tank illustrated, when the indicator shows it is half full?



Solution and Explanation:

The volume of this tank in cubic feet is found by multiplying the area of the bottom by the vertical height. As both these dimensions are given in inches, the cubical contents will be in cubic inches. This in turn must be reduced to cubic feet.

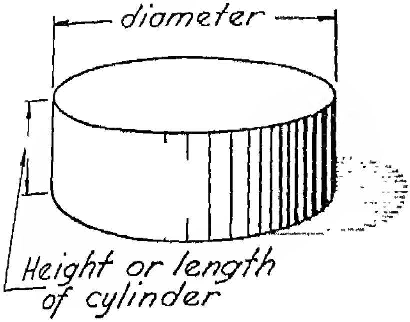
To avoid lengthy calculation, the volume is calculated and reduced to cubic feet in the process of cancellation as illustrated:

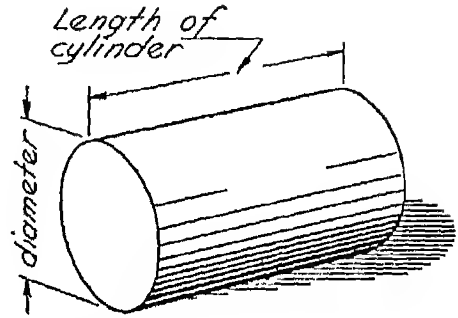
The area of the circular bottom equals $16 \times 16 \times .7854$.

The height is 27 in.

$$\frac{16 \times 16 \times .7854 \times 27}{1728} = 3.1416, \text{ or } 3.14 \text{ cu. ft., full capacity.}$$

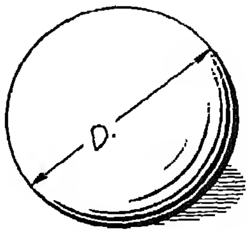
If the tank is half filled with the liquid, it will contain but ½ of the above volume, 3.14 cu. ft., or 1.57 cu. ft. of liquid.





3. Spheres, or Solids Having Spherical Form

The volume of a round ball, or a sphere as it is called, is found by multiplying the diameter by itself, and then multiplying that product by the diameter times 0.5236.



Expressed in abbreviated form this equals, $D \times D \times D \times 0.5236$, or, $D^3 \times 0.5236$, where D equals the diameter of the sphere.

Calculations based on this rule are worked out as in the following example.

Example:

A cast-iron ball 5 in. in diameter is to be used in an athletic contest known as "putting the shot." What is the weight of

this ball if the cast iron weighs approximately $\frac{1}{4}$ lb., to the cubic inch?

Solution and Explanation:

Calculated according to the above formula, the volume of a 5-in. ball equals:

$5 \times 5 \times 5 \times 0.5236$, or 65.45, which is approximately $65\frac{1}{2}$ cu. in.

Since 1 cubic inch of this material weighs approximately $\frac{1}{4}$ lb. then $65\frac{1}{2}$ cu. in. will weigh $65\frac{1}{2} \times \frac{1}{4}$, or $16\frac{3}{8}$ lb.

That is, the 5-in. iron ball weighs approximately $16\frac{3}{8}$ lb.

Volumes, or weights of regular forms resembling the rectangular, cylindrical, or spherical shapes, are calculated as explained. These may include such items as tanks, columns, metal rods and bars, excavations, and construction materials.

Problems Involving Rectangular, Cylindrical, and Spherical Shapes

1. How many loads of dirt must be removed in making an excavation that measures 12 ft. in diameter and 6 ft. deep, if the truck used for cartage holds $2\frac{1}{2}$ cu. yd.?

2. A tank that measures 8 ft. long and 6 ft. wide, holds 1,260 gal. when filled to the top. How deep is this tank, assuming that there are $7\frac{1}{2}$ gal. to the cubic foot?

3. In an effort to determine the number of gallons of oil in a cylindrical tank, 3 ft. in diameter, a measuring stick is lowered into the tank through a hole in the circular top until it touches the bottom. When the stick is withdrawn the oil mark shows a depth of $1\frac{1}{2}$ ft. How many gallons of oil are in the tank?

4. At $4 per cubic yard what is the cost of making 4 concrete ornamental columns $1\frac{1}{2}$ ft. square and 9 ft. long?

5. A tank that measures 8 ft. square on the bottom and 10 ft. deep is to be filled with water. If there are approximately $7\frac{1}{2}$ gal. to the cubic foot how many gallons of water will this tank hold?

6. Determine the weight of 500 steel ball bearings $\frac{1}{4}$ in. in diameter.

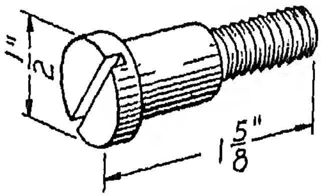
7. How many cubic yards of dirt will be removed in digging a trench that is to measure 2 ft. wide, $4\frac{1}{2}$ ft. deep, and 42 ft. long?

8. A tank with a square bottom, 5 ft. on the side, is filled with water to a depth of $3\frac{1}{2}$ ft. If 1 cubic foot of water weighs approximately $62\frac{1}{2}$ lb. what is the approximate weight of the water in this tank?

9. A piece of steel 8 in. long is cut from a 2-in.-square bar. If this material is listed as weighing .28 lb. per cubic inch, what is the weight of this 8-in. piece?

10. A solid concrete ball 18 in. in diameter is used as a garden ornament. What does this ball weigh?

11. What is the weight of a $\frac{1}{2}$-in. rass rod needed in making 500 crews as per the following drawng, allowing $\frac{1}{8}$ in. to each screw for utting off and finishing?



12. In order to hold 450 gallons hen completely filled, what should be the depth of a tank that easures 4 ft. square on the bottom? In this calculation ssume that there are $7\frac{1}{2}$ gallons to the cubic foot.

13. How many cubic yards in a box measuring 4 ft. 6 in. ng, 2 ft. 9 in. wide, and 2 ft. deep?

14. Two strips of sheet lead each $\frac{3}{16}$ in. thick, one measuring in. wide and 32 in. long, the other 14 in. wide and 32 in. ng, are used in lining a box. If lead weighs .41 lb. per cubic ch what is the total weight of these two sheets?

15. What is the capacity in gallons of a hemispherical copper ettle that measures $2\frac{1}{2}$ ft. inside diameter?

16. Determine the weight of 24 hemlock timbers each measing 6 in. thick, 8 in. wide, and 12 ft. long.

17. Calculate the weight of 3,000 one-quarter-inch steel balls such as are used in ball bearings.

18. The standard size brick measures $8 \times 3\frac{3}{4} \times 2\frac{1}{4}$ in. When used in construction work they average $19\frac{1}{2}$ bricks to the cubic foot. How many bricks will be needed in building a wall 24 in. thick, 6 ft. high, and 80 ft. long?

19. A concrete post used in building a highway fence measures 8 in. in diameter and is 6 ft. long. How much does this size post weigh?

20. Determine the number of truckloads of dirt that must be removed in making an excavation 12 ft. wide, 18 ft. long, and $4\frac{1}{2}$ ft. deep. The truck used for cartage holds 2 cu. yd. What is the equivalent number of tons of dirt removed?

21. What is the weight of 6 maple table tops that measure 2 in. thick, 4 ft. wide, and 6 ft. long?

22. Calculate the weight of 12 wooden balls made from white pine and measuring 10 in. in diameter.

23. Determine the number of bricks that are needed in building 8 piers each 3 ft. square and 5 ft. high.

24. What is the weight of 2 yellow pine timbers each 6 in. $\times$ 12 in. $\times$ 14 ft.?

25. Calculate the weight of 52 fir planks each 2 in. $\times$ 8 in. $\times$ 14 ft.

26. At the rate of 8¢ per pound, what is the cost of a flat cast-iron plate 4 in. thick and 20 in. in diameter?

27. With the price of copper 38¢ per pound determine the cost of a copper bar that measures $\frac{1}{2}$ in. thick, $1\frac{1}{2}$ in. wide, and 4 ft. long.

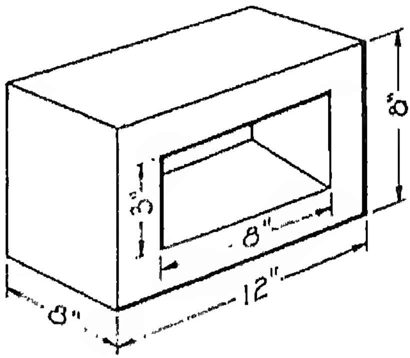
28. Approximately how many tons of dirt must be removed in making an excavation for a cellar that is to be 6 ft. deep, 18 ft. wide, and 22 ft. long?

29. Oil is drawn from a tank that measures 4 ft. in diameter until the level of the oil drops $1\frac{1}{4}$ ft. How many gallons are withdrawn?

30. Twelve pieces of stock each 8 in. long are cut from a steel bar that measures $1\frac{1}{2}$ in. in diameter. What is the weight of these pieces?*

CALCULATIONS RELATING TO HOLLOW RECTANGULAR SOLIDS

1. The volumes or weights of hollow rectangular forms or solids based on the rectangular form, such as hollow concrete blocks, rectangular shaped castings, and the like, may be calculated by using the previous rules relating to volumes of rectangular solids. This is illustrated in the following problem.



Example 1:

Calculate the weight of a concrete block made according to the dimensions in the illustration to the right.

Solution and Explanation:

The weight of such a solid is determined by first finding the volume of the entire piece, then subtracting from this the volume occupied by the hole.

Considering this block as a rectangular solid without a hole in it the volume would be:

$$8 \times 12 \times 8, \text{ or } 768 \text{ cu. in.}$$

The volume occupied by the rectangular hole equals $8 \times 3 \times 8$, or 192 cu. in. The actual volume occupied by the material then equals: $768 - 192$, or 576 cu. in.

*Answers to these problems will be found on page 294.

Since concrete weighs 150 lb. per cubic foot, as explained in the previous table, then the weight of this block becomes:

$$(576 \times 150) \div 1728, \text{ which equals } \frac{576 \times 150}{1728}, \text{ or } 50 \text{ lb.}$$

2. Volumes resembling the *hollow cylinder*, such as pipes, tubing, hollow shafting, and the like, are determined much the same as the hollow rectangular forms. In these calculations the volume of the outside cylinder is first determined, and from this there is subtracted the volume of the cylindrical hole. The difference between these two volumes equals the volume of the material in the piece. How such problems are worked out is illustrated in the following problem.

Example 2:

A cast-iron pipe measuring 4 in. outside diameter and 3 in. inside diameter is 5 ft. long. Determine the weight of this pipe.

Solution and Explanation:

As explained, the volume of a cylinder 4 in. in diameter and 5 ft. long is first determined. Because the weight of cast iron is given per cubic inch, this volume is calculated directly in cubic inches.

$$4 \times 4 \times .7854 \times 5 \times 12 = 753.984 \text{ cu. in.}$$

In the same way the volume of the hole is calculated.

$$3 \times 3 \times .7854 \times 5 \times 12 = 424.116 \text{ cu. in.}$$

The volume of the material in this pipe therefore equals the difference between the above two volumes. This equals:

$$753.984 - 424.116, \text{ or } 329.868 \text{ cu. in.}$$

Cast iron weighing .26 lb. per cubic inch, the weight o 329.868 cu. in. of cast iron, which is the volume of the pipe equals:

$$329.868 \times .26 = 85.76568, \text{ or approximately } 85\tfrac{3}{4} \text{ lb.}$$

That is, the weight of the above is approximately $85\frac{3}{4}$ lb

3. In calculating volumes of *hollow spherical shapes* such as hollow balls, and the like, the procedure is the same as that followed in calculating the volumes of hollow cylindrical shapes.

For example, to determine the weight of hollow balls, the volume of the ball is first calculated as though it were a solid ball. From this amount there is then subtracted the volume of a sphere which has for its diameter the diameter of the inside of the ball, or the diameter of the hollow portion.

The difference between these two volumes equals the actual volume of the material in the ball. This multiplied by the weight of a cubic inch of the material, equals the weight of the ball.

How this works out is illustrated in the following problem.

Example 3:

Determine the weight of a 4-in. hollow brass ball $\frac{1}{8}$ in. thick, which is to be placed upon the top of a flagpole.

Solution and Explanation:

Working this out according to the above suggestions, the volume of a 4-in. ball is first determined. This equals:

$$4 \times 4 \times 4 \times 0.5236, \text{ or } 33.5104 \text{ cu. in.}$$

The shell being $\frac{1}{8}$ in. thick would make the diameter of the inside surface $\frac{1}{4}$ in. less than the outside diameter of 4 in., or $3\frac{3}{4}$ in.

The volume of a sphere with a diameter of $3\frac{3}{4}$ in. equals:

$$3\tfrac{3}{4} \times 3\tfrac{3}{4} \times 3\tfrac{3}{4} \times 0.5236, \text{ or } 27.6117 \text{ cu. in.}$$

The volume of the *material* in the sphere would then be equal to the difference between 33.5104 and 27.6117, or 5.8987 cu. in.

At .30 lb. per cubic inch 5.8987 cu. in. of brass weigh:

$$5.8987 \times .30 = 1.86961, \text{ or approximately } 1\tfrac{7}{8} \text{ lb.}$$

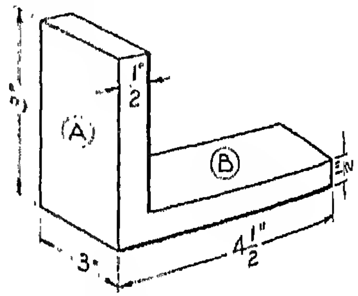
That is, a 4-in. brass ball $\frac{1}{8}$ in. thick weighs approximately $1\frac{7}{8}$ lb.

IRREGULAR FORMS

The volumes of *irregular solids*, however, may often be readily calculated by dividing them up to resemble regular forms that have already been illustrated.

The volumes, or weights, of these regular forms may then be determined as previously explained. The sum of these separate regular volumes will in turn equal the volume of the whole irregular piece.

The method of dividing up such irregular solids may be more clearly illustrated in the following example.



Example:

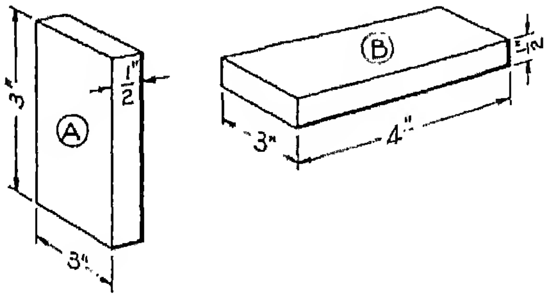
Calculate the weight of 25 cast-iron angle brackets made as in the sketch.

Solution and Explanation:

The shape of this piece, as may be seen from inspection, does not resemble any solid previously considered. A careful observation, however, will show that it may be divided up so as to resemble such solids, and as a result its volume may be readily determined. This particular piece may be divided as follows.

Referring to the drawing, the short leg (A), of the above angle iron, may be considered as a rectangular solid $\frac{1}{2}$ in. thick 3 in. wide, and 3 in. high. Its volume equals:

$$\tfrac{1}{2} \times 3 \times 3, \text{ or } 4\tfrac{1}{2} \text{ cu. in.}$$



In the same manner, the long leg (*B*), extending 4 in. out from the short leg may be considered as a rectangular solid measuring $\frac{1}{2}$ in. thick, 3 in. wide, and 4 in. long. Its volume in turn equals:

$$\tfrac{1}{2} \times 3 \times 4, \text{ or } 6 \text{ cu. in.}$$

The volume of part (*A*) and the volume of part (*B*), which make up the total volume of this angle iron, would then be equal to $4\frac{1}{2}$ cubic inches *plus* 6 cubic inches, which is $10\frac{1}{2}$ cubic inches.

Since cast iron weighs .26 lb. per cubic inch, then the weight of one angle bracket is:

$$10\tfrac{1}{2} \times .26, \text{ or } 2.73 \text{ lb.}$$

The weight of 25 such angle brackets would then be:

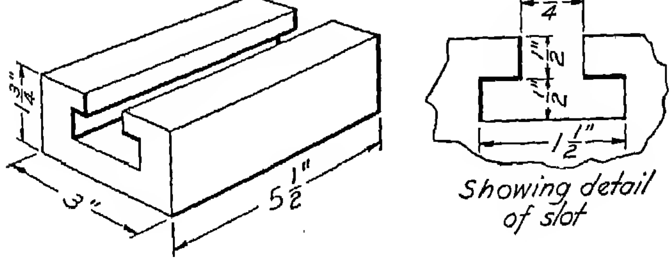
$$2.73 \times 25, \text{ which equals } 68\tfrac{1}{4} \text{ lb.}$$

In like manner, *other irregular shapes* may be divided up and as readily calculated. In each case the sum total of the separate volumes equals the total volume of the entire piece.

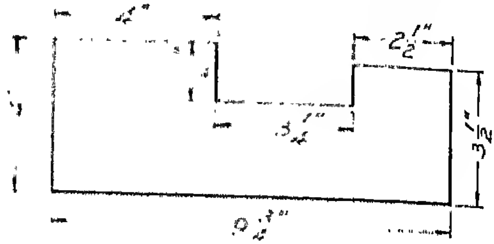
Problems Relating to Hollow and Irregular Forms

NOTE: Weights of various materials referred to in the following problems may be found in the table on page 270. Answers to these problems are to be correct to 2 decimal places.

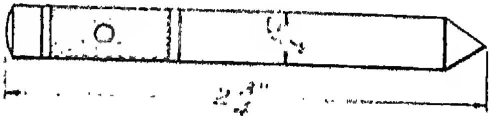
1. Calculate the weight of 8 T-slotted cast-iron blocks as per the dimensions in the illustration below.



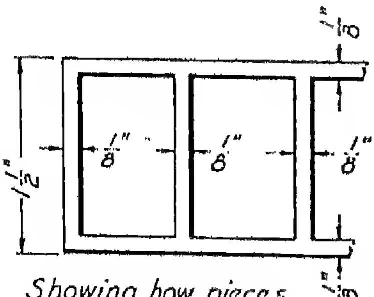
2. What is the weight of 24 steel plates $\frac{1}{4}$ in. thick, cut according to the dimensions in the following drawing?



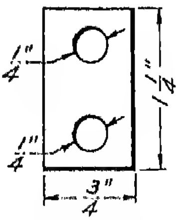
3. Determine the weight of $\frac{1}{4}$-in.-round stock needed in making 160 brass pins as shown below. Allow $\frac{1}{8}$ in. for cutting off each pin.



4. It is required to make 64 blanks on a foot press from sheet copper $\frac{1}{8}$ in. thick as per the following sketch. The strip of copper to be used measures $1\frac{1}{2}$ in. wide. An allowance of $\frac{1}{8}$ in. is to be made between the blanked pieces and at each end of the strip as shown. What is the total length of the strip of copper needed for these 64 blanks? What is the weight of the 64 blanks?

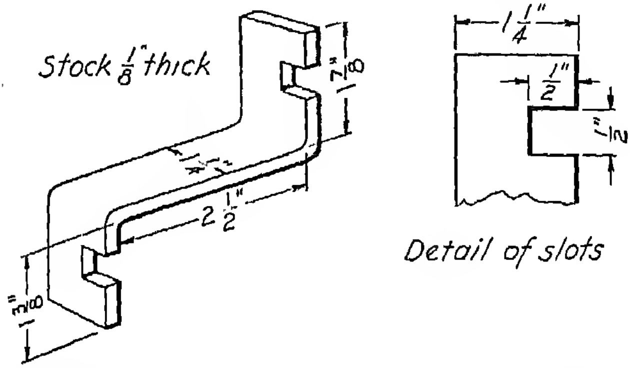


Showing how pieces are blanked from strip

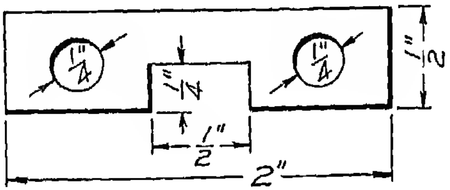


Dimensions of blank

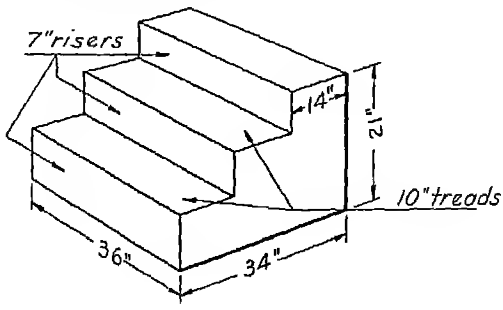
5. Determine the weight of material needed for 144 braces according to dimensions in the following drawing. There is a loss of $\frac{1}{8}$ in. of stock in cutting off each piece. Calculate the weight of the 144 braces if this material weighs .28 lb. per cubic inch.



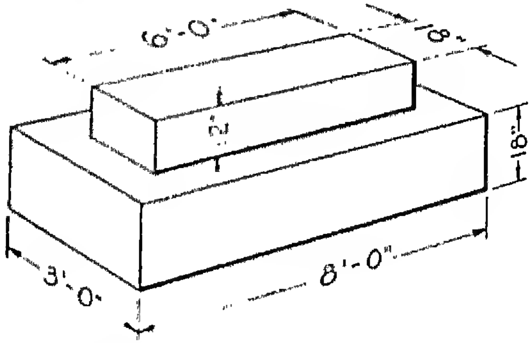
6. What is the weight of 1,500 blanks $\frac{1}{8}$ in. thick as shown below? The material in these weighs .22 lb. to the cubic inch.



7. From the dimensions in the following sketch, determine the weight of the concrete in the steps illustrated.

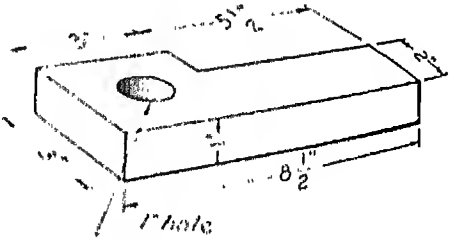


8. Calculate the number of tons of concrete in a footing of the following dimensions. This footing is to be calculated as having no reinforcements.

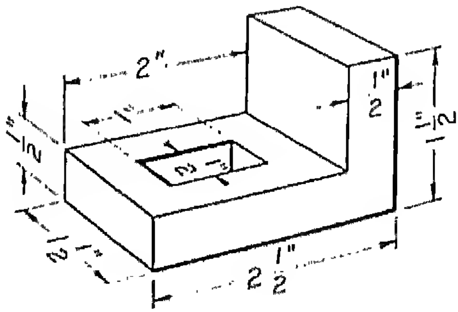


9. A copper half sphere $\frac{1}{4}$ in. thick has an outside diameter of 15 in. Determine its weight if copper weighs .32 lb. per cubic inch.

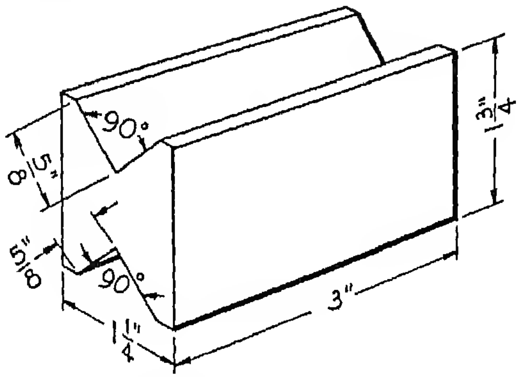
10. What is the weight of 25 brass castings as per the drawing to the right?



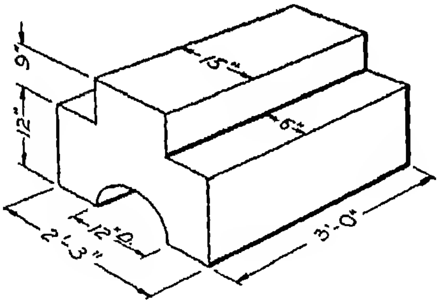
11. Determine the weight of 40 steel straps as illustrated in the following drawing.



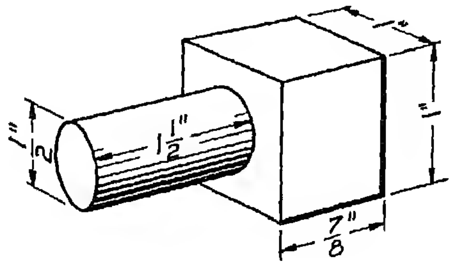
12. What is the weight of 16 steel V blocks like the one illustrated below?



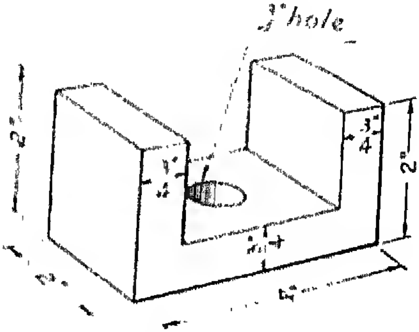
13. How many cubic feet of material are there in 2 concrete blocks made according to the following dimensions? What is the weight of these 2 pieces?



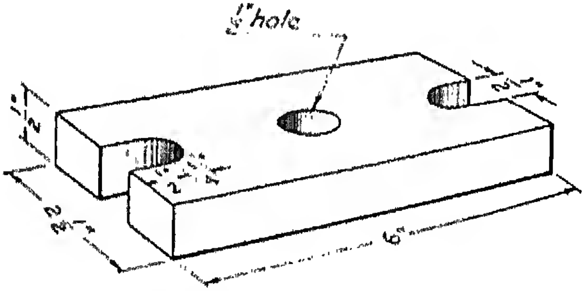
14. Determine the weight of 8 steel stop pins like the following.



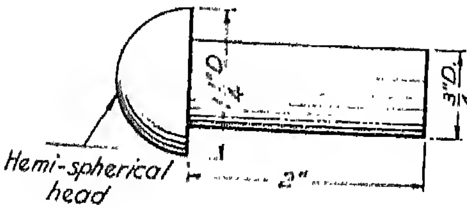
15. Calculate the weight of 10 cast iron blocks according to the dimensions in the drawing below.



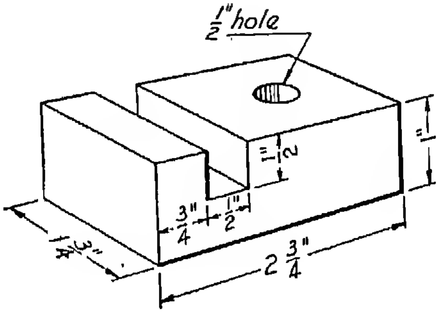
16. What is the weight of 5 steel straps made according to the following dimensions?



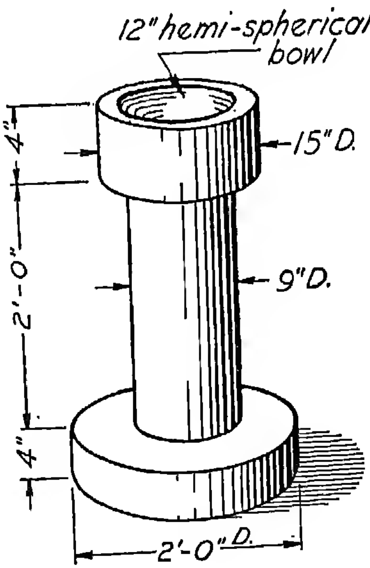
17. Four hundred aluminum pins are to be made according to the accompanying drawing. Determine their total weight.



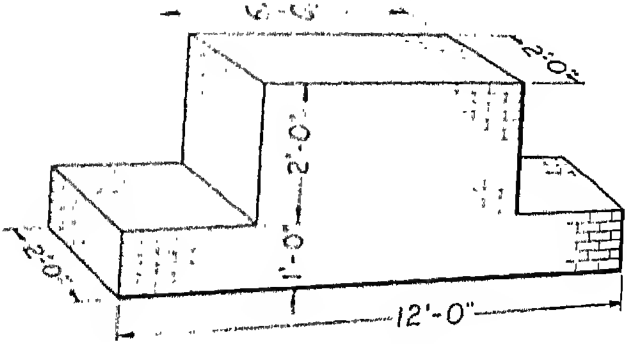
18. Calculate the weight of 16 brass pieces made according to the following drawing.



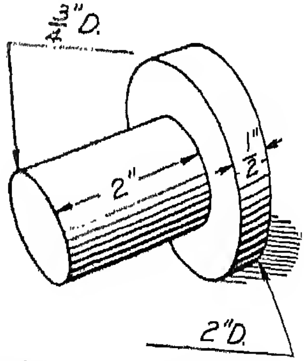
19. In constructing the following bird bath for a summer garden, $\frac{1}{8}$ of the total weight of the concrete piece is to be cement. How many pounds of cement will be needed for this job?



20. How many standard size bricks are needed to build the following construction, assuming that there are $19\frac{1}{2}$ bricks to the cubic foot?

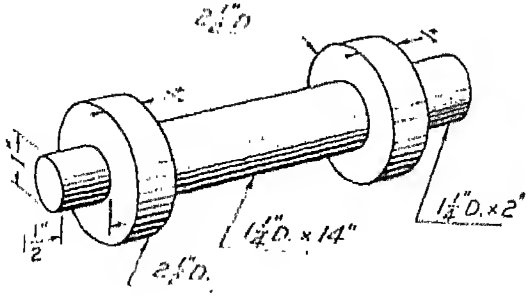


21. The pin in the drawing to the right is to be made of steel. Calculate the weight of 320 such pins.

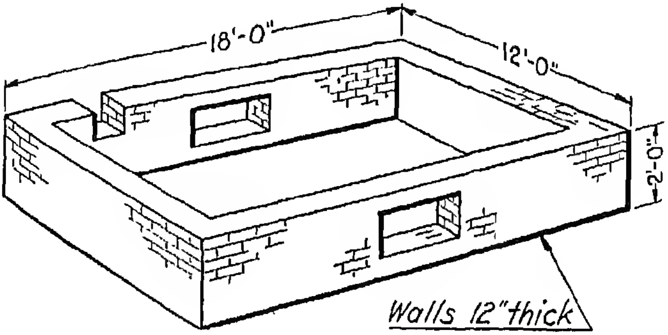


22. What is the weight of a hollow steel shafting which measures 4 in. outside diameter with a 2-in. circular hole running through its entire length of 6 ft.?

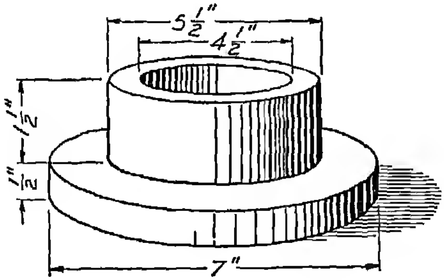
23. What is the weight of 10 steel spindles made as per the specifications in the following drawing?



24. Determine the number of bricks needed to build a foundation wall for a work shed according to the details in the following sketch. Four cubic feet are to be subtracted for the spaces occupied by the windows and the other openings.

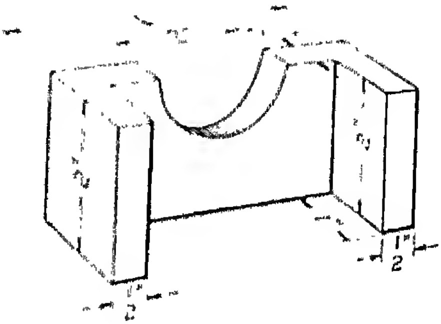


25. What is the weight of the bronze ring shown in the sketch? The $4\frac{1}{2}$-in. hole extends through the center of the ring.

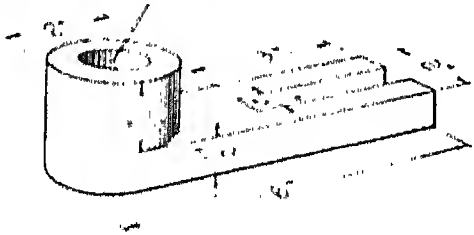


26. How many bricks are needed to build a cistern which is open at the top and whose outside dimensions are 8 ft. wide, 8 ft. long, and 5 ft. high? The inside dimensions are 6 ft. wide, 6 ft. long, and 4 ft. deep. Draw a complete plan of the cistern.

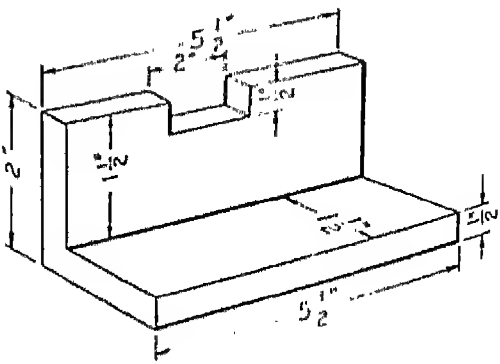
27. Calculate the weight of 50 steel pieces made as per the following dimensions.



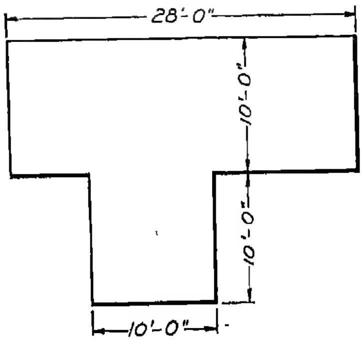
28. At the rate of 9¢ per pound what is the cost of 20 iron castings made according to the following drawing? The 1-in. hole extends through the casting.

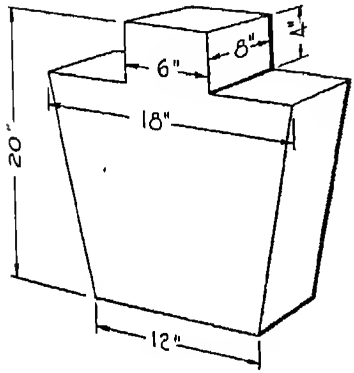


29. Determine the weight of 48 steel pieces made as per the specifications in the sketch.



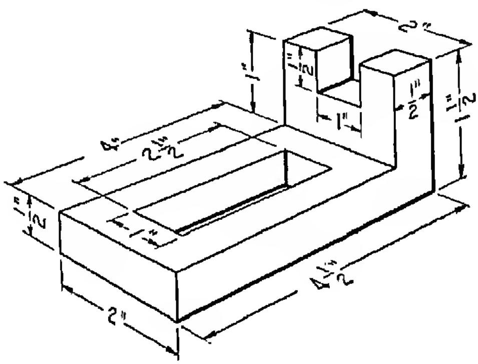
30. How many tons of earth must be removed in excavating for a cellar 5 ft. deep according to the plan in the sketch to the right?





31. What is the approximate weight of a granite keystone cut to the dimensions to the left? Granite weighs approximately 166 lb. to the cubic foot.

32. Calculate the weight of 10 brass castings made according to the measurements in the drawing below.



33. Determine the weight of the steel cover shown in the following sketch.*

ANSWERS TO PROBLEMS

Pages 270 and 271.

1. 472½ cu. ft.; 40,824 cu. in.; .052 cu. yd.; 1.07 cu. ft.; 9331.2 cu. in.
2. 864 cu. ft.
3. 30 cu. ft.
4. .258 lb.
5. 12.05 lb.
6. 40⅓ min.
7. 465.2 lb.
8. .029 lb.
9. 2⅔ cu. yd.
10. 15 min.

Pages 276 to 279.

1. 10 loads (approx.)
2. 3½ ft.
3. 79.5 gal.
4. $12.
5. 4,800 gal.
6. 1.145 lb.
7. 14 cu. yd.
8. 5468¾ lb.
9. 8.96 lb.
10. 265.065 lb.
11. 51.54 lb.
12. 3¾ ft.
13. .917 cu. yd.
14. 56.58 lb.
15. 30.68 gal.
16. 2,400 lb.
17. 6.87 lb.
18. 18,720 bricks.
19. 314.16 lb.
20. 18 loads; 48.6 tons.
21. 1,056 lb.
22. 109.08 lb.
23. 7,020 bricks.
24. 560 lb.
25. 2,588 lb.
26. $26.14.
27. $4.38.
28. 118.8 tons.
29. 117.8 gal.
30. 47.5 lb.

*Answers to these problems will be found on page 295.

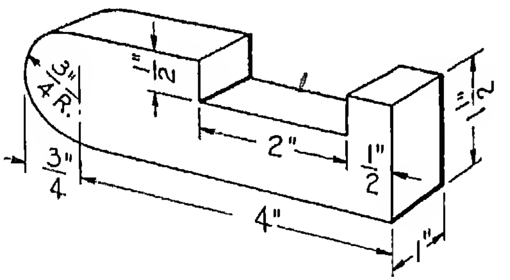
Pages 283 to 294.

1. 47.19 lb.
2. 55.23 lb.
3. 6.77 lb.
4. $56\frac{1}{8}$ in.; 2.15 lb.
5. 37.8 lb.; 34.5 lb.
6. 32.05 lb.
7. 1,575 lb.
8. $3\frac{3}{8}$ tons.
9. 27.34 lb.
10. 144.11 lb.
11. 26.6 lb.
12. 24.15 lb.
13. 16.77 cu. ft.; 2515.3 lb.
14. 2.62 lb.
15. 24.49 lb.
16. 9.88 lb.
17. 50.3 lb.
18. 19 lb.
19. 38.96 lb.
20. 936 bricks.
21. 219.9 lb.
22. 190 lb.
23. 85.77 lb.
24. 2,106 bricks.
25. 7.38 lb.
26. 3,432 bricks.
27. 73 lb.
28. $3.28
29. 122.64 lb.
30. 95 tons.
31. 202.9 lb.
32. 12 lb.
33. 21.9 lb.

Review Problems Involving Regular and Irregular Shapes

1. A hollow aluminum ball measuring 8 in. outside diameter is $\frac{3}{8}$ in. thick. Calculate its weight.

2. From the following drawing determine the weight of 45 brass castings made according to the given measurements.



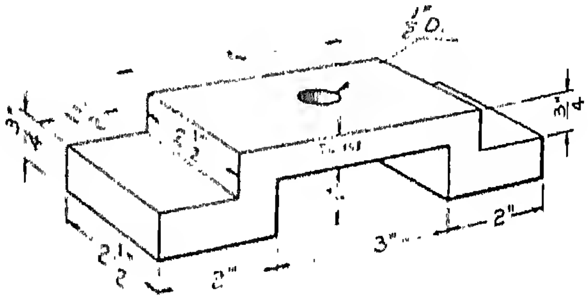
3. The following lumber is piled away to season. Because of the unseasoned condition of this lumber it is estimated as being 10% heavier than the weight of dry lumber as usually given. According to this what is the approximate weight of this lumber?

24 white pine boards $1\frac{1}{2}$ in. × 10 in. × 12 ft. long.
30 hemlock boards 2 in. × 6 in. × 10 ft. long.
18 spruce boards 4 in. × 8 in. × 10 ft. long.

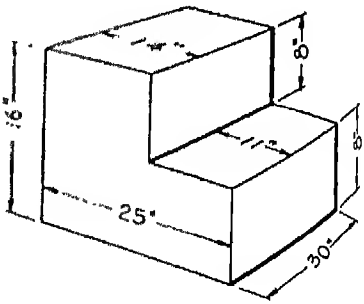
4. The average thickness of a concrete walk 6 ft. wide is 4 in. throughout its length of 142 ft. How many tons of concrete are there in this sidewalk?

5. A cylindrical iron tank weighing 24 lb. measures 18 in. in diameter and 24 in. deep inside. When it is one fourth filled with oil that weighs 7½ lb. per gallon, what is the weight of the tank and oil?

6. From the dimensions in the following drawing determine the weight of 15 cast iron clamping blocks.

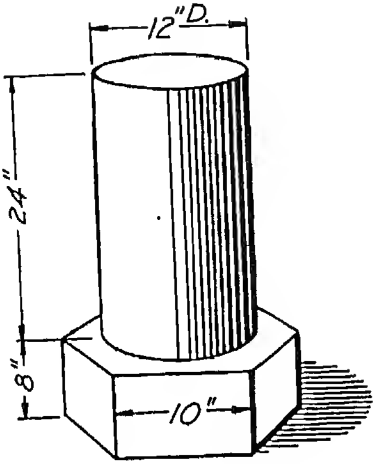


7. Calculate the weight of the concrete steps constructed according to the measurements in the drawing to the right.

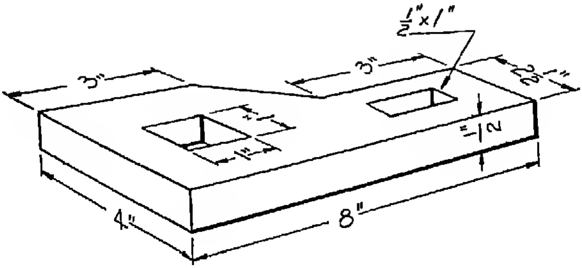


8. After a rainstorm it was noted that the water in a 6-ft. circular cistern had risen 8 in. The flat tin roof from which this rain was conducted to the cistern measures 18 by 32 ft. From this, calculate the approximate number of inches of rainfall per square foot.

9. What is the weight of the concrete pedestal in the following drawing?



10. Determine the weight of 64 forgings like that in the following sketch. The metal in this weighs .29 lb. per cubic inch.



LIQUID MEASURE

Liquid capacities are measured by cubical units of measure known as the *liquid measure*. These units are fixed by law and are related to each other according to the following table:

4 gills (gi.)	equal 1 pint (pt.)
2 pints	equal 1 quart (qt.)
4 quarts	equal 1 gallon (gal.), also 231 cu. in.
$31\frac{1}{2}$ gallons	equal 1 barrel (bbl.)

Such units are used in measuring fluids or liquids of all kinds, as water, oil, gasoline, turpentine, paint, vinegar, etc.

The sizes of barrels, although made a standard at $31\frac{1}{2}$ gallons, vary somewhat, up to 60 gallons per barrel.

One gallon, as fixed by United States law, measures 231 cubic inches. Unless otherwise stated, this measurement of a gallon will be used in the calculations that follow.

Knowing the number of cubic inches in one gallon, the number of cubic inches in one quart would be found by dividing 231 by 4, as there are 4 quarts to the gallon.

This equals: $231 \div 4$, or $57\frac{3}{4}$ cubic inches.

Changing Units of One Denomination to Units of Another Denomination

To change units of one denomination to units of another denomination the procedure is much the same as that followed in calculations relating to linear measure, square measure, and cubic measure.

The number of gallons in a given lot of barrels may be determined by multiplying the number of barrels by the number of gallons in each barrel. On the other hand, to find the number of barrels there are in a given number of gallons, the total number of gallons is divided by the number of gallons in one barrel.

To find the number of quarts in a given number of gallons, the figure representing the number of gallons is multiplied by 4, as there are 4 quarts in one gallon. Also, to determine the

number of gallons in a given number of quarts, the given number of quarts is divided by 4.

To find the number of pints in a certain number of quarts, the figure representing the number of quarts is multiplied by 2, as there are 2 pints in one quart. If it is desired to find the number of quarts in a given number of pints, that number is divided by 2.

To illustrate this method of changing units of one denomination into units of another denomination, the following examples are here worked out in detail.

Example 1:

How many quarts of oil in a 50-gal. barrel which is three-fourths full?

Solution and Explanation:

This problem is solved by first finding how many gallons there are in the barrel when three-fourths full. This amount is then changed to quarts.

Since there are 50 gallons in this barrel when it is full then three fourths of a barrel equals, $\frac{3}{4} \times 50$, or $37\frac{1}{2}$ gallons.

Then $37\frac{1}{2}$ gallons changed to its equivalent number of quarts becomes:

$$37\tfrac{1}{2} \times 4, \text{ or } 150 \text{ quarts.}$$

That is, there are 150 quarts of oil in the above 50-gallon barrel when it is three-fourths full.

Example 2:

During a period of 6 weeks there were collected in the drip pans under the oil barrels in a shop supply room, $34\frac{1}{2}$ quarts of oil. How many gallons does this amount equal?

Solution and Explanation:

Since there are 4 quarts in one gallon, then in $34\frac{1}{2}$ quarts there are as many gallons as 4 is contained in $34\frac{1}{2}$. This equals:

$$34\tfrac{1}{2} \div 4, \text{ or } 8\tfrac{5}{8} \text{ gallons.}$$

That is, $8\frac{5}{8}$ gallons of oil were collected in the drip pans.

Problems Relating to Liquid Measure

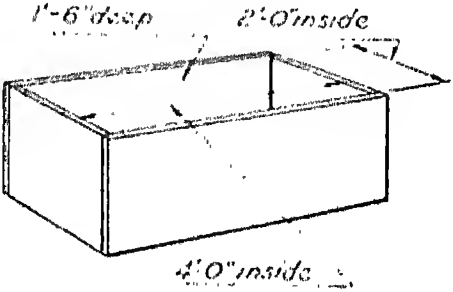
1. How many cubic feet are there in one $31\frac{1}{2}$-gal. barrel?

2. How many quarts of linseed oil can be put into an empty cylindrical tank 15 in. in diameter and 20 in. high?

When it is one-half full, what is the weight of the oil in the tank?

One gallon of linseed oil is listed as weighing 7.84 lb.

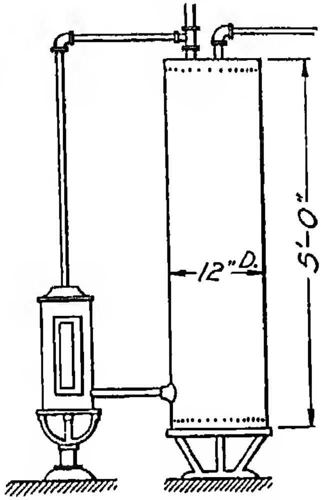
3. Two boys each carrying a bucket holding 10 quarts are assigned the job of filling the tank shown below. How many bucketfuls are required to completely fill this tank? How many gallons does this equal?



4. A cylindrical tank 18 in. in diameter, 30 in. high, and open at the top is three fourths filled with turpentine. The tank alone weighs 32 lb. If the turpentine weighs 7.26 lb. per gallon, what is the combined weight of the tank and the turpentine?

5. A cylindrical can used in a paint shop for linseed oil measures 18 in. in diameter and 24 in. high. To determine the amount of oil there was in the can a workman put a yardstick through the small opening in the top until the end of the stick touched the bottom. Upon withdrawing the stick, the oil mark showed that there was oil up to the 15-in. mark. From this determine the number of quarts of oil in the can.

6. What is the capacity in gallons of the tank used in connection with domestic water heating, as shown below? This tank measures 12 in. in diameter and is 5 ft. high.



7. A cylindrical bucket measures 15 in. in diameter, and is 20 in. deep. How many quarts will it hold?

8. Due to leakage and evaporation, $11\frac{1}{2}$ qt. of gasoline are lost from a 50-gal. barrel. At 16¢ per gallon, what does this loss equal?

9. What is the number of standard barrels capacity of a cylindrical tank which measures 6 ft. in diameter and $8\frac{1}{2}$ ft. high?

10. How deep must the tinsmith make a cylindrical bucket, if the diameter is to be 10 in. and it is to hold 18 qt. when completely filled?

11. From a tank of oil, 5 ft. long, 3 ft. wide, and 4 ft. deep there are withdrawn during a certain period 225 qt. of oil. How many gallons does this equal? How much did the level of the oil in the tank drop during this time?

12. A bucket that measures 11 in. in diameter and 15 in. deep inside, is used to fill a tank that measures 14 in. deep, 24 in. wide, and 6 ft. long. How many quarts does this bucket hold? How many gallons will the tank hold when completely filled?

13. To measure the amount of lubricating oil in a tank, 30 in. in diameter, a long stick marked off in inches is put down into the tank through a hole in the circular top. It showed that the oil is 14 in. deep. How many quarts of oil are in the tank?

14. A boy at work in a shop has the job of filling pint bottles with washing fluid from a tank that has a base measuring 24 × 36 in. After filling 50 bottles, how many gallons were withdrawn from the tank? How much did the level of the fluid in the tank drop?

15. How many ½-pint cans can be filled from a container holding 15 gallons of ready-mixed paint? A deduction of 5% is made for waste.*

DRY MEASURE

Dry measure is used in measuring commodities, such as corn, grain, potatoes, beans, apples, and the like.

The relation of the different units of measure to each other is explained in the following table:

2 pints (pt.)	equal	1 quart (qt.)
8 quarts	equal	1 peck (pk.)
4 pecks	equal	1 bushel (bu.)

The dry quart referred to above is fixed by law as containing 67.2 cubic inches. This differs slightly from the liquid quart referred to on page 298, which contains only 57.75 cubic inches.

From this, the number of cubic feet in a bushel is calculated as follows. Since there are 32 quarts in one bushel, then there

*Answers to these problems will be found on page 305.

are as many cubic inches in 32 quarts as 32 × 67.2, which equals 2150.4 cu. in. This amount divided by 1,728 gives the number of cubic feet in a bushel. This equals:

$$2150.4 \div 1728 = 1.244, \text{ or about } 1\tfrac{1}{4}$$

That is, in one bushel there are approximately $1\frac{1}{4}$ cubic feet. This measurement is sufficiently accurate to use in the problems which follow.

Aside from the daily uses to which these units are put, the measure of the capacities in either cubic inches, or cubic feet, is of great value in estimating, and in calculations relating to construction work. For example, in the building of a bin to hold a given number of bushels of corn, the measure of a bushel in cubic feet would aid in calculating the size of the bin and the amount of material to be used in the construction of this bin.

In reducing amounts from *higher* to *lower* denominations, or from *lower* to *higher* denominations, the process is carried out in dry measure the same as has already been explained in liquid measure. Because the explanation of this would be much the same as that already covered, it will not be reviewed again at this point.

Problems Involving Calculations in Dry Measure

1. How many bushels of apples will a bin hold that measures 6 ft. wide, 10 ft. deep, and $2\frac{1}{2}$ ft. high, when filled level with the top?

2. A young man who purchased 1 peck of corn noted that the grain dealer measured out the amount in a quart measure. He was rather surprised when, upon arriving home, he checked the purchase with his own quart measure and found that the grain dealer had used a liquid quart measure. How much was he short?

3. If a bushel of oats weighs 32 lb., how many tons of oats can be stored in a bin that measures 25 × 10 × 6 ft.?

4. A bushel of anthracite coal weighs on an average, 70 lb. If purchased in such quantities from a street peddler at 50¢ per bushel, how much money does a person lose in buying from time to time in this manner the equivalent of a ton of 2,000 lb., if this same coal sells at $12.50 per ton when purchased in ton lots?

5. Due to faulty construction, 112 quarts of oats "leaked out" of a large storage bin during a period of 8 weeks and were destroyed. Of how many bushels is this the equivalent?

6. In laying out a vegetable cellar, a man builds a storage bin 2 ft. high, 5 ft. wide, and 10 ft. long. Four feet from the short end there is erected a vertical partition, level with the top. Draw the plan of such a construction. How many bushels will each division of this bin hold when filled level with the top?

7. How many tons of anthracite coal will a bin hold, 8 ft. wide, 10 ft. long, and 4 ft. high, when filled level with the top, it being estimated that 1 cubic foot of this coal weighs 56 lb.?

8. Railroad box cars measuring $36 \times 8\frac{1}{2} \times 8$ ft. inside, are used in shipping wheat from an inland city to a grain elevator preparatory to shipping it abroad. If these cars are only half filled in shipping, how many will it take for an export shipment of 25,500 bushels? What is the weight of such a shipment if wheat weighs 60 lb. per bushel?

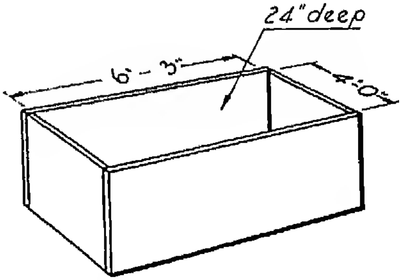
9. An ash can measures 25 in. high and is 18 in. in diameter. How many bushels will this can hold?

10. A sheet-metal worker has the job of constructing a cylindrical can that is to hold exactly one bushel. The only other specification given is that the can should be 15 in. in diameter. How deep should he make this can?

11. A peanut vendor who sells peanuts at the street corner buys them in half bushel bags. He then measures them into pint bags ready for sale. Out of a purchase of $1\frac{1}{2}$ bushels how many pint bags is he able to obtain?

12. A young man helping in a grocery store has the job of "putting up" quart measures of beans in paper bags. The store manager gives him a box containing 2 bushels and starts him on the job. How many quart bags is the boy able to "put up" from these 2 bushels?

13. How many bushels will the bin to the right hold when filled level with the top?



14. The curved side of a cylindrical can that is to measure 15 in. high is to be made by rolling up a piece of sheet metal 15 in. wide and 5 ft. 1 in. long. When this is rolled up into its curved shape an allowance of 1 in. is to be made for the lap seam. When the can is completely finished how many pecks will it hold?

15. How many bushels will a circular bin hold that measures $6\frac{1}{2}$ ft. in diameter and 20 in. deep, when the contents is level with the top of the bin?*

ANSWERS TO PROBLEMS

Pages 300 to 302.

1. 4.21 cu. ft.
2. 61.19 qt.; 59.43 lb.
3. 35.9 buckets; 89.77 gal.
4. 211.9 lb.
5. 66.1 qt.
6. 29.38 gal.
7. 61.2 qt.
8. 46¢.
9. 57.07 bbl.
10. 13.23 in. deep.
11. $56\frac{1}{4}$ gal.; 6.02 in. drop.
12. 24.68 qt.; 104.73 gal.
13. 171.36 qt.
14. $6\frac{1}{4}$ gal.; 1.67-in. drop.
15. 228 half pints.

Pages 303 to 305.

1. 120 bu.
2. $1\frac{1}{8}$ qt.
3. $19\frac{1}{5}$ tons.
4. $1.79.
5. $3\frac{1}{2}$ bu.
6. 32 bu.; 48 bu.
7. 8.96 tons.
8. 26 cars; 765 tons.
9. 3 bu. (approx.)
10. 12.17 in. deep.
11. 96 bags.
12. 64 bags.
13. 40 bu.
14. 2 bu. (approx.)
15. 44.4 bu.

*Answers to these problems will be found on this page.

Review Problems Involving Calculations in Liquid and Dry Measures

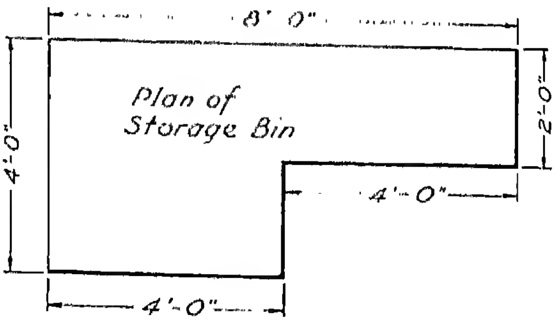
1. How many gallons in 48 quarts? How many pints in $7\frac{1}{2}$ gallons? How many quarts of vinegar in a barrel containing $31\frac{1}{2}$ gallons of vinegar? How many quarts of beans in a box containing $1\frac{1}{2}$ bushels of beans?

2. Determine the equivalent number of gallons of paint in two cartons each containing 24 pint cans.

3. In order to provide for the storage of 24 bushels of potatoes in his cellar, a grocer lays off on the cellar floor a space 4 ft. wide and $4\frac{1}{2}$ ft. long on which to construct a potato bin. How high should he build this bin in order to have the potatoes lie level with the top?

4. It is desired to construct a metal tank to hold 15 gallons. This tank, however, must be 15 in. deep. If 21 in. is selected as the length, how wide should the tank be?

5. The storage bin in the following plan is $2\frac{1}{2}$ ft. high. How many bushels will it hold?



6. A clerk in a grocery store has the job of filling paper bags with one peck of potatoes. How many of these paper bags can he fill from 4 large bags of potatoes each containing $1\frac{1}{2}$ bushels?

7. A young man who is planning to construct a coal bin large enough to hold 6 tons of anthracite coal, begins the task by laying out in a convenient place on the cellar floor, a space that is 7 ft. long and 6 ft. wide. It is his plan to have the coal level on the top so that the sides will extend up just even with the top of the coal in the bin. How high should the sides of this bin extend? In his efforts to obtain information that would aid him in working out this problem the young man finds that when 1 ton of anthracite is stored in this manner it takes up approximately 35 cubic feet. Upon this basis how high should this bin extend?

SHOP FORMULAS

Cutting speeds; R.P.M.; Belting; Screw Threads; Tap Drills; Bolts and Screws; Gear and Sprocket Drives; Tapers; Taper Turning.

Shop Formulas and Their Uses

Workmen in various trades are often called upon to do calculations that involve the use of what is known as a *shop formula.*

A shop formula is really a mathematical statement of a rule in which letters, symbols, or figures are used either separately or in combination with each other. Such formulas are used in the solution of problems by substituting for these letters or symbols their equivalent numerical values.

While there is a wide variety of formulas only the more elementary ones as applied to shopwork will be referred to in this text. The following is an example of such a formula.

$$C = 3.1416\ D$$

This is called the formula for *finding* the *circumference* of a *circle.* Expressed in words it means that the *length* of the circumference equals 3.1416 multiplied by the diameter. The *circumference* is expressed by the letter C, and the diameter by the letter D.

Sometimes the decimal 3.1416 is expressed by the symbol π, pronounced pī. By using this symbol instead of 3.1416, the formula becomes changed to $C = \pi D$, and is read "circumference equals pi times the diameter." More simply it is referred to as, pi D.

How this formula is applied is illustrated as follows.

Example 1:

If the diameter of a pulley is 8 in. what is its circumference?

Solution and Explanation:

$C = 3.1416\ D$.

Substituting the numerical value of D in this formula,

$C = 3.1416 \times 8$, or 25.1328.

That is, the circumference of an 8-in. pulley is 25.1328 in., or slightly more than $25\frac{1}{8}$ in.

Using this same formula, the *diameter* of a circle may also be found when its circumference is shown. This is determined by dividing the circumference by 3.1416.

Expressed as a shop formula this equals,

$D = C \div 3.1416$, or $D = \frac{C}{3.1416}$, which may also be expressed as

$$D = C \div \pi, \text{ or } D = \frac{C}{\pi}.$$

An application of this formula is seen in the following.

Example 2:

What is the outside diameter of a brass pipe that measures $9\frac{3}{4}$ in. around its circumference?

Solution and Explanation:

$D = C \div 3.1416.$

Substituting the numerical values of C in this formula,

$D = 9\frac{3}{4} \div 3.1416$, or $9.75 \div 3.1416$, which in turn reduces to,
$D = 3.103$ in.

That is, the outside diameter of the pipe that measures $9\frac{3}{4}$ in. around the circumference is 3.103 in.

NOTE: In using shop formulas, it is sufficient to carry the division to the second decimal place, unless the calculation involves very close accuracy, as in machine work or toolmaking. In such cases at least three decimal places should be used. If the dimension deals with a machine grinding operation it would be advisable to carry the division to the fourth decimal place. Measuring instruments common to machine shops and toolrooms may be used accurately for such close dimensions.

SURFACE SPEEDS AND CUTTING SPEEDS

The above formula is also used in calculating *surface speeds* of pulleys, and *cutting speeds* of lathe tools, circular saws, grinding wheels, and such revolving cutting tools.

Such speed is usually expressed in *feet per minute.* It equals the *distance in feet* through which a point on the circumference moves in one minute. This distance is equal to the *length* of the *circumference* in feet *multiplied* by the number of *revolutions* made *per minute.*

Using the letter S to represent *surface speed* in *feet per minute,* and "R.P.M." to represent *revolutions per minute,* this may be expressed in the following shop formula.

S = Circumference in feet × R.P.M., or,

$$3.1416 \times \text{Diameter in feet} \times \text{R.P.M.}$$

Since the diameter is usually given in inches, the quantity "3.1416 × Diameter" may be changed to its equivalent in feet by dividing it by 12. By representing the diameter in inches by D, this change becomes, $\frac{3.1416 \times D}{12}$, which in turn reduces to $.2618\ D$.

For convenience, this decimal form representing the circumference in feet will be used in problems that follow.

As a result of this change the above formula will then read,

$$S = .2618\ D \times \text{R.P.M.}$$

When this speed relates to the speed of a saw, or a revolving cutting tool, or to stock that revolves against a cutting tool, as in the case of a piece of metal, or a piece of wood being turned in a lathe, the speed becomes a *cutting speed.* In other such cases, as belt speeds, or revolving pulleys, it becomes *surface speed.*

This formula may be used in such problems as the following.

Example 1:

A 14-in. saw, as used on a variety saw in a woodworking shop, is listed as having 1800 R.P.M. At this rate what is its cutting speed?

Solution and Explanation:

The formula to be used in determining this cutting speed is,

$$S = .2618 \times D \times \text{R.P.M.}$$

In this problem 14 equals D, and 1800 equals R.P.M. Substituting these values in this formula,

$$S = .2618 \times 14 \times 1800, \text{ which reduces to } 6597.$$

That is, this saw has a cutting speed of 6,597 feet per minute.

This same rule may be used in finding the *Diameter* in inches when the Surface Speed and the Revolutions per Minute are known. This is done by dividing the surface speed by .2618 times the revolutions per minute. Using letters to represent these quantities this rule is expressed in the following shop formula:

$$D = \frac{S}{.2618 \times \text{R.P.M.}}$$

The following illustrates how this formula may be used.

Example 2:

What is the diameter of a line shaft pulley which has a surface speed of 1250 feet per minute when running at 500 R.P.M.?

Solution and Explanation:

In this problem S equals 1250, and R.P.M. equals 500.

Substituting these values in the above formula the diameter equals,

$$D = \frac{1250}{.2618 \times 500}, \text{ which in turn reduces to } 9.54.$$

That is, the diameter of the above pulley is 9.54 in.

In like manner this rule may also be used in calculating the Revolutions per Minute when the Surface Speed and the Diameter in inches are known. In this case, the surface speed is divided by, .2618 times the diameter.

Using letters to represent these quantities the formula for Revolutions per Minute equals:

$$\text{R.P.M.} = \frac{S}{.2618\, D}$$

The following is a typical application of this formula.

Example 3:

Determine the R.P.M. of a 12-in. pulley that has a surface speed of 1050 feet per minute.

Solution and Explanation:

In this problem S equals 1050, while D equals 12.

By using these figures in the above formula, the R.P.M. becomes:

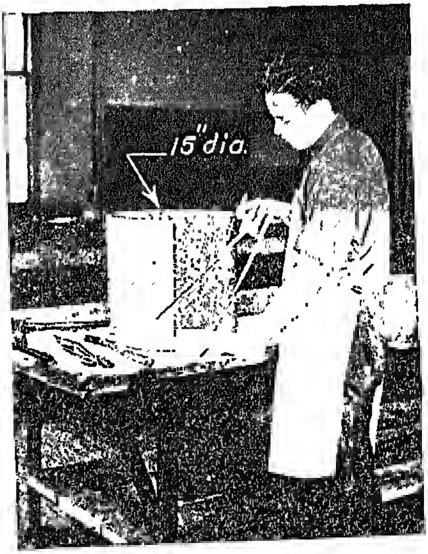
$$\text{R.P.M.} = \frac{1050}{.2618 \times 12},$$ which reduces to 334, approximately.

That is, the R.P.M. of the above pulley is 334.

Problems Involving the Formula of the Circle

1. A 12-in. circular saw as used in a cabinet shop is referred to as having a cutting speed of "a mile a minute." At this rate how many revolutions per minute does this saw make?

2. The workman in the picture to the right has the task of making a can 15 in. in diameter and is wondering if he has cut the piece of galvanized iron sheet long enough. If he were allowed $1\frac{1}{2}$ in. for lapping at the seam, how long should this piece of metal be?



3. What is the surface speed of a 10-in. pulley that makes 300 R.P.M.?

4. A grinding wheel 18 in. in diameter has a cutting speed of 3,200 feet per minute. How many revolutions per minute does this equal?

5. A piece of brass 4 in. in diameter is being turned in a lathe at the rate of 120 revolutions per minute. What is the cutting speed?

6. In checking the 16-in. grinding wheel in the picture it was found to turn at 1125 R.P.M. What would be its cutting speed at this rate?

7. A countershaft pulley 10 in. in diameter runs at 320 R.P.M. What is the surface speed of this pulley?

8. Calculate the cutting speed of a swing saw 14 in. in diameter that makes 1600 revolutions per minute.

9. The piece of metal in the engine lathe in the accompanying picture is being turned at the rate of 80 R.P.M. After the machinist *calipers* the diameter and finds that it measures 2 in., he decides to check the cutting speed. What do you figure the cutting speed to be?

10. A 6-in. cutter on a single spindle shaping machine, such as is used in cabinet shops, is checked as running at 3600 R.P.M. What is its cutting speed?

11. In order to obtain a cutting speed of 40 ft. per minute with a $2\frac{1}{2}$-in. milling cutter, what should be its R.P.M.?

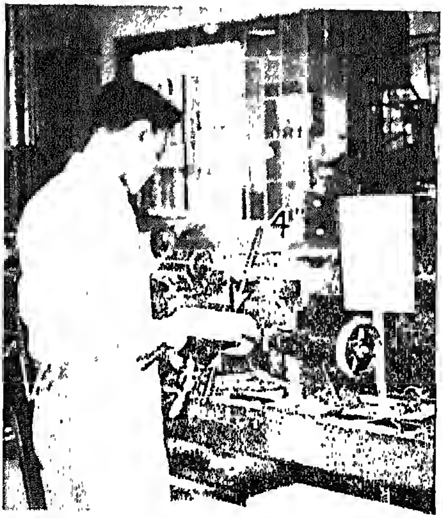
12. A young man visiting a lumber mill is quite surprised to learn of the high speeds at which some of the machines are run. He is told that a 36-in. band saw is listed at 1080 R.P.M. At this rate what would be the cutting speed of this band saw?

13. Compare the rim speeds of two pulleys, one 12 in. in diameter, the other 20 in. in diameter, running on the same shaft at 180 R.P.M.

14. The effective cutting diameter of the blades on the surfacing machine in the picture to the left is $4\frac{1}{2}$ in. This machine is listed as making 3000 R.P.M. According to this what is its cutting speed?

15. A grindstone 3 ft. in diameter is listed as having a surface speed of 450 ft. per minute. What is the equivalent number of R.P.M.?

16. At a spindle speed of 550 R.P.M. what is the cutting speed in feet per minute of the piece of wood 4 in. in diameter that is being turned in the wood lathe shown in the picture?

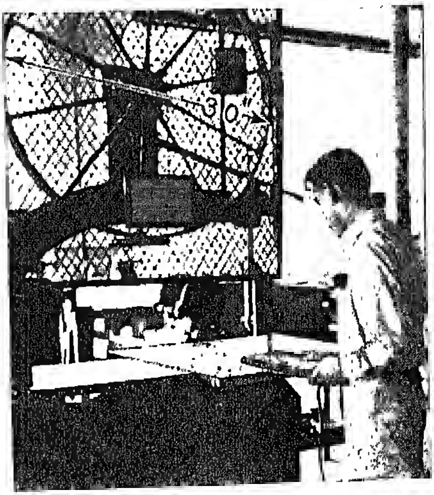


17. A 10-in. grinding wheel which has a surface speed of 4200 ft. per minute has become worn to 9-in. diameter. What is its cutting speed at this reduced diameter? What is the per cent reduction in speed?

18. In turning a round wooden stool seat in a wood lathe that has a spindle speed of 480 R.P.M. what would be the cutting speed when it becomes turned down to 14 in. in diameter?

19. In order to meet certain requirements it is necessary that the surface speed of a pulley should be 1200 ft. per minute, also that it should run at 320 R.P.M. In order to meet these requirements what should be the diameter of the pulley?

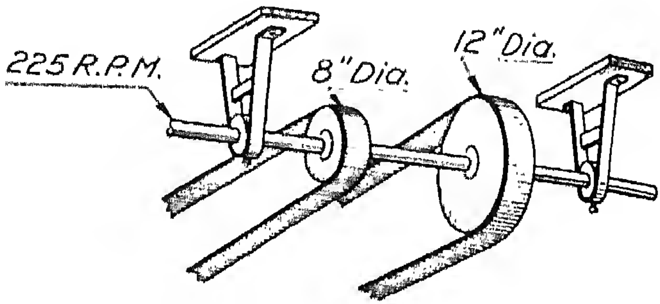
20. The 30-in. band saw in the accompanying picture is listed as having 900 revolutions per minute. What is the cutting speed that corresponds to this rating?



21. With the allowable cutting speed of 75 ft. per minute, what is the R.P.M. of a piece of steel $3\frac{1}{2}$ in. in diameter that is being turned in an engine lathe?

22. When the diameter of the piece in the above problem has become reduced to 2 in. what will be the cutting speed at the same R.P.M.?

23. Calculate the belt speeds of the pulleys as arranged on the shafting illustrated.



24. A wooden cylinder 8 in. in diameter is being turned to a smaller size in a wood-turning lathe running at 900 R.P.M. What is the cutting speed when this block is 6 in. in diameter?

25. What should be the diameter of a pulley on a motor making 1750 R.P.M. where the required surface speed is 2150 ft. per minute?

26. Determine the cutting speed of a 36-in. band saw running at 600 R.P.M.

27. If the allowable cutting speed of a piece of steel 3 in. in diameter is 85 ft. per minute what should be its equivalent R.P.M.?

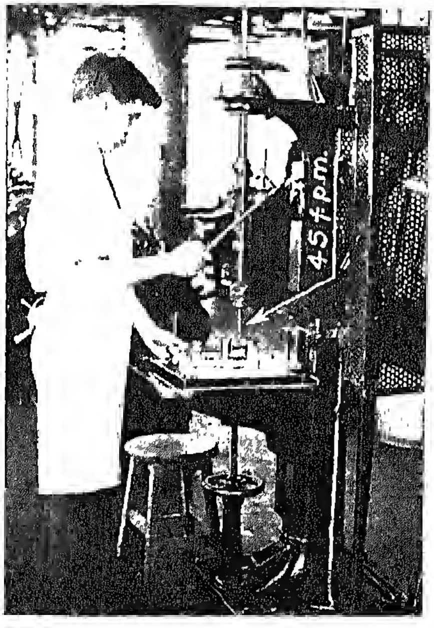
28. How large will be the diameter of a hoop made by bending into circular shape a strip of galvanized band iron that measures 42 in. long? An allowance of $1\frac{1}{2}$ in. is to be made for a riveted joint.

29. Calculate the surface speed of a pulley 18 in. in diameter that runs at 150 R.P.M.

30. At what R.P.M. should a piece of steel $1\frac{3}{4}$ in. in diameter be turned in an engine in order to meet the recommended cutting speed of 90 ft. per minute?

31. An 18-in. polishing wheel as used in a novelty manufacturing shop has a speed of 400 R.P.M. From this determine its polishing speed in feet per minute.

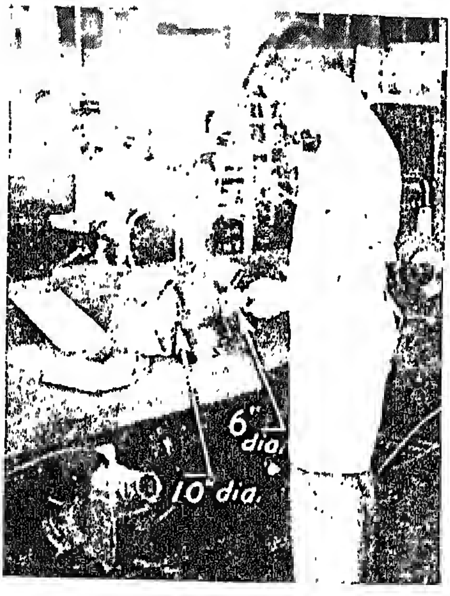
32. In order to have an allowable cutting speed of 45 ft. per minute, how many revolutions per minute should the $\frac{3}{4}$-in. drill make in the drill press shown in the picture?



33. An 8-in. pulley is the only one available to use on a motor making 1200 R.P.M. If the required surface speed of the pulley that is to be used on this motor is to be 1950 ft. per minute, to what diameter must this 8-in. pulley be reduced in order to meet this requirement?

34. A brass cylinder which is about to be turned in a lathe measures $3\frac{3}{4}$ in. in diameter. If the allowable cutting speed for this material is recommended as 90 ft. per minute, how many revolutions per minute should this piece make?

35. The cloth buffs on the polishing wheel illustrated in the picture were originally 12 in. in diameter and were rated as having a surface speed of 4500 ft. per minute. What are the surface speeds after they have worn down as shown?



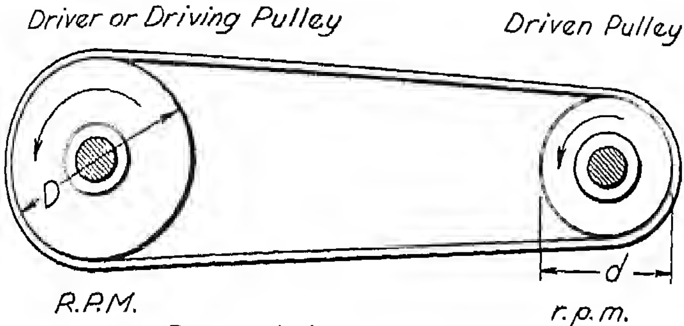
36. If the allowable cutting speed of a 12-in. grinding wheel used in a machine shop is specified as 6,000 ft. per minute, what should be the corresponding R.P.M.?*

PULLEYS CONNECTED BY BELTS

Among other simple shop calculations that a workman is called upon to make are those relating to pulleys that are driven by belts.

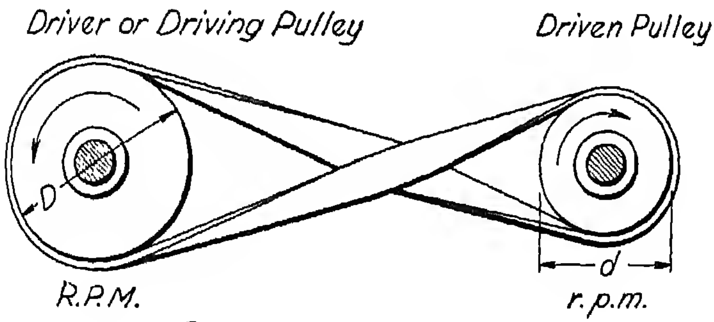
*Answers to these problems will be found on page 330.

In the following illustration of the "open" type of belt drive, both the *driving* pulley and the *driven* pulley rotate in the *same* direction as indicated.



Open Style of Belt Drive

In the "cross" type of belt drive as illustrated below, the *driven* pulley rotates in the *opposite* direction to the *driver.*



Cross Style of Belt Drive

The diameter of the driving pulley in each type is usually indicated by the *capital* letter "*D*"; while the diameter of the driven pulley is indicated by the *small* letter "*d.*" Also, the revolutions per minute of the driving pulley are indicated by the *capital* letters "R.P.M."; and those of the driven pulley by the *small* letters "r.p.m."

Since these pulleys are connected by a belt, their surface speeds are the same, for all practical purposes. That is, the surface speed of the driving pulley *equals* that of the driven pulley. Because of this equality, we may express such a condition by the following shop formula: Surface Speed of Driving Pulley = Surface Speed of Driven Pulley, or using the above abbreviations, $D \times$ R.P.M. $= d \times$ r.p.m.

This means that if the diameter of the driving pulley were multiplied by the number of revolutions it makes per minute, the product would equal that obtained by multiplying the diameter of the driven pulley by its revolutions per minute.

From this general formula, any one of the 4 items, D, d, R.P.M., or r.p.m., in the formula may be determined when the other three are known.

For example, the diameter of the driver D may be calculated by multiplying the diameter of the driven pulley by its revolutions per minute, and then dividing this amount by the revolutions per minute of the driver. Expressed as a shop formula this becomes:

$$(4) \qquad D = \frac{d \times \text{r.p.m.}}{\text{R.P.M.}}$$

This formula is applied in such problems as the following.

Example 1:

A small pulley is needed on a motor running at 1750 R.P.M. to drive a 20-in. pulley on a jack shaft that runs at 350 r.p.m. What size pulley must be used on the motor?

Solution and Explanation:

In this problem the diameter of the driven pulley d is 20 in., while its r.p.m. is 350. The R.P.M. of the driving pulley D equals 1750.

Using these values in the above formula for the diameter of the driver D, this works out as follows,

$$D = \frac{d \times \text{r.p.m.}}{\text{R.P.M.}} = \frac{20 \times 350}{1750} = 4.$$

That is, the diameter of the motor pulley should be 4 in.

The revolutions made per minute by the driving pulley are found by multiplying the diameter of the driven pulley by its revolutions per minute, and then dividing this amount by the diameter of the driver.

Expressed as a shop formula this becomes:

$$\text{(2)} \qquad \text{R.P.M.} = \frac{d \times \text{r.p.m.}}{D}$$

This formula is applied as follows.

Example 2:

What is the R.P.M. of a 15-in. pulley that drives a 6-in. pulley at 450 r.p.m.?

Solution and Explanation:

In this case $d = 6$ in.; r.p.m. $= 450$; $D = 15$ in.

Using these in the above formula, the revolutions of the driver equal

$$\text{R.P.M.} = \frac{d \times \text{r.p.m.}}{D} = \frac{6 \times 450}{15} = 180$$

That is, the R.P.M. of the driving pulley is 180.

In the same manner the diameter of the driven pulley may be found by multiplying the diameter of the driver by its revolutions per minute, and then dividing this amount by the revolutions per minute of the driven pulley. This is expressed in the following shop formula:

$$\text{(3)} \qquad d = \frac{D \times \text{R.P.M.}}{\text{r.p.m.}}$$

The following shows an application of the above formula.

Example 3:

What is the diameter of the pulley on a jack shaft running at 240 r.p.m. and driven by a 6-in. pulley which runs at 600 R.P.M.?

Solution and Explanation:

In this problem r.p.m. = 240; D = 6 in.; R.P.M. = 600.

Using these values in the above formula, the diameter of the driven pulley becomes:

$$d = \frac{D \times \text{R.P.M.}}{\text{r.p.m.}} = \frac{6 \times 600}{240} = 15.$$

That is, the diameter of the pulley on the jack shaft should be 15 in.

To determine the revolutions made per minute by the driven pulley, multiply the diameter of the driving pulley by the revolutions it makes per minute, and then divide this amount by the diameter of the driven pulley.

Expressed as a shop formula:

$$\text{(4)} \qquad \text{r.p.m.} = \frac{D \times \text{R.P.M.}}{d}$$

Problems like the following are solved by using this formula.

Example 4:

If a 12-in. pulley making 250 R.P.M. drives an 8-in. pulley by means of a belt, how many revolutions per minute does this 8-in. pulley make?

Solution and Explanation:

In this problem D = 12 in.; R.P.M. = 250; d = 8 in.

Using these figures in the above formula:

$$\text{r.p.m.} = \frac{D \times \text{R.P.M.}}{d} = \frac{12 \times 250}{8} = 375.$$

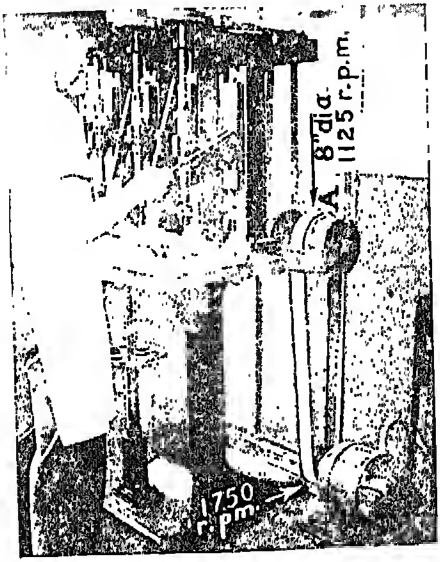
That is, the revolutions of the 8-in. pulley would be 375 per minute.

Problems Involving the Application of the Formulas on Belt Drive

1. A pulley 7 in. in diameter is driven by a pulley 10 in. in diameter which turns at 350 R.P.M. What is the r.p.m. of the driven pulley?

2. It is desired to reduce the revolutions of a jack shaft to 320 R.P.M. What size pulley must be used if this jack shaft is driven by a 5-in. pulley making 720 R.P.M.?

3. In the belt drive on the 4-spindle drill press in the accompanying picture it is necessary that the driven pulley *A* make 1125 R.P.M. This pulley measures 8-in. in diameter and is to be driven from a pulley on the motor which turns at 1750 R.P.M. What size pulley must be used on the motor?



4. An 8-in. pulley on a motor running at 870 revolutions per minute drives a 24-in. pulley. What are the revolutions per minute of the driven pulley?

5. If it is desired to obtain a speed of 400 r.p.m. on the driven pulley in the above problem, what size pulley should be used?

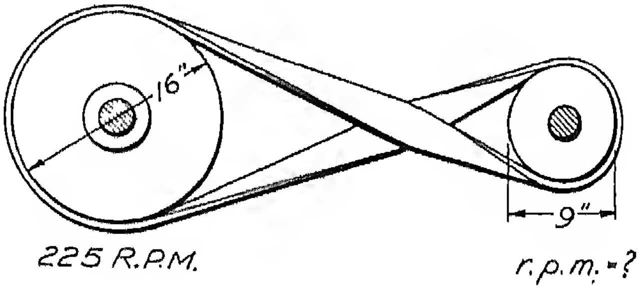
6. With each revolution of pulley *A* on the jig saw in the picture on page 328 one complete cutting stroke is made.

The pulley on the motor makes 1120 R.P.M. and is 1 in. in diameter. The driven pulley to which the belt is attached is $3\frac{1}{2}$ in. in diameter. How many complete cutting strokes per minute are made according to this setup?

7. What are the revolutions per minute of a driving shaft using a 12-in. pulley which in turn drives a 20-in. pulley at 210 revolutions per minute?

8. What size pulley must be used on a motor running at 1500 R.P.M. in order to drive a 15-in. pulley at 480 r.p.m.?

9. What is the R.P.M. of the 9-in. pulley as driven by the 16-in. pulley in the following setup?



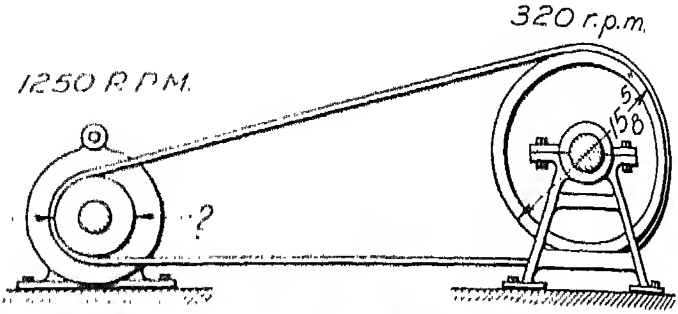
10. On the filing machine in the accompanying picture one filing stroke is made each time pulley A makes one revolution. The pulley on the motor is $1\frac{1}{4}$ in. in diameter and makes 1750 R.P.M. Pulley A measures $6\frac{1}{2}$ in. in diameter. How many filing strokes does this machine make per minute?

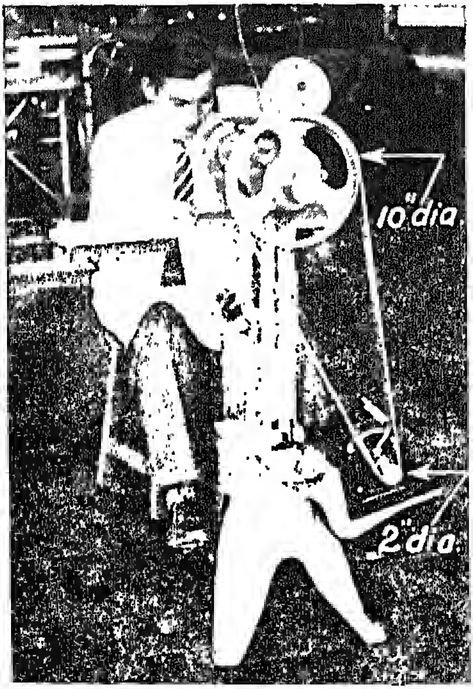
11. A driven pulley 4 in. in diameter has a speed of 1800 r.p.m. What is the diameter of its driving pulley which turns at 450 R.P.M.?

12. Calculate the diameter of a pulley necessary to obtain 168 r.p.m. when connected with a 12-in. pulley making 280 R.P.M.?

13. Determine the revolutions per minute of a $7\frac{1}{2}$-in. pulley which is driven by a 10-in. pulley running at 285 revolutions per minute.

14. What size pulley is used on the motor in the following?





15. On the same shaft with the $15\frac{5}{8}$-in. pulley above is a 12-in. pulley which in turn drives an 8-in. pulley. What are the revolutions per minute of this 8-in. pulley?

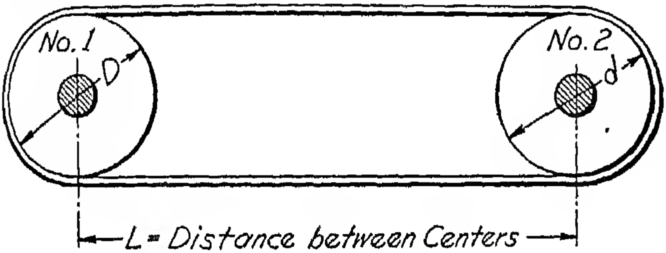
16. The capacity of the power stitcher in the picture to the left is indicated by the number of wire staples it can stitch in one minute. Each time the larger pulley makes one revolution the machine makes one stitch. On this particular machine the small driving pulley is 2 in. in diameter and makes 1140 R.P.M. The larger pulley measures 10 in. in diameter. How many stitches is this machine capable of making in one minute?

17. A 16-in. pulley has a surface speed of 840 ft. per minute. It is driven by a 7-in. pulley. What is the R.P.M. of each pulley?*

BELTING

Length of Open-Style Belt

In calculating the length of belt required for open pulley drives the following formula may be used. It gives approximate measurements and serves for general practical purposes, especially where there is not a wide difference in the sizes of the diameters.



Referring to the above sketch, the length of the open belt is seen to be equal to one half the circumference of pulley No. 1, plus one half the circumference of pulley No. 2, plus twice the distance between the centers of the pulleys.

Using: D for the diameter of pulley No. 1, expressed in inches;
d for the diameter of pulley No. 2, expressed in inches;
L for the length between centers, expressed in inches;
the above rule may be readily expressed in the following shop formula:

$$\frac{3.1416\ D}{2} + \frac{3.1416\,d}{2} + 2\,L$$

Changing this to simpler form it equals:

$$\frac{3.1416}{2}\,(D + d) + 2\,L, \text{ or,}$$

$$1.5708\,(D + d) + 2\,L$$

*Answers to these problems will be found on page 339.

To avoid confusion in calculations involving this rule, all these dimensions should be given in inches. The final result can then be reduced to feet by dividing the total length in inches by 12, as there are 12 inches in one foot.

The use of this rule is illustrated in the following problem.

Example 1:

Two pulleys, one 18 in. in diameter, the other 20 in. in diameter are 10 ft. between centers and are driven by an open belt. What is the length of this belt?

Solution and Explanation:

The diameter of the large pulley D equals 20 in.

The diameter of the small pulley d equals 18 in.

The length between centers equals 10 ft. This changed to inches equals 10×12, or 120 in. The length L then becomes 120 in.

As the first step in using such formulas, the values of D, d, and L, should be substituted for these letters. With these changes the result then becomes:

Length of open belt $= 1.5708\ (20 + 18) + 2 \times 120.$

As the next step, the quantity within the parentheses ( ) should be simplified. This changes $1.5708\ (20 + 18)$, to 1.5708×38, or 59.6904 in.

This gives the length of the open belt as $59.6904 + 240$, or 299.6904 in.

By changing 299.6904 in. to its equivalent in feet, there results:

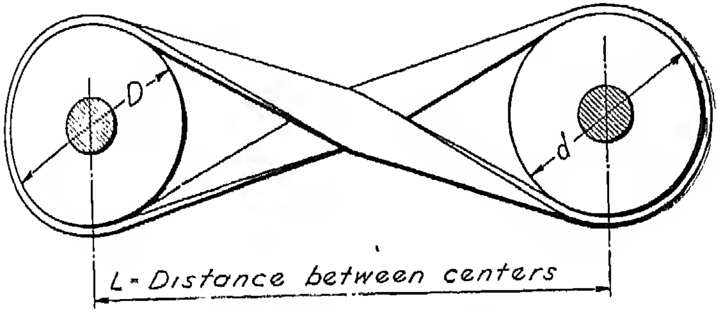
$$299.6904 \div 12, \text{ or } 24.9742 \text{ ft.}$$

Reducing to inches the decimal portion of a foot .9742, this equals $.9742 \times 12$, or 11.6904 in. This is approximately $11\frac{3}{4}$ in.

This results in making the total length of open belt required in this problem as, 24 feet — $11\frac{3}{4}$ inches.

In calculating the length of a crossed belt like that in the sketch on page 333, the previous formula becomes changed slightly to provide for the increased length of belt needed for this style of drive.

While there are several formulas for the length of crossed belts, the following is among the more simple ones and is sufficiently accurate for ordinary purposes. Instead of the num-



ber 3.1416, as used in the previous formula, the number $3\frac{1}{4}$, or 3.25, is used. This gives as a result the following new formula.

$$3.25\,\frac{(D + d)}{2} + 2\,L, \text{ which in turn reduces to,}$$

$$1.625\,(D + d) + 2\,L$$

How this formula is used is shown in the following problem.

Example 2:

Determine the length of crossed belt needed in connecting two pulleys that measure 12 in. in diameter and 14 in. in diameter and whose centers lie 18 ft. apart.

Solution and Explanation:

In this problem D equals 14; d equals 12, and the distance between centers L expressed in inches equals 18×12, or 216.

By substituting these values in the formula, it becomes, Length of crossed belt $= 1.625\,(14 + 12) + 2 \times 216$.

Following the same plan as in the previous problem, the bracketed expression, $1.625\,(14 + 12)$, becomes changed to 1.625×26, which equals 42.25 in.

In turn $2 \times L$, or 2×216, becomes 432 in.

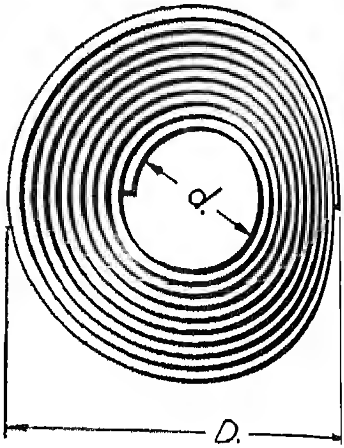
Adding these two amounts together the length becomes, 42.25 in. + 432 in., or 474.25 in. Reduced to feet this equals 39.5208 feet.

By changing the decimal portion .5208 feet to inches there results .5208 × 12, or 6.2556 in., which is approximately $6\frac{1}{4}$ in.

This gives the final result as 39 ft. $6\frac{1}{4}$ in. for the length of the crossed belt required.

Length of Belt in Coil

Should it be necessary to determine the number of feet of belting in a coil, as in the sketch to the right, the following shop formula is used.



Referring to the sketch,

N = number of laps in coil

D = outside diameter of coil in inches

d = inside diameter of coil, or hole, in inches.

Using these as symbols, the formula for the length of a coiled belt in feet equals,

$$3.1416\ \frac{(D + d)}{24} \times N.$$

The quantity $\frac{(D + d)}{24}$ equals one half the sum of the diameters reduced to feet. Expressed in lowest terms, this formula becomes:

$$\text{Length of belt in coil} = .131\ (D + d) \times N.$$

This is applied as follows:

Example 3:

A coil of belt measures 15 in. outside diameter. The hole, or inside diameter is 5 in. There are 20 laps in the coil. How many feet of belting in this coil?

Solution and Explanation:

Length of belt in coil = .131 $(D + d) \times N$.

In the problem $D = 15$; $d = 5$; $N = 20$.

Substituting these values in the above formula the length of belt in the coil equals,

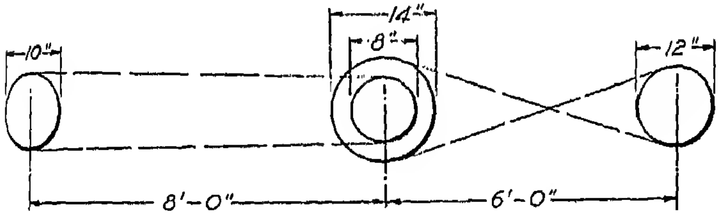
.131 (15 + 5) × 20, or .131 × 20 × 20, which equals 52.4 ft.

Reduced to inches, the decimal portion .4 ft. is approximately $4\frac{3}{4}$ in.

The length in feet and inches of the belt in the above coil then becomes equal to 52 ft. $4\frac{3}{4}$ in.

Problems Involving the Calculation of Belt Lengths

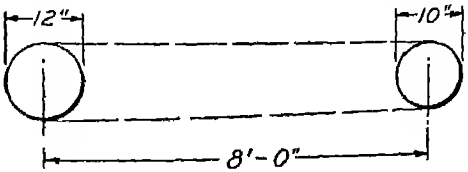
1. In the following drawing calculate the length of belt needed for each drive.



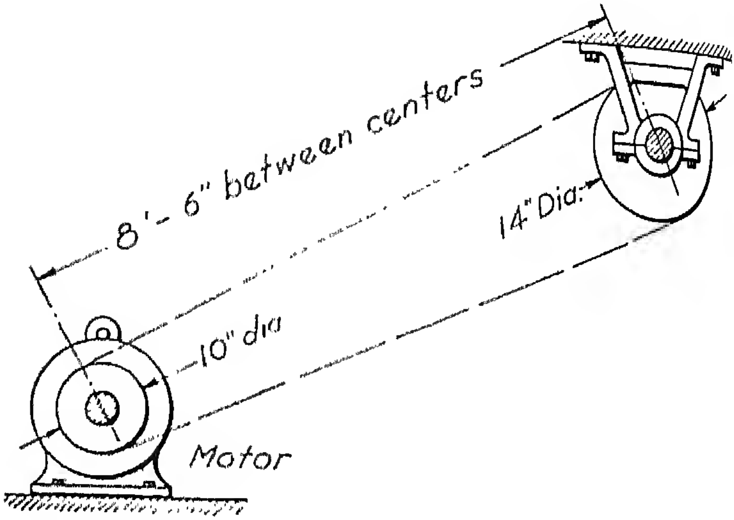
2. What is the approximate length of belting needed to connect two open drive pulleys, 20 in. in diameter and 24 in. in diameter, the centers of which are $11\frac{1}{2}$ ft. apart?

3. Determine the length of belt in a coil that measures 19 in. outside diameter and 7 in. inside diameter with 20 laps.

4. Calculate the approximate length of belt needed in connecting the pulleys in the following drawing.



5. What is the approximate length of belt needed in the following setup?



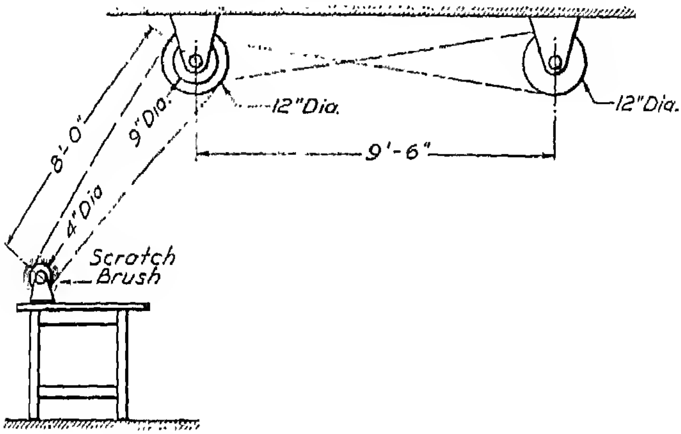
6. Two pulleys measuring 28 in. and 32 in. in diameter lie 18½ ft. on centers. If a crossed-belt drive were used on these pulleys, determine the approximate length of belt needed. If an open belt were used, approximately how long would that be?

7. In checking over the shop inventory, 4 rolls of belting are listed as follows. The apprentice doing this checking is asked to calculate the length of each roll. What is the length of each roll?

One roll has 9 laps and measures 8 in. outside diameter and 3½ in. inside diameter.
The second roll has 12 laps and measures 12½ in. outside and 5 in. inside.
The third roll has 10 laps and measures 14 in. outside and 8 in. inside.
The fourth roll has 24 laps and measures 19½ in. outside and 6 in. inside.

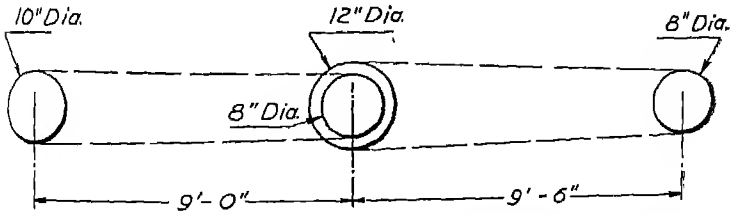
8. Two pulleys 14 ft. on centers are connected by a crossed belt. One pulley measures 24 in. in diameter and the other measures 22 in. What is the approximate length of the belt required?

9. Determine the approximate length of belting needed for the following arrangement of pulleys.

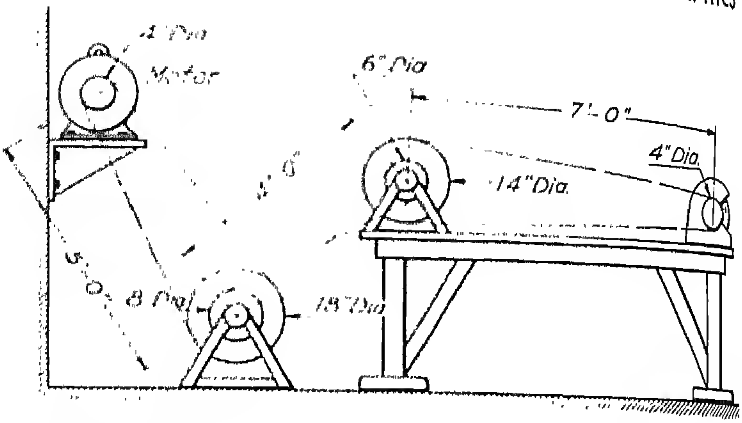


10. A coil of belting that measures $14\frac{1}{2}$ in. outside diameter and $5\frac{1}{2}$ in. inside diameter has 15 laps. Calculate the length of this belt.

11. What is the approximate length of belt required for each of the following pulley drives?

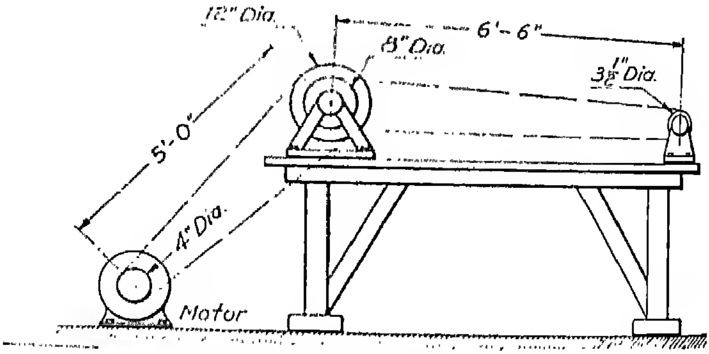


12. Should it be decided to use 2 crossed-belt drives in the above arrangement instead, what would be the approximate length of belt required?



13. Referring to the above drawing, it is found necessary to disconnect the motor for this setup. The 18-in. pulley is then driven by a crossed belt from a 12-in. pulley on a jack shaft that is 9½ ft. center distance from it. Make a rough sketch of this changed condition and show the center distance with the new arrangement of the 18-in. pulley and the 12-in. pulley. Determine the length of belt needed for each drive. What is the approximate length of the crossed belt required in making this change?

14. Determine the approximate length of belting needed for each of the drives in the following setup of a shopwork bench.*



*Answers to these problems will be found on page 339.

ANSWERS TO PROBLEMS

Pages 315 to 322.

1. 1680.7.
2. $48\frac{5}{8}$ in. approx.
3. 785.4 f.p.m.
4. 679.1 r.p.m.
5. 125.7 f.p.m.
6. 4712.4 f.p.m.
7. 837.8 f.p.m.
8. 5864.3 f.p.m.
9. 41.9 f.p.m.
10. 5654.9 f.p.m.
11. 61.1 r.p.m.
12. 10178.8 f.p.m.
13. 565.5 f.p.m.; 942.5 f.p.m.
14. 3534.3 f.p.m.
15. 47.9 r.p.m.
16. 575.96 f.p.m.
17. 3780 f.p.m.; 10%.
18. 1759.3 f.p.m.
19. 14.3 in. dia.
20. 7068.6 f.p.m.
21. 81.85 f.p.m.
22. 42.86 f.p.m.
23. 471.2 f.p.m.; 706.9 f.p.m.
24. 1413.7 f.p.m.
25. 4.69 in. dia.
26. 5654.9 f.p.m.
27. 108.2 r.p.m.
28. 12.9 in. dia.
29. 706.9 f.p.m.
30. 196.3 r.p.m.
31. 1885 f.p.m.
32. 229.2 r.p.m.
33. 6.21 in. dia.
34. 91.7 r.p.m.
35. 2250 f.p.m.; 3750 f.p.m.
36. 1909.9 r.p.m.

Pages 326 to 331.

1. 500 r.p.m.
2. $11\frac{1}{4}$ in. dia.
3. 5.143 in. dia.
4. 290 r.p.m.
5. 17.4 in. dia.
6. 320 strokes.
7. 350 r.p.m.
8. 4.8 in. dia.
9. 400 r.p.m.
10. 336.5 strokes.
11. 16 in. dia.
12. 20 in. dia.
13. 380 r.p.m.
14. 4 in. dia.
15. 480 r.p.m.
16. 228 stitches.
17. 200.5 r.p.m.; 458.4 r.p.m.

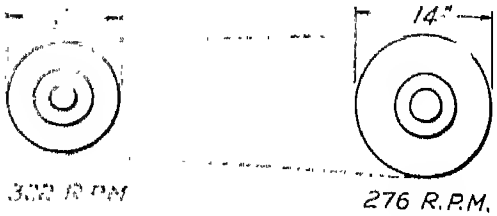
Pages 335 to 338.

1. 18 ft. $4\frac{1}{4}$ in.; 15 ft. $6\frac{1}{4}$ in.
2. 28 ft. $9\frac{1}{8}$ in.
3. 68 ft. $1\frac{1}{2}$ in.
4. 18 ft. $10\frac{1}{2}$ in.
5. 20 ft. $1\frac{5}{8}$ in.
6. 45 ft. $1\frac{1}{2}$ in.; 44 ft. $10\frac{1}{4}$ in.
7. 13 ft. $6\frac{3}{4}$ in.; 27 ft. $6\frac{1}{8}$ in.; 28 ft. $9\frac{7}{8}$ in.; 80 ft. 2 in.
8. 34 ft. $2\frac{3}{4}$ in.
9. 18 ft. $1\frac{1}{8}$ in.; 21 ft. $10\frac{1}{8}$ in.
10. 39 ft. $3\frac{5}{8}$ in.
11. 20 ft. $4\frac{1}{4}$ in.; 21 ft. $7\frac{3}{8}$ in.
12. 20 ft. $5\frac{1}{4}$ in.; 21 ft. $8\frac{1}{2}$ in.
13. 12 ft. $10\frac{1}{2}$ in.; 9 ft. $10\frac{3}{4}$ in.; 16 ft. $4\frac{1}{4}$ in.; 23 ft. $\frac{3}{4}$ in.
14. 12 ft. $1\frac{1}{8}$ in.; 14 ft. 6 in.

Review Problems on Pulleys, Surface Speeds, and Belt Lengths

1. A piece of stock in a lathe measures 2¼ in. in diameter. It turns at 160 R.P.M. What is its cutting speed?

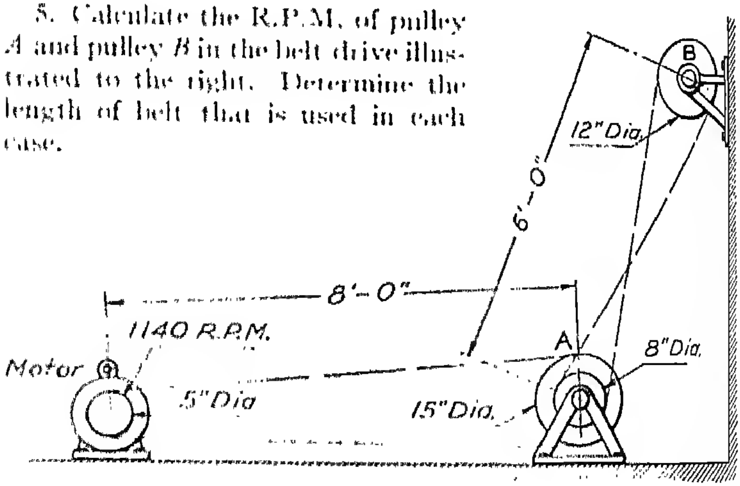
2. What size pulley is used in the following setup?



3. On the same shaft with the 14-in. pulley in the above problem, is an 18-in. pulley. What is the surface speed of this 18-in. pulley?

4. Calculate the cutting speed used in turning a cast-iron pulley 16 in. in diameter on an engine lathe which runs at 15 R.P.M.?

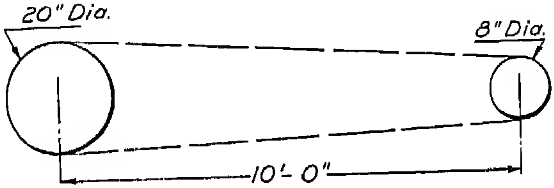
5. Calculate the R.P.M. of pulley *A* and pulley *B* in the belt drive illustrated to the right. Determine the length of belt that is used in each case.



6. What is the length of leather belting in the following rolls: One roll measures 12 in. outside diameter and 3 in. inside diameter and has 16 laps; the other roll measures 15 in. outside diameter and 4 in. inside diameter and has 20 laps.

7. A notation on a blueprint specifies that when drilling the $\frac{1}{2}$-in. hole indicated the cutting speed of the drill should be 30 ft. per minute. At this rate what should be the R.P.M. of the drill?

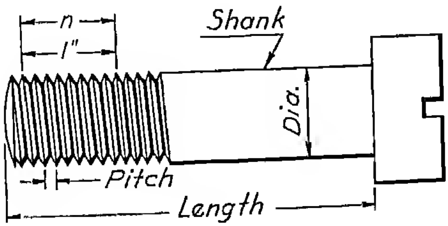
8. Calculate the length of the belt needed to connect the pulleys in the following drawing.



9. Determine the length of a roll of belting that measures $4\frac{1}{2}$ in. inside diameter and $16\frac{1}{2}$ in. outside diameter and has 24 laps.

SCREW THREADS

One of the everyday jobs in a machine shop is that of cutting screw threads on a lathe. These threads have definite proportions which may be calculated by the use of simple shop formulas.



Screw threads are usually referred to by the number of threads per inch, as for example, 8 threads per inch. This means that for each inch of length of the screw there are 8 threads. The length of the screw is the length of the "shank" measured from underneath the head of the screw as shown in the sketch on page 341.

The distance from the top of one thread to the top of the next thread is called the "pitch." This is determined by dividing 1 in. by the number of threads per inch.

Expressed as a shop formula this becomes:

$p = \frac{1}{n}$; where "p" represents pitch, and "n" the number of threads per inch.

Should it be desired to find the number of threads per inch when the pitch only is known, this formula becomes:

$$\text{Number of threads per inch, or } n = \frac{1}{p}.$$

These formulas are used as follows.

Example 1:

What is the pitch of the thread on a bolt having 8 threads per inch?

Solution and Explanation:

Since the pitch equals 1 in. divided by the number of threads per inch, or $\frac{1}{n}$, then in this case it would be equal to $1 \div 8$, or $\frac{1}{8}$, or .125.

This indicates that on a bolt having 8 threads to the inch, the pitch is $\frac{1}{8}$ in., or 0.125 in.

Example 2:

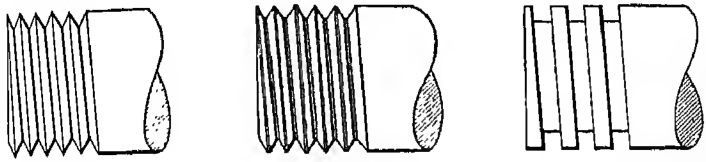
How many threads per inch on a screw having a pitch of .0625 in.?

Solution and Explanation:

Number of threads per inch equals $1 \div p$, which is, $1 \div .0625$, or 16.

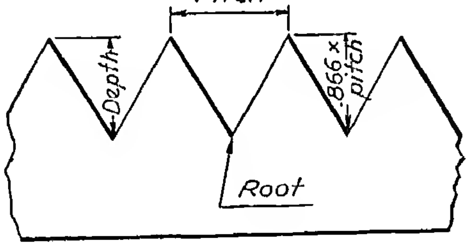
That is, there are 16 threads per inch on a screw having a pitch of .0625 in.

Among the more common forms of screw threads are the sharp "V" thread, the American National form of thread, and the "square" thread.



Sharp V thread   American National thread   Square thread

The sharp "V" thread is usually cut on smaller screws. This thread has a sharp top and a sharp bottom as shown below.



Outline of sharp V thread

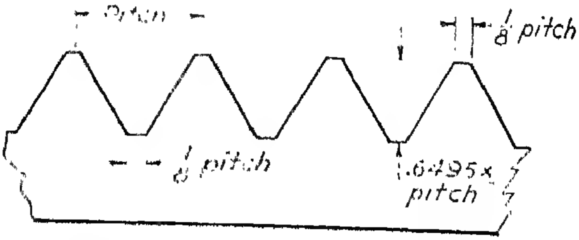
The bottom of the "V" is frequently called the "root" of the thread. The *depth* of the sharp "V" thread is found by *multiplying* the pitch by .866; or, by *dividing* .866 by the number of threads per inch.

Expressed as shop formulas these rules become:

$H = p \times .866$; where "H" is the depth of the sharp "V" thread, and "p" is the pitch; or

$H = \frac{.866}{n}$, where "n" equals number of threads per inch.

The American National form of screw thread has the same "groove," or outline as the sharp "V" thread, except that it has a flat top and a flat bottom, as illustrated below. This gives a stronger thread than the "V" thread.



Outline of American National thread

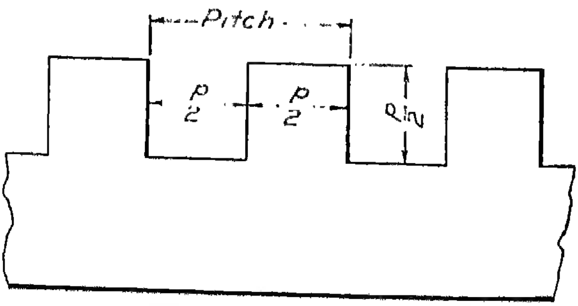
The flat on the top, also on the bottom, is $\frac{1}{8}$ of the pitch, or $\frac{1}{8} \times p$.

Such flattening of the top and the bottom makes the depth less than that of the sharp "V" thread. It is .6495 times p, as shown on the above drawing.

The shop formula used in this calculation is:

Depth of American National thread = h = p × .6495, Pitch × .6495, or when expressed in terms of the number of threads to the inch it becomes:

$$h = \frac{.6495}{n}.$$



Outline of Square thread

Of the heavier, or coarser, threads, the *square* thread is quite common. This thread gets its name from the shape of the outline of the thread as illustrated on page 344.

The square-shaped groove and the square-shaped thread each have a width equal to one half of the pitch. The depth is also estimated as one half of the pitch, although practical use requires that this depth be slightly greater than this amount.

For ordinary calculations the following is acceptable.

$$\text{Depth of square thread} = \frac{p}{2}.$$

How the above formulas on the depth of threads are applied is illustrated in the following problems.

Example 3:

Determine how deep a cut must be taken in cutting a sharp "V" type thread, 10 to the inch, on a special brass bolt.

Solution and Explanation:

Formula for depth of "V" thread, $H = \dfrac{.866}{n}$

Applied to this problem it works out as, $\dfrac{.866}{10}$, or .0866.

That is, the depth of the above "V" thread is 0.0866 in., or 0.087 in.

Example 4:

If it were decided to cut an American National screw thread 10 to the inch, what would be the depth of the thread?

Solution and Explanation:

The depth of American National thread is $h = \dfrac{.6495}{n}$.

Using this formula the depth of the above thread becomes, $\dfrac{.6495}{10}$, or .06495 in. which equals 0.065 in. when carried to 3 decimal places.

Example 5:

If the pitch of a sharp "V" thread is 0.125 in., what is the depth of this thread?

Solution and Explanation:

The depth of "V" threads in terms of the pitch equals, $H = p \times .866$.

As used in this problem it equals, $.866 \times .125$, or .10825.

That is, the depth of this thread equals 0.10825 in. or 0.108 in.

Example 6:

How deep a cut must be taken in cutting an American National thread on a bolt having a pitch of 0.20 in.?

Solution and Explanation:

The depth of American National threads equals, $h = p \times .6495$.

As applied to this problem it works out as follows:

$$.20 \times .6495, \text{ or } .1299$$

That is, the depth of this thread is 0.1299 in., or 0.130 in.

Example 7:

A square threaded screw on a bench vise has 4 threads to the inch. What is its pitch?

Solution and Explanation:

The formula for the pitch of a square thread is, Pitch $= \frac{1}{n}$.

Using this in the above problem the pitch equals:

$$\tfrac{1}{4}, \text{ or } 0.250 \text{ in.}$$

Example 8:

What is the depth of cut required in making the above square thread?

Solution and Explanation:

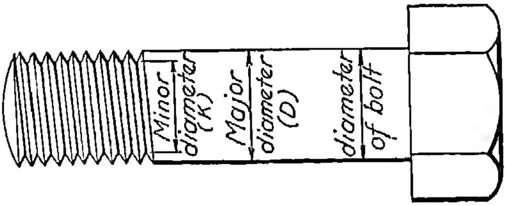
The formula for the depth of a square thread is,

$$\text{Depth} = \frac{p}{2}.$$

The depth of the cut required in making the foregoing square thread therefore is:

$$\frac{.250}{2}, \text{ or } 0.125 \text{ in.}$$

When a thread is cut on a screw, or on a bolt, as shown in the following sketch, the distance between the roots of the thread across the shank, is called the *minor diameter* of the thread.



This minor diameter is determined by subtracting from the major diameter, twice the depth of the thread. Expressed as a shop formula this becomes:

$$K = D - (2 \times H).$$

For the sharp "V" thread this would equal:

$$K = D - \left(2 \times \frac{.866}{n}\right) \text{ or,}$$

$$= D - (2 \times p \times .866).$$

For American National threads this would become:

$$K = D - \left(2 \times \frac{.6495}{n}\right) \text{ or,}$$

$$= D - (2 \times p \times .6495)$$

For the square thread this would be:

$$K = D - \frac{1}{n}, \text{ or,}$$

$$= D - p$$

Twice the depth of the thread of any screw is often referred to as the *double depth* of the thread.

The standard form of the American National thread is generally used by machine-tool manufacturers and by automobile manufacturers.

However, the Society of Automobile Engineers (S.A.E.), has adopted for automobile use a finer pitch thread for the same diameter screw. This means that automobile screws have more threads per inch than machine screws of the same diameter.

This finer pitch thread as used in automobile work, was formerly referred to as the S.A.E. standard thread, meaning the standard as adopted by the Society of Automobile Engineers. It is now known as the American National fine-thread (NF) series. The coarser pitch threads, commonly used on bolts, nuts, machine screws, and threaded parts in industry, and which were formerly referred to as the U.S.S. are now designated as the American National coarse-thread (NC) series of screw threads.

The method of calculating thread sizes and proportions in both of these series of screw threads is the same.

The following table illustrates the variations in the number of threads per inch of these two standards.

Dia. of screw	*Threads per inch*		*Dia. of screw*	*Threads per inch*	
	NC	*NF*		*NC*	*NF*
$\frac{1}{4}$ in.	20	28	$\frac{5}{8}$ in.	11	18
$\frac{5}{16}$ in.	18	24	$\frac{11}{16}$ in.	11	16
$\frac{3}{8}$ in.	16	24	$\frac{3}{4}$ in.	10	16
$\frac{7}{16}$ in.	14	20	$\frac{7}{8}$ in.	9	14
$\frac{1}{2}$ in.	13	20	1 in.	8	14
$\frac{9}{16}$ in.	12	18			

The older terms "Outside Diameter" and "Root Diameter," also have been changed. They are now known as the "Major Diameter" and "Minor Diameter," respectively.

Dimensional Symbols for Use in Formulas

Problems Illustrating the Use of the Above Formulas for Double Depth, and Diameters of Threads

Example 1:

The American National coarse-thread series standard on a $1\frac{3}{4}$-in. bolt is 5 threads per inch. What is the double depth of this thread, also the minor diameter?

Solution and Explanation:

Double depth $= 2 \times \dfrac{.6495}{n}$

Applied to this case the double depth equals:

$$2 \times \frac{.6495}{5}, \text{ or } .2598.$$

This gives the double depth of the above thread as .2598 in.

The minor diameter (K) = major diameter (D) −

$$\left(2 \times \frac{.6495}{n}\right)$$

As applied to the above problem this equals:

$$K = 1\tfrac{3}{4} \text{ in.} - \left(2 \times \frac{.6495}{5}\right), = 1.4902$$

The minor diameter of this bolt therefore equals 1.490 in.

Example 2:

Determine the minor diameter of a 1-in. bolt with a "V" type thread having a pitch of 0.125 in.

Solution and Explanation:

Minor diameter K = D − (2 × p × .866).

In this case the minor diameter equals, 1 − (2 × .125 × .866) which reduces to .7835.

That is, the minor diameter of this bolt equals 0.7835 in., or 0.784 in.

Example 3:

What is the minor diameter of a square threaded screw having 4 threads per inch and measuring $1\frac{3}{4}$ in. in diameter?

Solution and Explanation:

Minor diameter (K) of square thread $= D - p$.
In this problem the solution becomes, $1\frac{3}{4} - \frac{1}{4}$, or $1\frac{1}{2}$.
That is, the minor diameter of the above screw is $1\frac{1}{2}$ in.

Problems on Screw-Thread Calculations

NOTE: Thread dimensions such as $\frac{3}{4}$ in.—10 NC, or $\frac{3}{4}$ in.—16 NF, are used to indicate that threads of the American National form are used. The 10 NC means 10 threads per inch of the national-coarse thread series, and 16 NF means 16 threads per inch of the national fine-thread series.

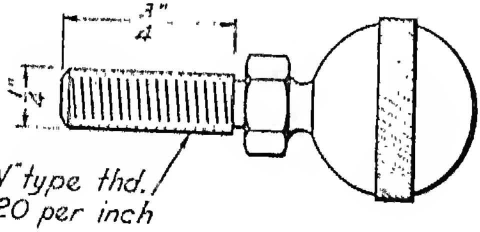
1. Determine the pitch of the threads on a machine bolt having 14 American National form threads per inch.

2. Compare the pitch of a $\frac{3}{8}$-in.—16 NC screw with that of a $\frac{3}{8}$-in.—24 NF screw.

3. Calculate the depth of thread as cut on a 12 NC screw.

4. What is the pitch, also the depth, of a square thread cut 6 threads to the inch, on a piece of stock that measures $1\frac{3}{4}$ in. in diameter?

5. After cutting threads on the end of the knob in the following sketch, what will be the minor diameter of the screw part?



6. Is there any difference in the minor diameters of a 1-in. bolt having 8 "V" threads per inch, and the same diameter bolt with 8 NC threads per inch? What is the difference in square inch area?

7. Determine the depth of cut made in cutting 10 "V" type threads to the inch on a screw measuring $\frac{3}{4}$ in. in diameter.

8. What is the minor diameter of the screw in the above problem?

9. In order to cut $2\frac{1}{2}$ square threads to the inch, how wide must the point of the lathe tool be that is used in cutting this thread?

10. How far does the tool cut into the stock in forming the thread in the above problem?

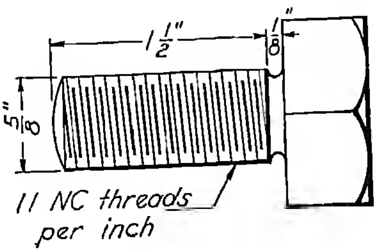
11. How many threads per inch on screws having 0.0625-in. pitch; 0.05-in. pitch; and 0.125-in. pitch?

12. How deep will the lathe tool cut in making "V" type threads of the above proportions?

13. The screw stem on a valve measures $\frac{7}{8}$ in. in diameter and has 9 square threads per inch. Calculate the pitch, the depth of thread, and the minor diameter.

14. A shaft $2\frac{1}{2}$ in. in diameter has been set up in a lathe so that 4 NC threads to the inch may be cut on one end. How deep must this thread be cut? What is the diameter across the roots of the thread?

15. What is the pitch of the thread in the following square-head setscrew? What is the depth of this thread?

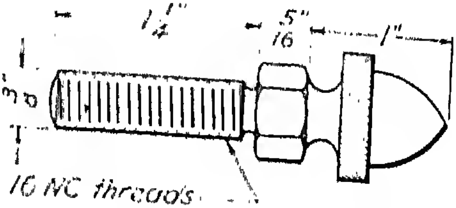


16. To what measurement should the point be ground on the tool that is used in cutting the above screw thread?

17. In order to fasten a bracket on an automobile frame it becomes necessary to make $\frac{7}{16}$-in. 24 NF threads per inch. What is the pitch of this thread? What is the minor diameter of this bolt?

18. What is the double depth of a "V" thread cut on a 2-in. spindle having $4\frac{1}{2}$ threads to the inch? Give the minor diameter.

19. Calculate the area at the minor diameter of the following special binding screw.

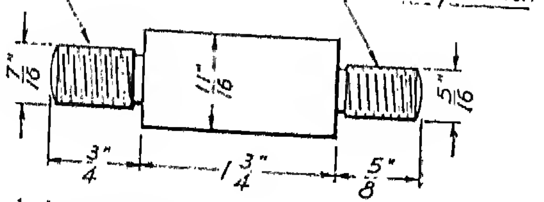


20. What is the width of the point on the threading tool used in cutting a $1\frac{1}{8}$-in. 7 NC thread?

21. Determine the pitch, also the minor diameter, of the threaded portion of a $\frac{3}{8}$-in. 24 NF screw. What is the area of the minor diameter?

22. Calculate the pitch, also the double depth, of a $1\frac{1}{2}$-in. 6 NC screw.

23. Determine the pitch and the minor diameter of each of the threaded ends on the following spindle.



24. Find the width of the point of the threading tools used in cutting 10 threads per inch, and 8 threads per inch, American National form threads. How deep will these tools cut in each case?

25. Calculate the pitch, also the double depth, of a 20 NF screw.

26. A "V" thread is to be cut on a special stud $\frac{3}{8}$ in. in diameter. How deep should this cut be made if there are 16 threads to the inch?

27. A $1\frac{1}{2}$-in. pipe has $11\frac{1}{2}$ threads to the inch. What is the pitch of this thread?

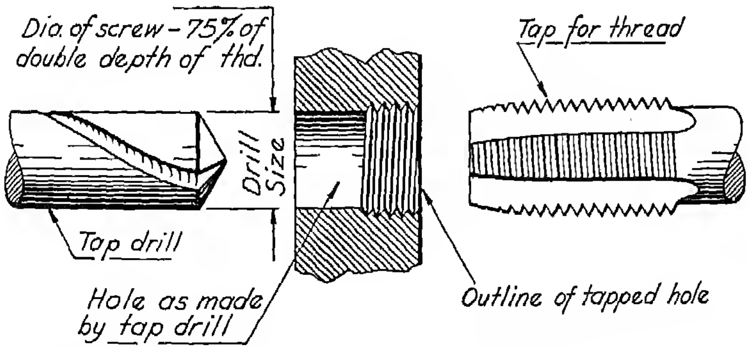
28. Calculate the pitch of a square thread cut $2\frac{1}{2}$ threads to the inch. How deep does the tool cut in making this thread?

29. Calculate the depth of a "V" thread having $\frac{1}{16}$-in. pitch.

30. What is the pitch, the depth of the thread, and the root diameter of a square threaded screw 1 in. in diameter having 5 threads to the inch?*

TAP-DRILL CALCULATIONS

When it becomes necessary to insert a screw or a bolt into a piece of metal, a hole of the proper size is first drilled in the metal and this hole is then threaded to suit the screw thread. In calculating the size of this hole for a given thread, the



*Answers to these problems will be found on pages 366 and 367.

major diameter of the screw and the pitch, or the number of threads to the inch, should be known.

The drill used for this purpose is called a "*tap drill.*" The thread is referred to as a "*tapped thread.*"

How such a hole appears, when drilled with a *tap drill* and partly tapped with the corresponding *tap*, is illustrated on page 355.

Theoretically, a hole drilled to the size of the minor diameter should be correct for "tapping" the thread. But for practical purposes such a hole would be too small, or too "tight," as it does not allow sufficient working clearance. Because of this the drilled hole must be slightly larger than the minor diameter.

One generally accepted rule for this is that the diameter of the tapped drilled hole should be approximately equal to the *major diameter, D,* of the screw, *less 75%* of the double depth of the thread.

Expressed in a general formula this would be:

$$\text{Tap drill size} = D - (.75 \times 2 \times H)$$

This size hole is referred to as "allowing 75% of a full thread."

For the American National form threads this calculated tap-drill size, expressed in terms of the Pitch, p, and the number of threads, n, per inch, becomes:

$D - \left(.75 \times \dfrac{1.3}{n}\right)$, which reduces to, $D - \dfrac{.97}{n}$, or $D - .97\,p$.

For the sharp "V" type of thread, this is changed to:

$D - \left(.75 \times \dfrac{1.732}{n}\right)$, which reduces to, $D - \dfrac{1.3}{n}$, or $D - 1.3\,p$.

From this *calculated size*, the *commercial* drill size in 64ths of an inch may be found by determining how many *full* 64ths there are in the calculated size. This is determined by multiplying the *decimal portion* of the calculated size by 64. The whole number in this result indicates how many *full* 64ths there are in this decimal.

This number of 64ths, combined with the original whole number, if any, in the calculated size, gives the commercial tap-drill size to the nearest 64th of an inch, that should be used.

While some tap-drill manufacturers may vary slightly from this, the above rule generally holds true.

The following problems illustrate how these formulas are used.

Example 1:

What is the calculated size of tap drill needed in making a square nut for a $1\frac{5}{16}$ in.—7 NC bolt?

Solution and Explanation:

The calculated sizes of tap drills for American National form threads $= D - \dfrac{.97}{n}$.

Applied to this problem this becomes, $1\frac{5}{16} - \dfrac{.97}{7}$, which reduces to $1.3125 - .1385$, or 1.174 in.

To determine the equivalent *commercial* size drill, the *decimal portion* of this number, 1.174, is multiplied by 64. This equals $.174 \times 64$, or 11.036.

In this result, the whole number 11 indicates that in this decimal portion .174 there are 11 full 64ths. Accordingly, in 1.174 in. there are $1\frac{11}{64}$ in. The commercial drill size, therefore, is $1\frac{11}{64}$ in., which also equals the size recommended by drill manufacturers.

Example 2:

Determine the size of the tap drill to be used in tapping a hole for a $\frac{3}{4}$-in. "V" threaded stud having a pitch of 0.10 in.

Solution and Explanation:

Calculated tap drill for "V" threads $= D - (p \times 1.3)$.

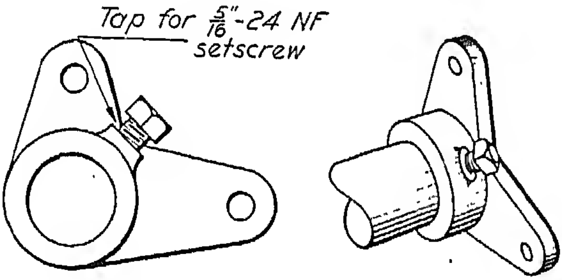
Used in this problem the tap-drill size becomes, $\frac{3}{4} - (.10 \times 1.3)$, or $.750 - .130$, or .620 in.

Following the procedure in the foregoing problem, the commercial drill size in 64ths is determined by multiplying .620 by 64. This gives .620 × 64, or 39.680.

Disregarding the decimal portion in this, the whole number 39 indicates that there are $\frac{39}{64}$ in. in the equivalent commercial size of tap drill. Therefore, a $\frac{39}{64}$-in. tap drill is the one to use for this job, and is the size recommended by drill manufacturers.

Example 3:

In attaching the "bell crank lever" to the shaft as illustrated below, what size tap drill should be used for the $\frac{5}{16}$-in.—NF setscrew?



Solution and Explanation:

Using the same formula as in calculating tap-drill sizes for the American National threads, the calculated size of tap drill for this $\frac{5}{16}$-in.—24 NF setscrew would be determined as follows:

$$\text{Calculated size of tap drill} = D - \frac{.97}{n}.$$

Applied to this particular problem the size becomes, $\frac{5}{16} - \frac{.97}{24}$, or .3125 − .0404 which equals .2721 in. as the calculated size.

The nearest 64th size to this would be found as in the foregoing problems by reducing .2721 to 64ths, which in this case equals $\frac{17}{64}$ in.

The commercial tap-drill size to use therefore, would be $\frac{17}{64}$ in. in diameter.

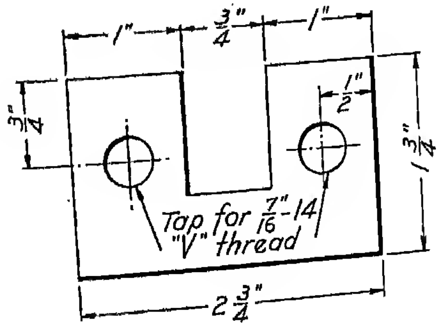
To aid in checking tap-drill sizes in the problems which follow, there are listed at the back of this book tables of commercial tap-drill sizes for NC and NF threads, and for the "V" type threads.

Problems Involving Tap-Drill Calculations

1. A special nut is to be made to fit a shafting 2 in. in diameter having $4\frac{1}{2}$ NC threads per inch. What is the calculated size of the tap drill that should be used?

2. Determine the drill needed in tapping a hole for a $\frac{1}{2}$-in. —13 NC stud.

3. The following drawing is submitted to a machine apprentice with the instructions to "go ahead and tap the holes indicated." What size tap drill should he select for this work?



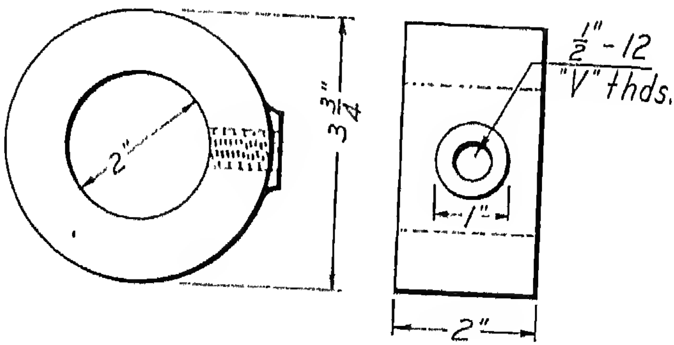
4. A clamp is to be refitted with a $1\frac{1}{4}$-in.—NC 7 screw. What is the calculated size of the tap drill needed? What standard size drill should be used?

5. What is the calculated size of the tap drill that should be used in tapping threads in a motor block for $\frac{3}{8}$-in.—NC 24 studs? What commercial size drill should be used?

6. A special bronze nut is to be made for a $2\frac{1}{4}$-in.—NF $4\frac{1}{2}$ screw. What is the calculated size of the tap drill needed? What is the equivalent size commercial drill?

7. Calculate the size of tap drill needed to make a nut for a $1\frac{3}{16}$-in. "V" threaded bolt, 7 threads to the inch. What is the standard size drill that is used?

8. Two cast-iron shaft collars like that in the following drawing were made in the machine shop for use on a countershaft. In tapping the holes for the setscrews what size drill was used?



9. In making a special brass nut for a "V" type thread as cut on a $1\frac{3}{8}$-in. shafting, what is the calculated size tap drill that would be used if there are 6 threads to the inch? What would be the standard size?

10. Is the tap drill that is needed for an $\frac{11}{16}$-in.—11 NC screw thread larger or smaller than that required for the same size "V" shaped thread, having the same number of threads to the inch. If there is a difference, what is it?

11. Calculate the diameter of the tap drill used in tapping a hole for a 1-in.—8 NC screw. What is the corresponding commercial size?

12. Cap screws $\frac{5}{16}$-in.—18 NC, are to be used in attaching a bracket to the pedestal of a grinder. What is the calculated size of the tap drill used for these holes? What drill is recommended?

13. What is the calculated size of the tap drill to use for a $\frac{7}{8}$-in.—9 NC cap screw?

14. Holes are to be tapped in a casting for four $\frac{9}{16}$-in.—12 NC machine screws. What is the calculated size of the tap drill? What size is recommended for the job?

15. A tapped hole for a $\frac{13}{16}$-in. "V" threaded screw with 10 threads per inch is drilled into a brass casting. What is the calculated size of the drill to be used? What is the equivalent standard size recommended?

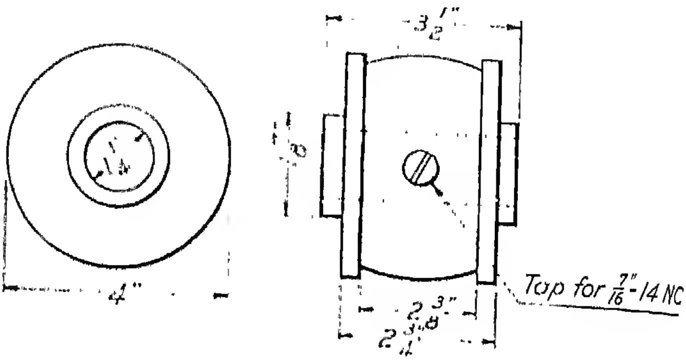
16. Compare the tap drill for a $\frac{5}{8}$-in.—18 NF screw with the tap drill used for a $\frac{5}{8}$-in.—11 NC screw.

17. A $\frac{7}{16}$-in.—20 NF screw is to be used in attaching a support to an automobile transmission housing. Calculate the size of the tap drill that should be used for this purpose. What is the nearest commercial size in 64ths of an inch?

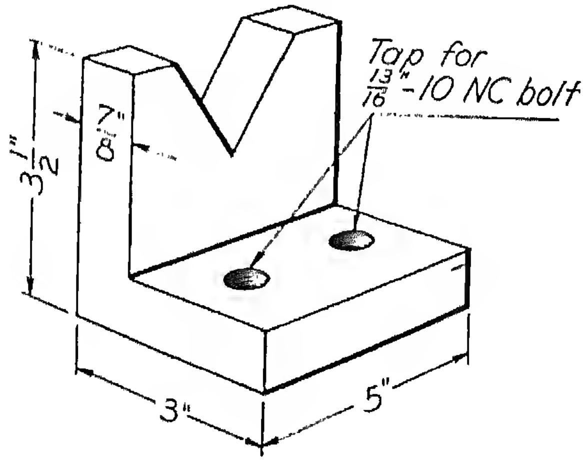
18. Compare the calculated tap-drill sizes, also the commercial sizes to the nearest 64th of an inch as used for a $\frac{9}{16}$-in. —18 NF setscrew, a $\frac{9}{16}$-in. 12 "V" type setscrew, and a $\frac{9}{16}$-in. —12 NC setscrew.

19. In order to keep the small cast-iron pulley that is illustrated on page 360 from slipping on its shaft, a slotted setscrew

is used as shown. Calculate the size of tap drill that is needed for drilling this setscrew hole. What is the standard size drill recommended?



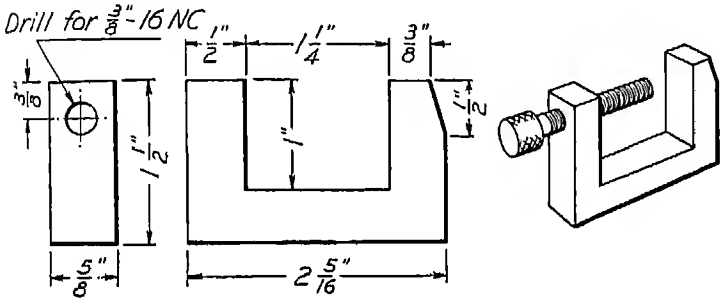
20. The following sketch is given a machinist with the request that he tap the holes referred to. Not having a tap-drill chart handy he was confused as to the size of drill to use. What would you figure it to be?



21. Calculate the size of tap drill to use for a $1\frac{1}{4}$-in.—7 NC "hex" head bolt.

22. A $\frac{3}{8}$-in.—24 NF screw is to be used in attaching a bracket to an automobile frame. What size tap drill in 64ths of an inch should be used for the hole into which the screw is to fit?

23. In the following shop drawing what size tap drill should be used in drilling the hole for the screw indicated?



24. The stud holes in an automobile engine block are to be retapped for a $\frac{1}{2}$-in.—20 NF stud. What size tap drill, to the nearest 64th of an inch, should be used for this purpose?

25. What size tap drill should be used in making nuts for a bolt $1\frac{1}{4}$ in. in diameter having 7 "V" threads to the inch?

26. A hole is to be tapped for a $\frac{5}{8}$-in. stud having 11 "V" threads per inch. What size tap drill is needed for this job?

27. A work order calls for the installing of a $\frac{3}{4}$-in.—16 NF plug in an engine base. What size tap drill to the nearest 64th of an inch, should be used for this?

28. Determine the size of the tap drill to the nearest 64th of an inch, that is to be used in making a 1-in.—14 NF nut.*

*Answers to these problems will be found on page 367.

AMERICAN STANDARD BOLTS AND NUTS

The American Standards Association has established formulas for calculating the dimensions of two series of standard bolts and nuts — the regular and the heavy. These formulas are subject to slight modifications in order to make them apply to *unfinished*, *semifinished*, and *finished* bolts and nuts in both the regular and the heavy class.

For the purposes of this book, the following formulas will suffice. All of them are based on the diameter of the bolt, and it is referred to in the formulas by the letter D.

Distance across the flats or parallel sides of both hexagon and square nuts and bolt heads $= W = 1\frac{1}{2} \times D$, reduced to sixteenths of an inch.

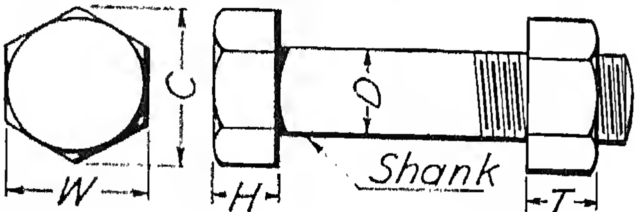
The thickness or height of both hexagon and square bolt heads $= H = \frac{2}{3} \times D$, reduced to the nearest sixty-fourth of an inch.

The thickness or height of both hexagon and square nuts $= T = \frac{7}{8} \times D$, reduced to the nearest sixty-fourth.

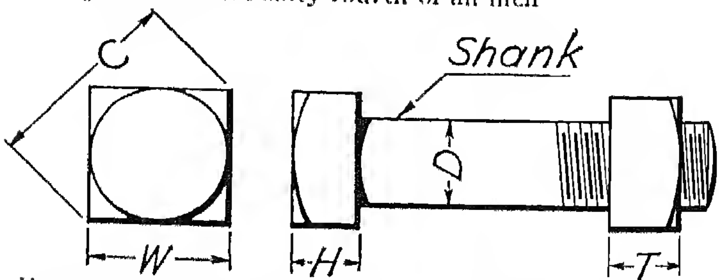
The distance across corners for hexagon nut and bolt heads $= C = 1.155 \times W$.

The distance across corners for square nut and bolt heads $= C = 1.414 \times W$.

The proportion of both the "hex" and square bolts and nuts are shown in the following illustrations.



$W = 1\frac{1}{2}\ D$ to sixteenths of an inch. $C = 1.155\ W$
$H = \frac{2}{3}\ D$ to sixty-fourths of an inch
$T = \frac{7}{8}\ D$ to nearest sixty-fourth of an inch



Formulas for W, H, T same as for hexagon head bolt and nut except that: $C = 1.414 \times W$.

The proportions of both the "hex" nut and the square nut are illustrated in the drawing on page 362.

The following show how these formulas are applied.

Example 1:

What size hexagon stock is needed to turn up a "hex" bolt with a shank that measures $\frac{3}{4}$ in. in diameter?

Solution and Explanation:

Distance across the flats, or across the parallel sides of a "hex" head bolt $= W = 1\frac{1}{2}\,D$.

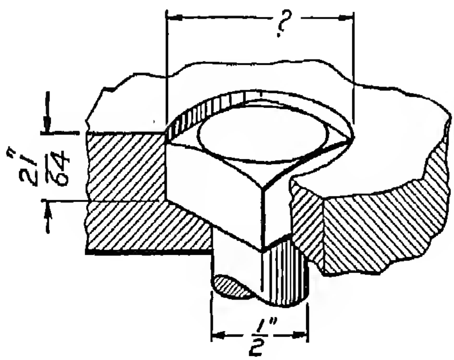
Where "D" is $\frac{3}{4}$ in., this becomes $1\frac{1}{2} \times \frac{3}{4} = 1\frac{1}{8}$ in.

That is, the hexagon stock needed in making a $\frac{3}{4}$-in. hexagon head bolt measures $1\frac{1}{8}$ in. across the flats.

Example 2:

In the following drawing determine the diameter of the hole into which is sunk the head of the $\frac{1}{2}$-in. square head bolt as illustrated.

How deep should this hole be in order that the top of the head of the bolt lie flush with the outside surface as shown?



Solution and Explanation:

The diameter of this hole should be equal to the distance across the corners of the square head. This is determined by first finding the distance across the parallel sides of the head.

Distance across the flats, or across the parallel sides of a square head bolt $= W = 1\frac{1}{2}\ D$.

Where "D" is $\frac{1}{2}$ in., this becomes $1\frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$ in.

Distance across the corners = Distance across flat $\times$ 1.414.

This works out as $\frac{3}{4} \times 1.414 = 1.0605$ in.

That is, the diameter of the hole that will just contain the $\frac{1}{2}$-in. square head bolt in the foregoing problem is 1.0605 in.

To obtain the depth of the hole required so that the top of the head will be just flush with the surface, use the formula:

Thickness of head $= H = \frac{2}{3} \times D = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ in. Changing this to the nearest sixty-fourth makes this depth $\frac{21}{64}$ in.

If this hole is drilled $\frac{21}{64}$ in. deep, the top of the head will be just flush with the surface.

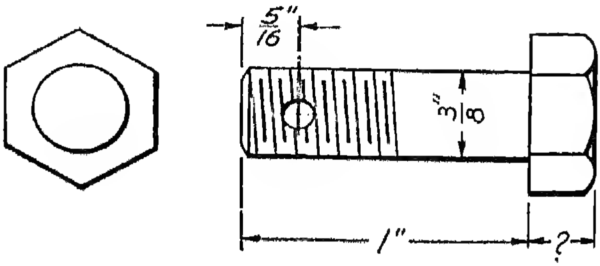
Problems Involving Applications of Formulas on "Hex" Head Bolts and Square Head Bolts

1. Calculate the distance across the parallel sides of a hexagonal head bolt that is $1\frac{3}{4}$ in. in diameter.

2. What size stock is needed to turn up square head bolts the shank of which measures $\frac{7}{8}$ in. in diameter?

3. What must be the diameter of a hole to be drilled in a casting that will just hold the head of a $\frac{1}{2}$-in. hexagon bolt?

4. Determine the size stock required for the following bolt. How thick should the head be?



5. It is desired to sink a $\frac{9}{16}$-in. hexagon head bolt below the surface of the casting into which it is about to be placed. How deep should this hole be drilled in order to have the top of the head come just even with the surface of the casting?

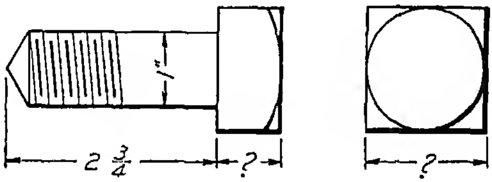
6. What should be the diameter of the drill that is to be used in drilling the hole for the head in the above case?

7. What size stock is needed in turning up a $\frac{3}{8}$-in. hexagon head bolt?

8. Determine the difference in the measurements across the corners of stock used in making $\frac{1}{4}$-in. hexagon head bolts and $\frac{1}{4}$-in. square head bolts.

9. A shop order calls for one gross each of "hex" head bolts of the following diameters: 1 in.; $\frac{7}{8}$ in.; $1\frac{1}{4}$ in., and $\frac{5}{16}$ in. What size hexagon stock is required in each case?

10. What size square stock is needed to turn up the bolt in the following drawing? What are the missing dimensions indicated by the question mark?

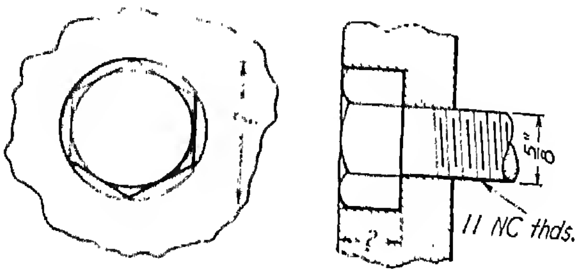


11. Calculate the approximate distance across the corners of square head bolts that measure $\frac{3}{4}$ in.; $\frac{5}{8}$ in.; $\frac{7}{8}$ in., and $1\frac{1}{8}$ in. in diameter.

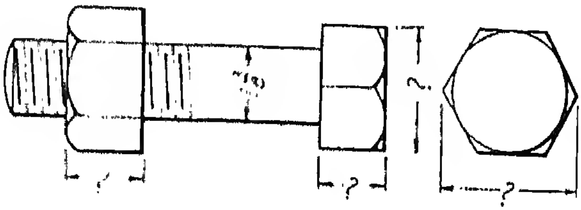
12. What is the size of the square stock needed to make a $1\frac{5}{8}$-in. square head bolt? What is the distance across the corners of the head of the bolt?

13. How large in diameter and how deep must be the hole that will just contain the head of a $\frac{5}{16}$-in. square head bolt?

14. Determine the missing dimensions that are questioned in the following drawing. What is the size of the tap drill that is used in drilling the holes for the threaded portion of the screw?



15. What are the missing dimensions as questioned on the following drawing?



16. Calculate the distance across the parallel sides of a square head bolt ¾ in. in diameter. What is the head thickness?*

ANSWERS TO PROBLEMS

Pages 350 to 353.

1. .0714 in.
2. .0625 in.; .0417 in.
3. .0541 in.
4. .1667 in.; .0833 in.
5. .1634 in.
6. Yes; .0541 in.; .069 sq. in.
7. .0866 in.
8. .5768 in.
9. .20 in.
10. .20 in.
11. 16; 20; 8.
12. .0541 in.; .0433 in.; .1082 in.
13. .111 in.; .0556 in.; .764 in.
14. .1624 in.; 2.175 in.
15. .0909 in.; .059 in.
16. .0114 in.

*Answers to these problems will be found on page 367.

17. .0417 in.; .2584 in.
18. .3849 in.; 1.6151 in.
19. .0678 sq. in.
20. .0179 in.
21. .0417 in.; .3209 in.; .0809 sq. in.
22. .1667 in.; .2165 in.
23. .0714 in.; .3138 in.; .0556 in.; .2162 in.
24. .0125 in.; .0156 in.; .065 in.; .0812 in.
25. .05 in.; .0649 in.
26. .0541 in.
27. .0869 in.
28. .40 in.; .20 in.
29. .0541 in.
30. .20 in.; .10 in.; .80 in.

Pages 357 to 361.

1. 1.785 in.
2. $\frac{27}{64}$ in.
3. $\frac{11}{32}$ in.
4. 1.111.4 in.; $1\frac{7}{64}$ in.
5. .335 in.; $\frac{21}{64}$ in.
6. 2.0344 in.; $2\frac{1}{32}$ in.
7. 1.002 in.; 1 in.
8. $\frac{25}{64}$ in.
9. 1.158 in.; $1\frac{5}{32}$ in.
10. Larger; $\frac{1}{32}$ in.
11. .879 in.; $\frac{7}{8}$ in.
12. .259 in.; $\frac{1}{4}$ in.
13. .767 in.
14. .482 in.; $\frac{15}{32}$ in.
15. .6825 in.; $\frac{11}{16}$ in.
16. $\frac{37}{64}$ in.; $\frac{17}{32}$ in.
17. .389 in.; $\frac{25}{64}$ in.
18. .5086 in.; $\frac{33}{64}$ in.; .4542 in.; $\frac{29}{64}$ in.; .4817 in.; $\frac{31}{64}$ in.
19. .3682 in.; $\frac{3}{8}$ in.
20. $\frac{23}{32}$ in.
21. $1\frac{7}{64}$ in.
22. $\frac{21}{64}$ in.
23. $\frac{5}{16}$ in.
24. $\frac{29}{64}$ in.
25. $1\frac{1}{16}$ in.
26. $\frac{1}{2}$ in.
27. $\frac{11}{16}$ in.
28. $\frac{15}{16}$ in.

Pages 364 to 366.

1. $2\frac{5}{8}$ in.
2. $\frac{9}{16}$ in.
3. 0.8625
4. $\frac{9}{16}$ in.; $\frac{1}{4}$ in.
5. $\frac{3}{8}$ in.
6. $\frac{61}{64}$ in. approx.
7. $\frac{9}{16}$ in.
8. 0.097 in.
9. $1\frac{1}{2}$ in.; $1\frac{5}{16}$ in.; $1\frac{7}{8}$ in.; $\frac{1}{2}$ in.
10. $1\frac{1}{2}$ in.; $\frac{43}{64}$ in.
11. $1\frac{19}{32}$ in.; $1\frac{21}{64}$ in.; $1\frac{55}{64}$ in.; $2\frac{25}{64}$ in.
12. $2\frac{7}{16}$ in.; $3\frac{29}{64}$ in.
13. $\frac{23}{32}$ in.; $\frac{7}{32}$ in.
14. $1\frac{3}{32}$ in.; $\frac{27}{64}$ in.; $\frac{17}{32}$ in.
15. $\frac{59}{64}$ in.; $1\frac{13}{64}$ in.; $2\frac{1}{16}$ in.; $2\frac{25}{64}$ in.
16. $1\frac{1}{8}$ in.; $\frac{1}{2}$ in.

Review Problems on Screws, Bolts, and Tap-Drill Sizes

1. Determine the proportions of square nuts for the following size bolts: $\frac{9}{16}$ in.; $1\frac{1}{4}$ in.; $1\frac{1}{2}$ in.

2. What is the pitch, also the depth of thread, for bolts having $4\frac{1}{2}$ and $5\frac{1}{2}$ U.S. Std. threads per inch?

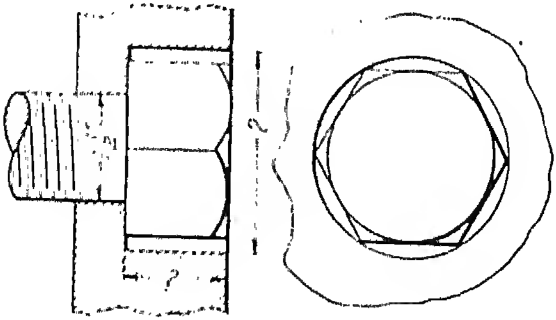
3. What are the tap-drill sizes for the following "V" thread screws: $\frac{7}{8}$ in.—9; $1\frac{3}{4}$ in.—5; $\frac{11}{16}$ in.—9; $\frac{9}{16}$ in.—12; and 1 in.—8.

4. A shop drawing calls for a 1-in. square threaded screw to replace a broken one in a bench vise. The screw is to have 4 threads to the inch. What is the depth of this thread, also the root diameter?

5. What size hexagon stock is needed to make a $1\frac{5}{8}$-in.—$5\frac{1}{2}$ NC bolt? What is the root diameter of such a bolt? What size tap drill would be used in drilling a hole to be tapped for this size bolt?

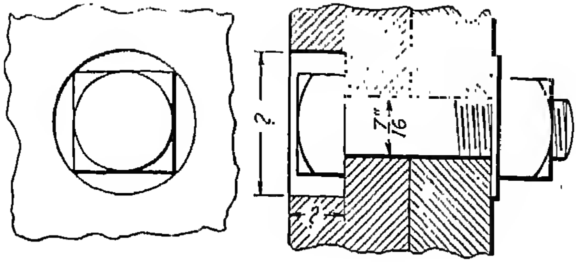
6. What is the difference in the root diameters of a $\frac{3}{4}$ in.—10 "V" thread and a $\frac{3}{4}$-in.—10 NC thread? What size tap drill would be used for each?

7. What should be the diameter of the hole in the following sketch to take the head of the hexagon-head bolt shown? How deep should this hole be drilled in order to have the top of the head just flush with the top of the hole?

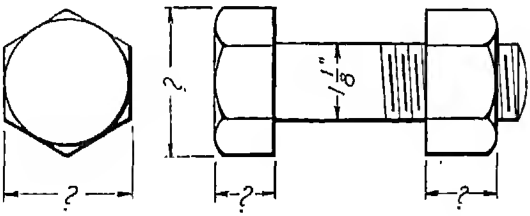


8. Determine the minor diameter of a $1\frac{1}{8}$-in. bolt having 7 "V" threads per inch. What size tap drill would be used in drilling a hole for such a bolt?

9. What is the diameter of the hole to be drilled to just take the head of the bolt shown in the following drawing? If the head of this bolt lies $\frac{1}{8}$ in. below the level of the top of the hole how deep should this hole be drilled?



10. Determine the proportions of the hexagon-head bolt as indicated by the question marks on the following sketch.



11. Determine the distance across the parallel sides, and across the corners of a nut for a $1\frac{1}{8}$-in. square-head bolt.

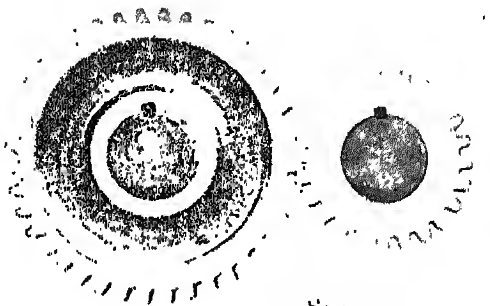
12. Determine the distance across the parallel sides, and across the corners of a nut for a $\frac{7}{8}$-in. hexagon-head bolt.

13. Find the double depth of a "V" thread on a bolt having 6 threads per inch.

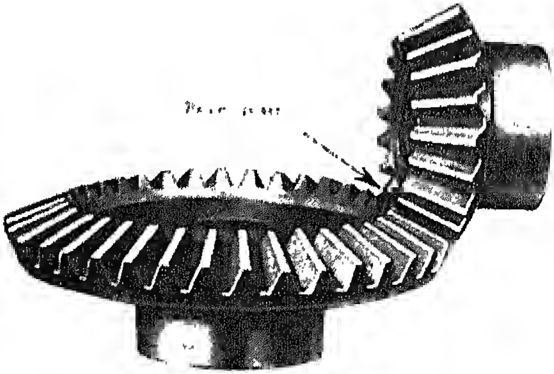
GEAR DRIVE

Where the driving distance, or the distance between centers is very short, or conditions do not warrant the use of belt drive, the power, or motion, may be transmitted by gears.

Among the more common types of gears, is that known as the *spur gear*. The gears in the following illustration are of this kind.



Spur gears



As illustrated, the teeth project out radially at the rim of the gear, and *mesh* with the teeth of the other gear. This gives a very positive drive, with no slippage. When two gears are

arranged in this manner they are frequently referred to as "a pair of gears."

Another kind of gear is the *bevel gear* as illustrated at the bottom of page 370. This style of gear is used where the *drive* shaft is at an angle with the *driven* shaft.

In these different arrangements of gears, or gearing, as it is sometimes called, the gear that does the driving is known as the *driving gear*, or *driver*. The gear that is driven is referred to as the *driven gear*. This is much the same as where pulleys are driven by belts, as explained on page 322.

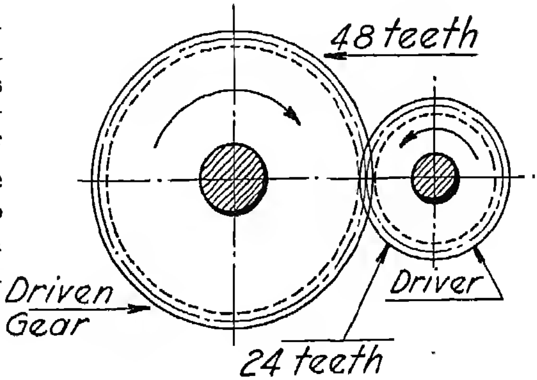
GEAR-DRIVE CALCULATIONS

The calculations relating to gear drives are similar to those relating to pulley drives. In the case of the pulleys, it was found that the diameter of the driving pulley multiplied by its R.P.M. was equal to the diameter of the driven pulley multiplied by its r.p.m.

This same rule holds true with gearing, *except* that in place of the diameters, the number of teeth in each gear is considered. While the revolutions per minute might be used in gear-drive calculations, it is sufficient to refer to the *number of revolutions* for any given length of time, or to merely the *number of turns*. This is illustrated in the following problem.

Example 1:

If a 24-tooth gear drives a 48-tooth gear as shown, how many turns of the driver are necessary to make one turn of the driven gear?



Solution and Explanation:

Sketches of gears are sometimes made like those illustrated on page 371, showing the teeth of the gears *only* at the point where they are in contact with each other. This practice, however is *not* usually followed in making shop drawings. Instead, only broken circles are drawn representing the pitch circle, and the circles at the outside of the teeth and the base of the teeth, and *no* tooth outline is shown. This practice will be followed in the remaining drawings, except where special illustration is necessary.

Referring to the previous drawing, as the 24-tooth gear makes one full turn in the direction given, it will engage 24 teeth on the driven gear turning it as indicated. Since this is only one half the number of teeth on the driven gear, then this driven gear will turn but half way around. In order that the driven gear may make one complete turn, therefore, it will be necessary for the *driving* gear to make *two* full turns.

These facts as explained above result in the following rule: *The revolutions of the driving gear multiplied by its number of teeth, are equal to the revolutions of the driven gear multiplied by its number of teeth.* Or more briefly, this may be expressed as:

Teeth on the driving gear × revolutions of driving gear = teeth on the driven gear × revolutions of driven gear.

In the above problem this would work out as follows:

$$24 \times 2 = 48 \times 1$$

Representing the number of teeth on the driving gear as T, the revolutions of the driving gear as R, the number of teeth on the driven gear as t, and the number of revolutions of the driven gear as r, this rule may be expressed by the following formula:

$$T \times R = t \times r$$

From this equality any one of the quantities T, R, t or r, may be determined when the three remaining quantities are given. This enables one to work out the following rules which,

from observation, may be seen to resemble similar rules in pulley-drive calculations.

Rule 1. The number of teeth on the driving gear may be determined by multiplying the number of teeth on the driven gear by the number of revolutions it makes, and then dividing this product by the revolutions of the driving gear.

Expressed as a shop formula this becomes:

$$T = \frac{t \times r}{R} = \text{Teeth on the driving gear, or driver.}$$

This is applied in such problems as the following.

Example 2:

Determine the number of teeth on a driving gear which revolves 4 times while it turns a 96-tooth gear $1\frac{1}{2}$ times.

Solution:

$$T = \frac{t \times r}{R} = \frac{96 \times 1\frac{1}{2}}{4} = \frac{96 \times 3}{4 \times 2} = 36$$

That is, there are 36 teeth on the driving gear.

Rule 2. The revolutions made by the driving gear may be found by multiplying the number of teeth on the driven gear by the revolutions of the driven gear, and then dividing that product by the number of teeth on the driving gear.

Expressed as a shop formula, this becomes:

$$R = \frac{t \times r}{T} = \text{Revolutions of driving gear, or driver.}$$

This is used in solving such problems as the following.

Example 3:

Calculate the number of revolutions made by a driving gear with 60 teeth, while it turns a 24-tooth gear 5 revolutions.

Solution:

$$R = \frac{t \times r}{T} = \frac{\overset{2}{\cancel{24}} \times \cancel{5}}{\underset{\cancel{12}}{\cancel{60}}} = 2$$

That is, the driving gear makes 2 turns, while the driven gear makes 5 turns.

Rule 3. The number of teeth on the driven gear may be determined by multiplying the number of teeth on the driving gear by the revolutions of the driving gear, and then dividing that product by the revolutions of the driven gear.

Expressed as a shop formula this becomes:

$$t = \frac{T \times R}{r} = \text{teeth on the driven gear.}$$

This is applied in such cases as the following.

Example 4:

How many teeth should there be in a gear that makes $3\frac{1}{2}$ revolutions while the driving gear makes 2 revolutions? The driving gear has 70 teeth.

Solution:

$$t = \frac{T \times R}{r} = \frac{70 \times 2}{3\frac{1}{2}} = \frac{\overset{10}{\cancel{70}} \times 2 \times 2}{\cancel{7}} = 40$$

That is, the driven gear should have 40 teeth.

Rule 4. The revolutions of the driven gear may be calculated by multiplying the number of teeth on the driving gear by its revolutions and then dividing that product by the number of teeth on the driven gear.

Expressed as a shop formula, this becomes:

$$r = \frac{T \times R}{t} = \text{revolutions of the driven gear.}$$

This is applied in such examples as the following.

Example 5:

How many revolutions will a driven gear of 40 teeth make while the driving gear with 112 teeth makes 5 revolutions?

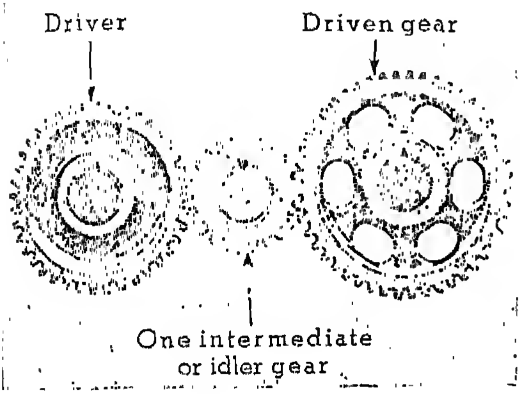
Solution:

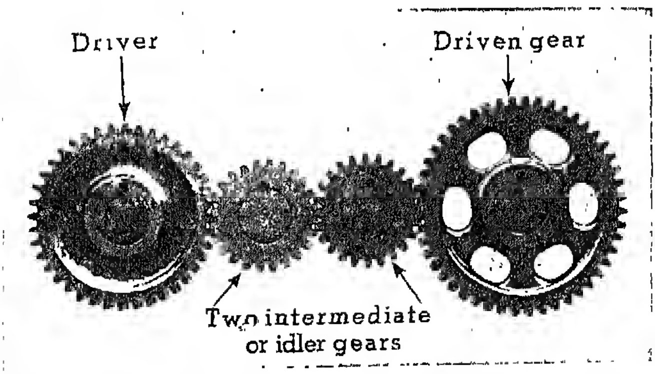
$$r = \frac{T \times R}{t} = \frac{\overset{14}{\cancel{112}} \times \cancel{5}}{\underset{\cancel{8}}{\cancel{40}}} = 14$$

That is, the driven gear makes 14 revolutions, the driving gear 5.

SIMPLE GEAR TRAINS

Where the driving takes place through one or more intermediate gears, or through idler gears, like that illustrated below, the arrangement is called a *simple* gear train. This type of drive is known as simple gearing.





From the above sketches it is seen that:

1. Where *one* intermediate gear, or one idler gear, is used, the driven gear revolves in the *same* direction as the driver.

2. Where two intermediate gears, or two idler gears, are used the driven gear revolves in the *opposite* direction to the driver.

3. Where the gear train is larger, and there is an odd number of intermediate gears, such as 3 or 5, the *direction* of rotation of the driven gear will be found to be the *same* as that of the driver.

4. Where an even number of intermediate gears, as 2, 4, or 6, is used however, the *direction* of rotation of the final gear will be found to be *opposite* to that of the driver.

Idler gears are used for either of two reasons. When the driver and driven gears intermesh, they rotate in opposite directions. When both are to rotate in the same direction, an idler placed between them, will accomplish the purpose.

At times the centers of the shafts on which the driver and driven gears are to be mounted, are so far apart that unusually large gears would be required to connect them. To overcome this, one or more idlers may be used between the gears on the shafts, thus solving the problem.

GEAR-TRAIN CALCULATIONS

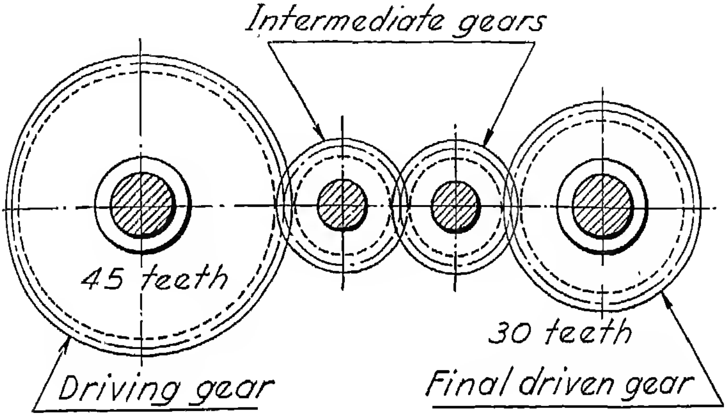
In *simple* gear-train calculations involving only the *first driving gear* and the *final driven gear* neither the intermediate gears nor the idler gears enter into the calculations, unless, of course, it is desired to know their direction or number of revolutions. Instead, only the *final driven* gear and the *first driving* gear are considered, much the same as though only *two* gears were concerned.

In such cases in order to determine the number of revolutions that the *final* gear makes for *one* of the *driving* gear, the number of teeth on the driving gear is divided by the number of teeth on the final driven gear. In other words, the formulas as expressed on pages 373 and 374 are to be used for *simple gear-train calculations*. The method of finding the direction of rotation of such gears has already been explained.

The following problem is typical of those involving simple gear trains and illustrates the application of this rule.

Example:

Determine the number of revolutions, also the direction of rotation, of the final driven gear for one revolution of the driving gear in the following simple gear train.



Solution and Explanation:

The driving gear has 45 teeth and moves clockwise as shown.

The driven gear has 30 teeth.

As explained above, the formula to be used in the solution of this problem would be:

$$r = \frac{T \times R}{t}$$

Where,

T = Number of teeth on driving gear
t = Number of teeth on final driven gear
R = Number of revolutions of driving gear
r = Number of revolutions of final driven gear

In this problem $T = 45$; $t = 30$; $R = 1$.

As a result, the revolutions of the final driven gear become:

$$r = \frac{45 \times 1}{30}, \text{ or } 1\tfrac{1}{2}$$

That is, the final driven gear makes $1\frac{1}{2}$ revolutions while the driving gear, or the driver, makes one revolution.

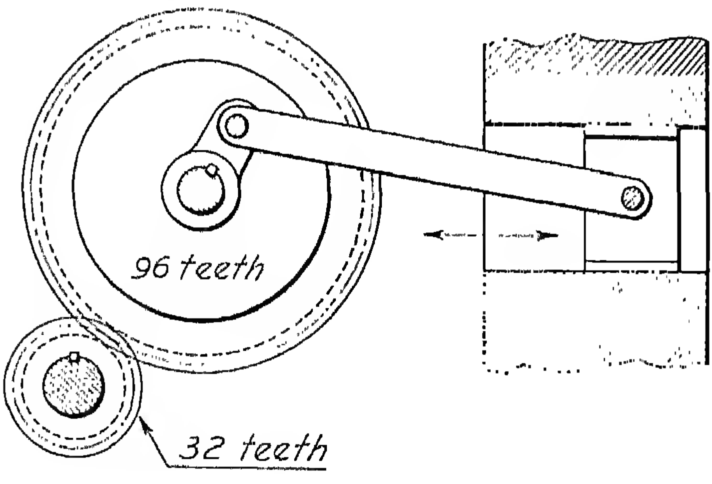
There being an *even* number of intermediate gears in this train, the direction of rotation of the final driven gear will be *opposite* that of the driving gear, or *counterclockwise*.

Problems Relating to Calculations in Simple Gearing

1. How many teeth are necessary on a gear which must run at 126 R.P.M. when driven by an 84-tooth gear running at 30 R.P.M.?

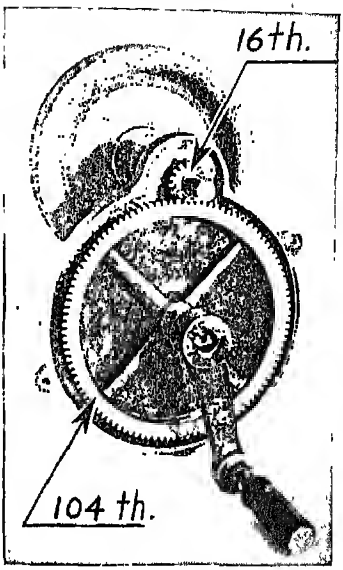
2. How many turns will a 64-tooth spur gear make for 8 turns of a 48-tooth gear that meshes with it? Prove this.

3. The following setup shows the gear drive on a special cutting machine. The small gear makes 120 R.P.M. Each time the larger gear makes one revolution one complete cutting stroke is made. At this rate how many cutting strokes are made per minute?

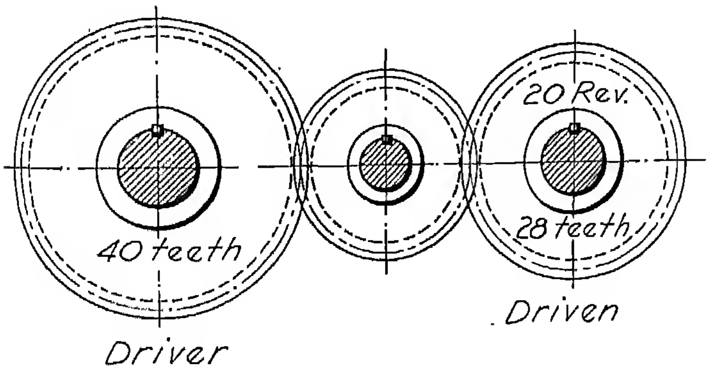


4. In two minutes how many revolutions will a 32-tooth gear make that drives a 104-tooth gear 14 revolutions in 30 seconds?

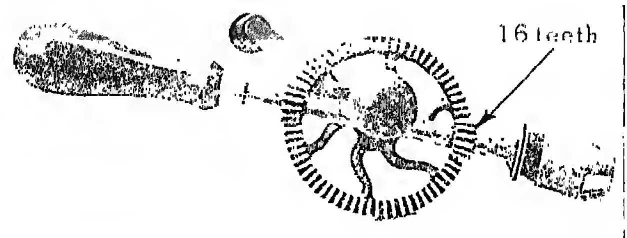
5. On the small hand-operated bench grinder illustrated to the right, the small gear is fixed to the shaft on which the grinding wheel revolves. If the large gear, to which the handle is fixed, is turned at 10 R.P.M. how many revolutions will the grinding wheel make in this time?



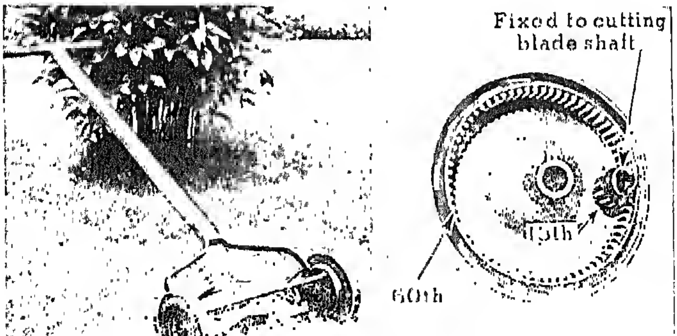
6. Calculate the direction and the revolutions of the driving gear for 20 turns of the driven gear in the following gear train.



7. The gearing on a standard carpenter's hand drill is arranged as in the following sketch. If the spindle gear turns $4\frac{1}{2}$ times for one turn of the handle, how many teeth are there on the larger gear to which the handle is attached?

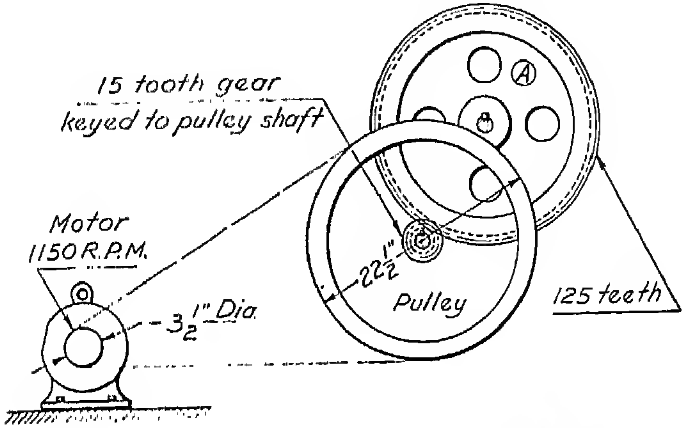


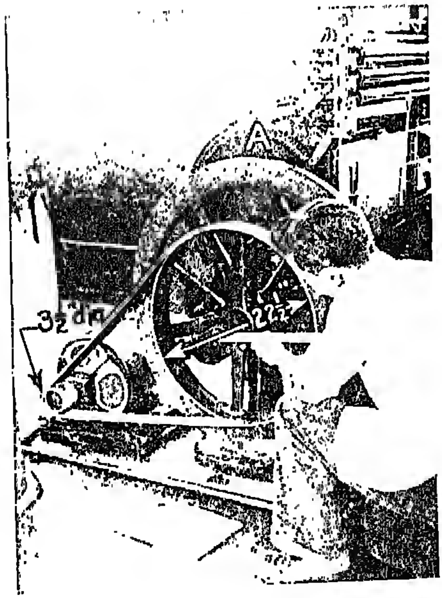
8. The ordinary home lawn mower is geared up as follows. How many revolutions will the cutting blades make for one revolution of the larger gear? It is to be noted in this that the smaller gear is a spur gear in which the teeth are on the *outer* rim of the gear and *extend out* from the center. On the larger gear, however, the teeth are on the *inside* of the gear and *extend inward*. This latter type of a gear is known as an *internal* gear.



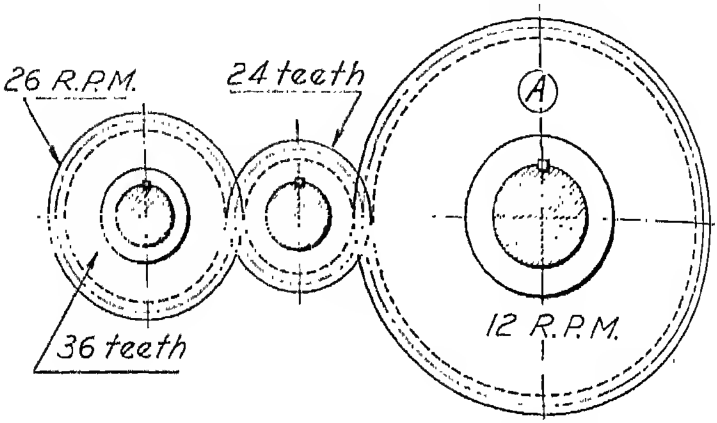
Arrangement of gearing on a lawn mower

9. The following is an illustration of a combination belt and gear drive on a small printing press. Each time gear (A) revolves one printing impression is made. How many impressions are made per minute on this press under these conditions?



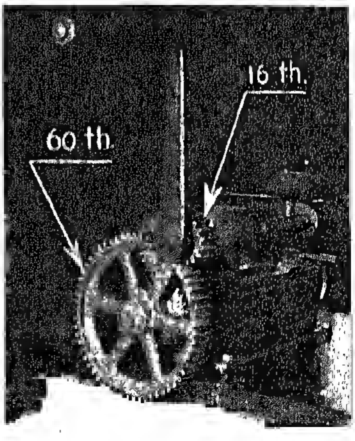


10. What should be the number of teeth on gear (*A*) in the following gear train in order that it make 12 R.P.M.?



11. In the *differential* of a well-known automobile the *drive-shaft pinion gear* has 11 teeth. This engages with the *rear-axle drive gear* which in turn has 46 teeth. For each turn of this axle drive gear the rear wheels of the automobile make one complete revolution. At this rate determine the R.P.M. of the drive shaft when the car is running at 30 miles per hour, assuming that the outside diameter of the tires on the rear wheel remain at 28 inches.

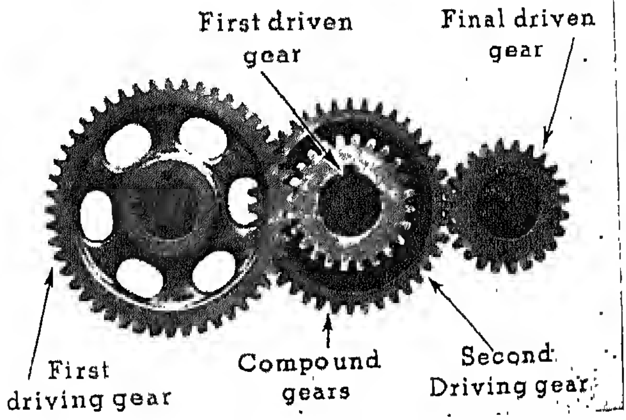
12. The machine in the picture to the right is used for slitting sheet metal. It is operated by a handle which is attached to the driving-gear shaft. The driving gear has 16 teeth and the larger gear has 60 teeth. How many turns of the driving gear are necessary for one turn of the driven gear?*



*Answers to these problems will be found on page 396.

COMPOUND-GEAR TRAINS

When gears are arranged as shown below, that is, where a driving gear is on the same shaft as a driven gear, the combination is referred to as *compound* gearing.



Compound-gear drives of this sort are used where it is desired to increase or to decrease speeds. They are found in the transmission gearing of an automobile; in gearing on engine lathes; also in the ordinary 8-day clock, and in many other mechanical drives.

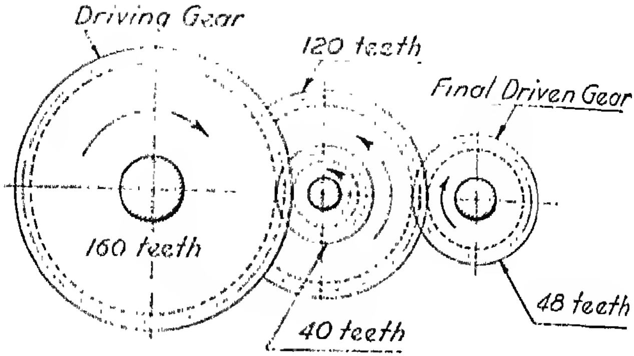
COMPOUND-GEAR-TRAIN CALCULATIONS

In *compound*-gear-train calculations *all* gears must be considered. How such problems are solved may be readily understood if each step is worked out as in simple gearing.

This is explained in the solution of the following problem.

Example 1:

In the compound-gear train illustrated on page 384, determine the number of revolutions made by the final driven gear for one revolution of the first driving gear.



Solution and Explanation:

As in the previous problems on simple gearing, the 40-tooth gear is the *first* driven gear. Its revolutions are calculated as follows.

$$r = \frac{T \times R}{t}, \text{ or } \frac{\overset{4}{\cancel{160}} \times 1}{\cancel{40}} = 4$$

The 120-tooth gear being fixed to the same shaft as the 40-tooth gear likewise turns with, and also makes 4 revolutions. This 120-tooth gear meshes with the 48-tooth gear and drives it.

The revolutions which this 48-tooth gear makes while the 120-tooth gear makes 4 turns is also found in a similar manner, using the above formula.

$$r = \frac{T \times R}{t}, \text{ or } \frac{\overset{10}{\cancel{120}} \times \cancel{4}}{\underset{12}{\cancel{48}}} = 10$$

That is, the 48-tooth gear, or the *final* gear, makes 10 turns while the 120-tooth gear makes 4 turns. During this time, however, as above explained, the first driving gear, or the 160-tooth gear, has made but *one* revolution. The *final* gear therefore, makes 10 revolutions while the *first* driving gear makes *1* revolution.

This process may be very much shortened, however, by multiplying the *product* of the *teeth* on the *driving gears* by the *revolutions* of the *first driver*, and then *divide* that amount by the *product* of the *teeth* on the *driven gears*.

Expressed as a shop formula this becomes:

$$r = \frac{(T \times T_1 \times T_2 \times T_3, \text{etc.}) \times R}{(t_1 \times t_2 \times t_3 \times t_4, \text{etc.})} = \text{Revolutions of final driven gear.}$$

The letters in this formula represent the quantities noted as follows:

T, T_1, T_2, T_3, etc. = number of teeth on the different driving gears

t, t_1, t_2, t_3, etc. = number of teeth on the different driven gears

R = number of revolutions of first driving gear

r = number of revolutions of final driven gear

t_r = symbol to represent the number of teeth on the *final* driven gear

From the above shop formula the number of teeth T, on the first driver in a gear train may also be determined. This works out as:

$$T = \frac{\text{Product of teeth on all driven gears} \times \text{rev. of final driven gear}}{\text{Revolutions of first driver} \times \text{product of teeth on all other drivers}}$$

The formula for the revolutions R, of the first driver would be:

$$R = \frac{\text{Product of teeth on all driven gears} \times \text{rev. of final driven gear}}{\text{Product of teeth on all drivers}}.$$

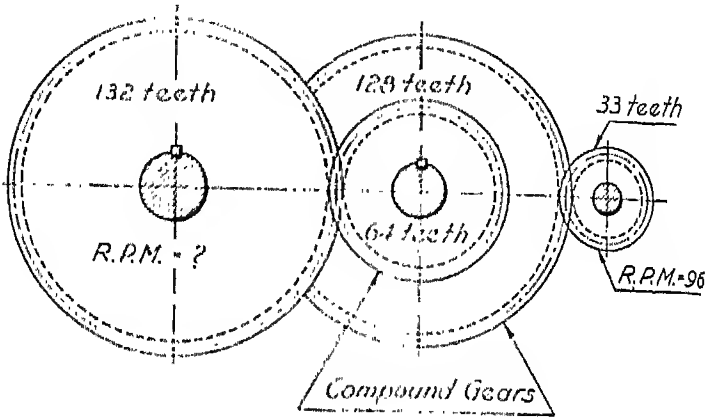
The formula for the number of teeth t_r, on the final driven gear is:

$$t_r = \frac{\text{Product of teeth on all drivers} \times \text{revolutions of first driver}}{\text{Product of teeth on all other driven gears} \times \text{rev. of final driven gear}}$$

These formulas are applied in such problems as the following.

Example 2:

How many revolutions per minute must be made by the driving gear in the following gear train in order to obtain the result noted?



Solution and Explanation:

In this problem $t = 64$; $t_1 = 33$; $r = 96$; $T = 132$; $T_1 = 128$. Using these values in the formula for the revolutions R, of the first driver, the result becomes:

$$R = \frac{t \times t_1 \times r}{T \times T_1} = \frac{64 \times 33 \times 96}{132 \times 128} = 12$$

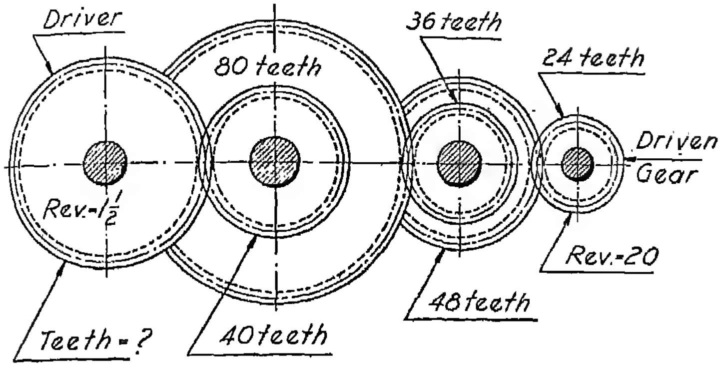
That is, the first driver makes 12 revolutions per minute.

Example 3:

How many teeth on the *first driver* in the following gear train illustrated at the top of page 387?

Solution and Explanation:

In this problem $t = 40$; $t_1 = 36$; $t_2 = 24$; $r = 20$; $T_1 = 80$; $T_2 = 48$; $R = 1\frac{1}{2}$.



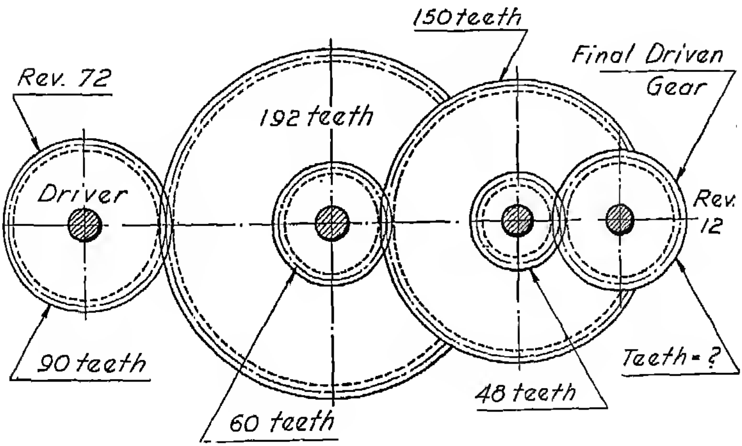
Using these values in the formula for the number of teeth T, in the first driver, there results:

$$T = \frac{t \times t_1 \times t_2 \times r}{T_1 \times T_2 \times R} = \frac{40 \times 36 \times 24 \times 20}{80 \times 48 \times 1\frac{1}{2}} = 120$$

That is, there are 120 teeth in the first driver.

Example 4:

How many teeth on the last driven gear in the following arrangement of gears?



Solution and Explanation:

In this problem $t = 192$; $t_1 = 150$; $r = 12$; $T = 90$; $T_1 = 60$; $T_2 = 48$; $R = 72$.

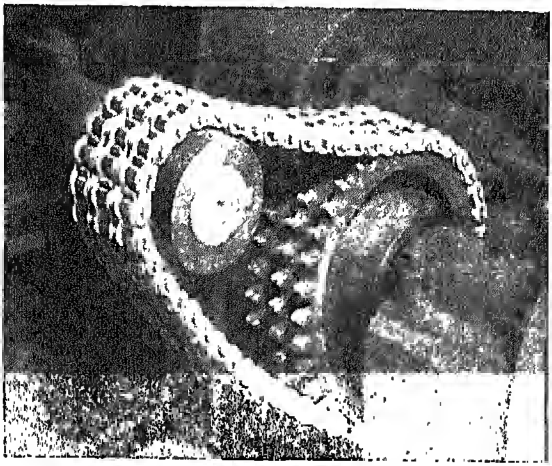
The figures used in the formula for determining the number of teeth t_f, in the final driven gear, give the following result:

$$t_f = \frac{T \times T_1 \times T_2 \times R}{t \times t_1 \times r} = \frac{90 \times 60 \times 48 \times 72}{192 \times 150 \times 12} = 54$$

That is, there are 54 teeth in the final driven gear.

SPROCKET AND CHAIN DRIVE

Sometimes conditions will not permit the use of either gear drive or belt drive. In such cases the power and motion may be transmitted by what is known as a *sprocket and chain drive.*



The teeth of a sprocket, also called a *sprocket gear* or a *sprocket wheel*, project out radially at the rim and are especially designed for the purpose they serve. They are so arranged as to allow a special kind of link belt, or chain, to fit over them as illustrated above, connecting the driving sprocket with the

driven sprocket. A familiar type of sprocket and chain drive is seen on the ordinary bicycle. Another type is found on the timing chains on automobiles.

SPROCKET-AND-CHAIN-DRIVE CALCULATIONS

Calculations relating to sprocket and chain drives are the same as those relating to gears that drive direct. Problems involving such methods of driving are solved in the same manner as those explained on pages 376 and 377 using the same formula: $T \times R = t \times r$.

In these calculations:

T = teeth on driving sprocket
R = revolutions of driving sprocket
t = teeth on driven sprocket
r = revolutions of driven sprocket

The following problem in chain and sprocket drive illustrates how this formula is applied.

Example:

On a bicycle, the large sprocket has 24 teeth while the smaller one has 10 teeth. How many turns will the rear wheel make for 5 turns of the large sprocket?

Solution and Explanation:

In this problem $T = 24$; $R = 5$; $t = 10$.

The formula to use for revolutions of the driven sprocket is similar to that for revolutions of a driven gear, and works out as follows:

$$r = \frac{T \times R}{t}, \text{ or } \frac{24 \times 5}{10} = 12$$

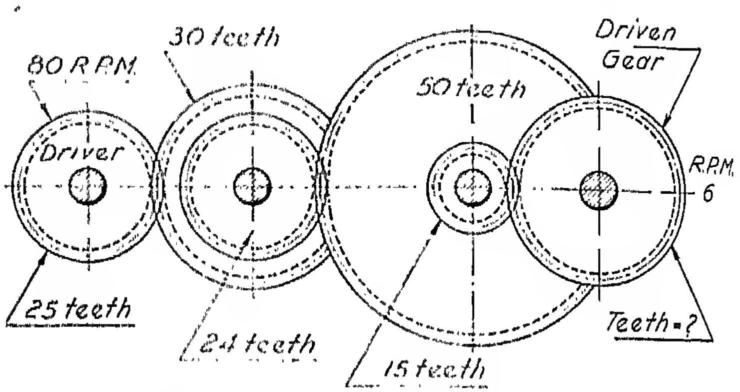
That is, the rear wheel makes 12 turns for 5 turns of the driving sprocket.

Problems in Compounding Gearing and Sprocket Drives

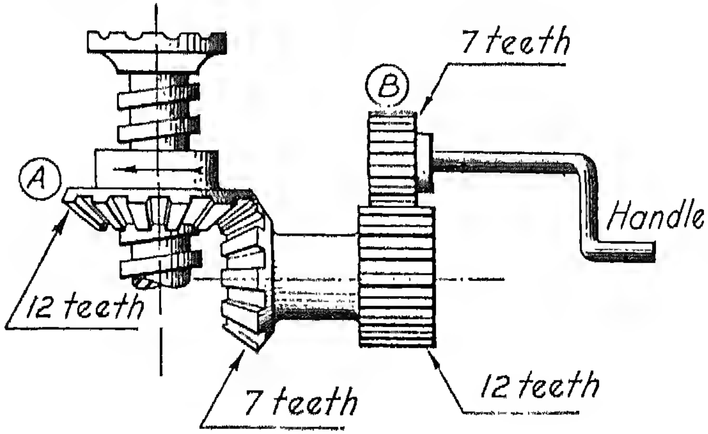
1. A bevel gear having 48 teeth and running at 42 R.P.M. drives a 36-tooth bevel gear. On one end of the shaft on which this 36-tooth gear is located there is also a 75-tooth gear which

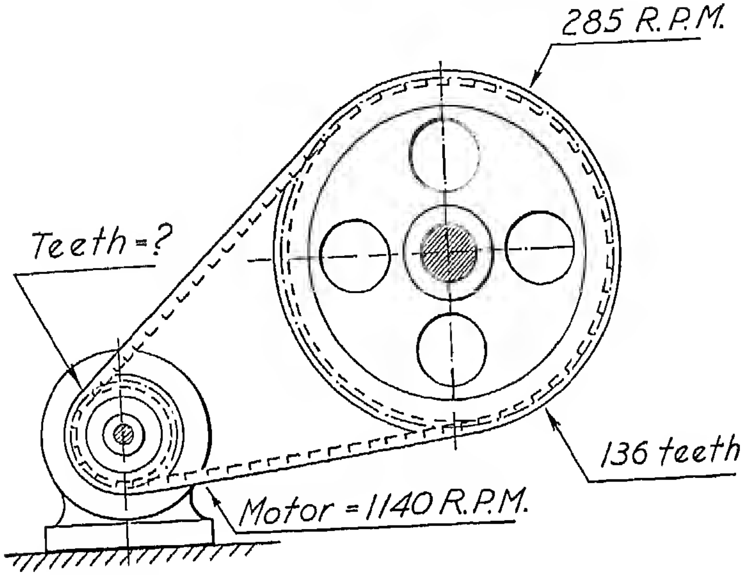
meshes with a 30-tooth pinion. How many revolutions does this pinion gear make per minute? Make a sketch showing such an arrangement of gears.

2. The driver in the following gear train makes 80 R.P.M. Determine the number of teeth on the driven gear.



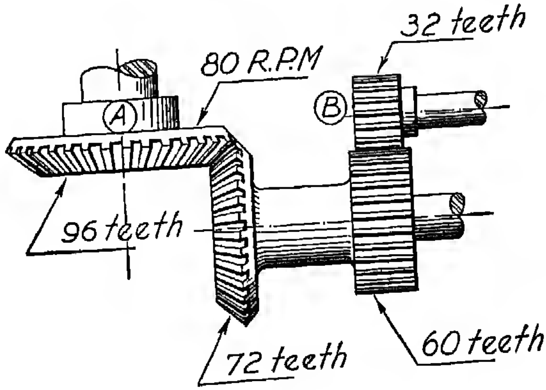
3. A familiar style of automobile jack is operated as shown in the following gear train. Each time gear (*A*) makes one revolution in the direction indicated the screw moves up $\frac{1}{8}$ in. How many revolutions of gear (*B*) are needed to move the screw up $\frac{1}{4}$ in.?

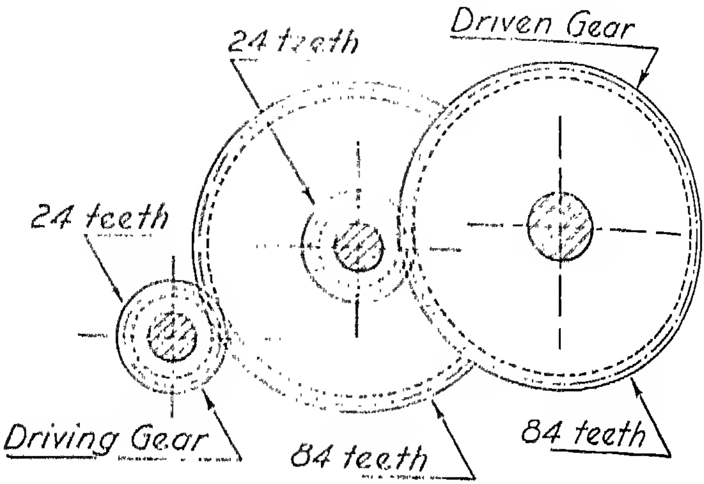




4. A belt-driven machine is to be changed over to a "silent" chain drive as illustrated above. The speed of the motor is rated at 1140 R.P.M. What size gear must be used on the motor in order to obtain the results indicated?

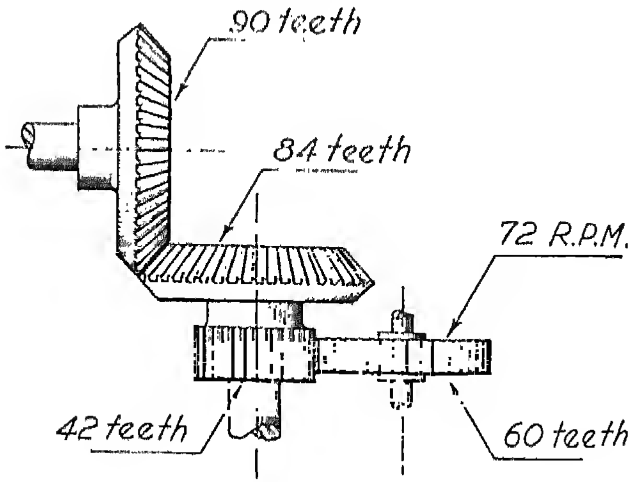
5. What is the R.P.M. of gear (*B*) in the gear train as illustrated below? The driving gear (*A*) makes 80 R.P.M.



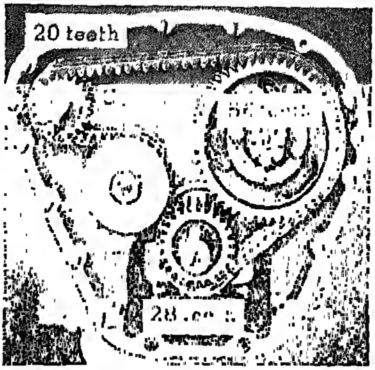


6. In the arrangement above of reducing gears, determine the number of revolutions made by the driving gear for 4 revolutions of the final driven gear.

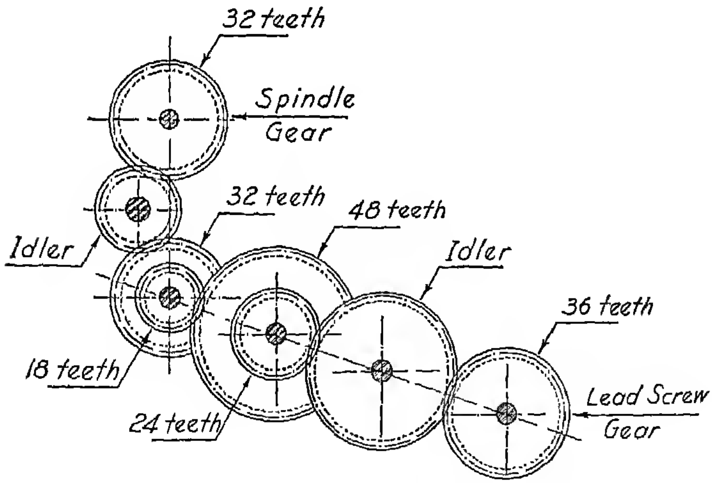
7. Determine the revolutions made per minute by the 90-tooth bevel gear in the following gear drive.



8. The timing-gear drive to the right is typical of that on many automobile engines. When the crankshaft A is turning at 1200 R.P.M. what is the R.P.M. of the camshaft sprocket B, also the R.P.M. of the sprocket C on the generator shaft?

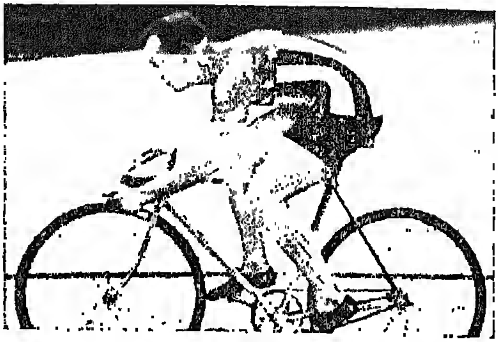


9. The gear train on a lathe set up for screw cutting is arranged as follows. During one turn of the screw gear how many turns will be made by the spindle gear?

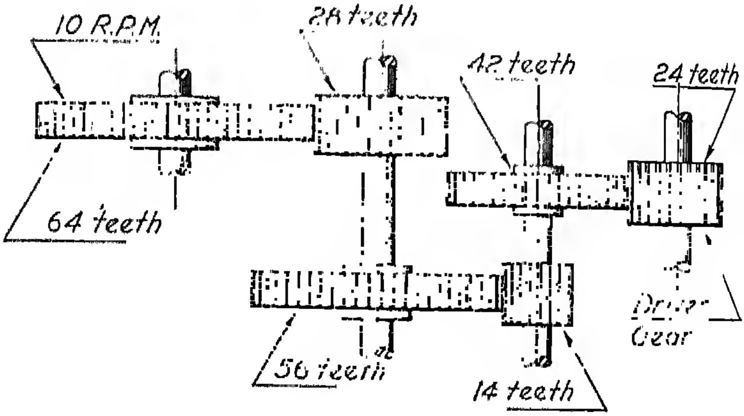


10. The young man, William (Bill) Honeman, in the picture on page 394, holds a $\frac{1}{8}$-mile record as the American Spring Bicycle Champion. His time for that distance is $11\frac{3}{5}$ seconds. In making such a record how fast must he "push" his pedals if the driving sprocket on his bicycle has 24 teeth, the small

sprocket 7 teeth, and the rear wheel is 28 in. in diameter?

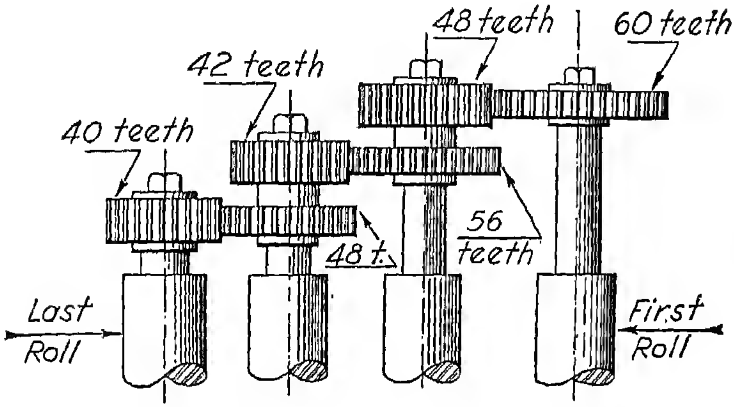


11. From the following sketch of a compound-gear train determine the R.P.M. of the final driven gear.

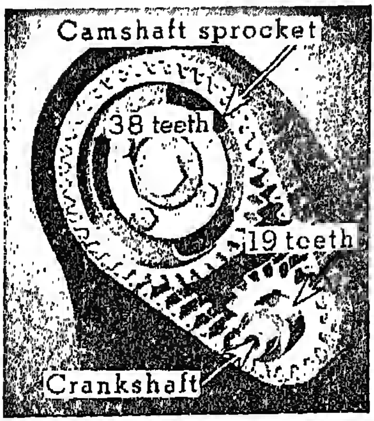


12. Gears on the ends of rolls on a drawing machine are arranged as shown in the drawing on page 395. From this

determine the relative increase in speed of the last roll as noted.*



13. The timing gears on a popular priced automobile are arranged as in the following drawing. How many revolutions of the small gear are made for 4 revolutions of the larger gear?



*Answers to these problems will be found on page 396.

ANSWERS TO PROBLEMS

Pages [illegible] to [illegible].

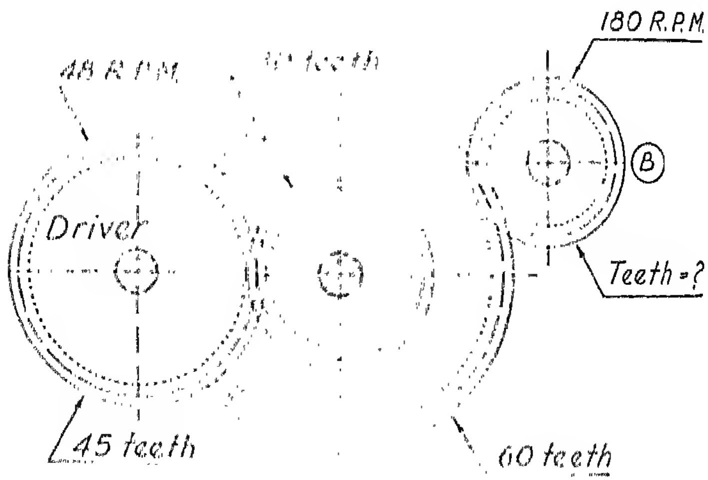
1. 20 teeth.
2. 6 turns.
3. 40 strokes.
4. 182 rev.
5. 65 rev.
6. 14 rev.
7. 72 teeth.
8. $4\frac{8}{11}$ turns.
9. $21\frac{7}{15}$ impressions.
10. 78 teeth.
11. 1500 r.p.m.
12. $3\frac{1}{4}$ turns.

Pages 389 to 395.

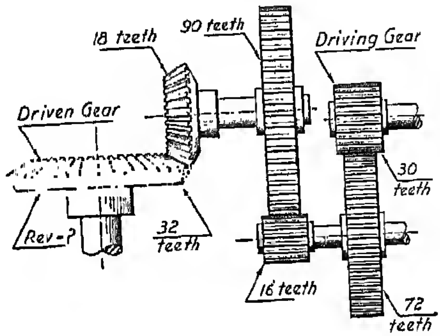
1. 140 rev.
2. 80 teeth.
3. 5.9 rev.
4. 34 teeth.
5. 200 r.p.m.
6. 49 rev.
7. 96 r.p.m.
8. 600 r.p.m.; 1680 r.p.m.
9. 4 turns.
10. 2.26 rev. per sec.
11. 160 r.p.m.
12. Twice that of first roll.
13. 8 rev.

Review Problems in Gear Drives

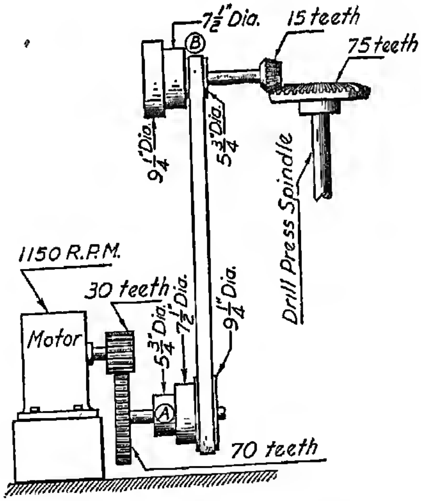
1. The driver on the gear train illustrated below makes 48 R.P.M. It is desired that the driven gear (*B*) make 180 R.P.M. In order to accomplish this how many teeth should there be on gear (*B*)?



2. In the following gear train calculate the number of revolutions made by the final driven gear for 48 revolutions of the first driving gear.

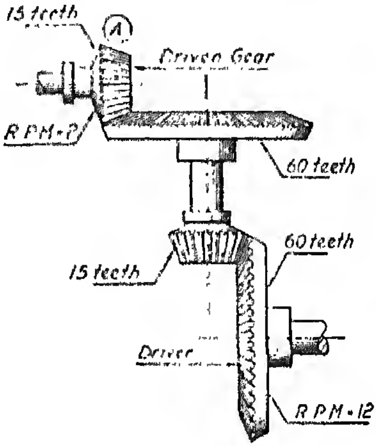


3. The drive on a standard drill press is arranged as follows:

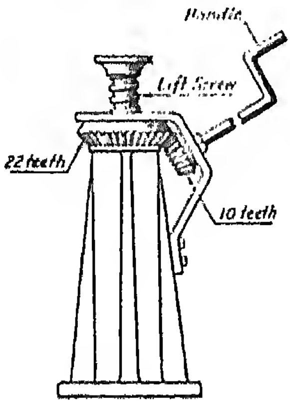


How many R.P.M. does the spindle make when the belt has been shifted to step (*A*) on the lower cone pulley? What is the R.P.M. when the belt is moved to step (*B*)?

4. Determine the R.P.M. of the shaft (A) in the gear train as shown in the following sketch.



5. In the automobile jack shown in the following sketch, the lifting screw moves $\frac{1}{8}$ in. for each turn of the larger gear. How many turns of the handle are needed to raise the screw $\frac{5}{8}$ in.?



TAPER TURNING

Another job frequently done in machine shops, also woodturning and patternmaking shops, is that of *turning tapers.*

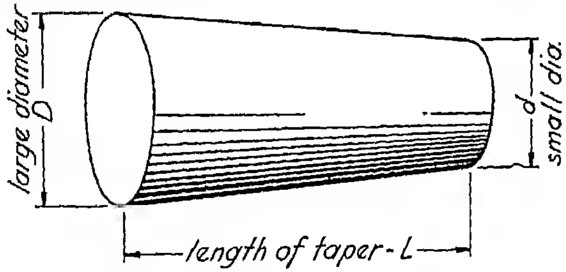
A piece is said to *taper* when it has a gradual, or uniform difference in thickness or diameter, throughout its length, or throughout a portion of its length. It is common shop practice to refer to this as "taper per inch," or "taper per foot." As for example, a taper of $\frac{3}{4}$ in. in the length of 6 in., would be referred to as $\frac{1}{8}$-in. taper per inch. This might also be expressed as taper per foot, which would, of course, be 12 times the taper per inch. In this case the taper per foot would be $12 \times \frac{1}{8}$ in., or $1\frac{1}{2}$ in.

The formulas used in calculating such tapers are as follows:

$$\text{Taper per inch} = \frac{(\text{Large dia.} - \text{Small dia.})}{\text{length in inches}}$$

$$\text{Taper per foot} = \frac{(\text{Large dia.} - \text{Small dia.})}{\text{length in inches}} \times 12$$

By referring to the following drawing of a tapered piece, these various quantities may be abbreviated as shown.



If the large diameter $= D$, the small diameter $= d$, and the length in inches $= L$, then the above formulas become changed as follows:

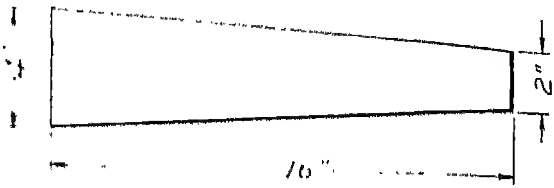
$$\text{Taper per inch} = \frac{D - d}{L}, \text{ and}$$

$$\text{Taper per foot} = \frac{D - d}{L} \times 12$$

These formulas are used in shop practice as illustrated in the following problem.

Example 1:

A piece about to be turned in a lathe has the following dimensions. What is its taper?



Solution and Explanation:

$$\text{Taper per inch} = \frac{D - d}{L}$$

Applied in the above problem this becomes, $\frac{4 - 2}{16}$, or $\frac{1}{8}$.

That is, the taper per inch of the piece in the above drawing is $\frac{1}{8}$ in.

If the taper per foot were desired the following formula would be used: $\text{Taper per foot} = \frac{D - d}{L} \times 12.$

Applying this to the above case the taper per foot would be:

$$\frac{4 - 2}{16} \times 12, \text{ or } 1\tfrac{1}{2}.$$

The above taper expressed in taper per foot therefore would be $1\frac{1}{2}$ in.

From the above formulas the length L, may also be determined when the taper per inch, or the taper per foot, also the large diameter D, and the small diameter d, are known.

The above formulas then become changed to:

$$\text{Length in inches} = \frac{(D - d)}{\text{taper per inch}}$$

$$\text{Length in feet} = \frac{(D - d)}{\text{taper per inch} \times 12}$$

A practical application of these formulas is as follows.

Example 2:

What must be the length of a piece of stock needed to turn a taper per inch of $\frac{1}{8}$ in. where the large diameter is 7 in. and the small diameter is 5 in.?

Solution and Explanation:

$$\text{Length in inches} = \frac{(D - d)}{\text{taper per inch}}$$

According to the above dimensions this equals, $\frac{7-5}{\frac{1}{8}}$, or

$\frac{7-5}{1} \times \frac{8}{1}$, which, reduced to simple form equals 16.

That is, the length of this piece in inches would be 16 in.

Should it be desired to determine this taper directly in feet, the following formula would be used.

$$\text{Length in feet} = \frac{D - d}{\text{taper per inch} \times 12}$$

Using the above dimensions in this formula the solution becomes:

$$\frac{7-5}{\frac{1}{8}} \times 12, \text{ or, } 2 \times \tfrac{8}{12}, \text{ which equals } 1\tfrac{1}{3}.$$

The length of this piece expressed in feet, therefore, is $1\frac{1}{3}$ ft., which, of course, equals 16 in. as in the above solution.

In this same manner the large diameter D may be found when the taper per inch, the length L in inches, and the small diameter d are known.

Also, the small diameter d may be found when the large diameter D, the taper inch and the length in inches L are known.

The above formulas then become changed as follows.

Large diameter D = (length in inches L) $\times$ (taper per inch) $+ d$.

Small diameter $d = D -$ (length in inches $\times$ taper per inch).

These are applied in such practical cases as the following.

Example 3:

What should be the large diameter of a metal piece 18 in. long with a taper of $\frac{1}{16}$ in. to the inch, and a small diameter of 2 in.?

Solution and Explanation:

Large diameter D = (length in inches $\times$ taper per inch) + d.

Applied to the above conditions this works out as follows.

Large diameter = $(18 \times \frac{1}{16}) + 2$, which reduces to:

$$1\tfrac{1}{8} + 2, \text{ or } 3\tfrac{1}{8}.$$

That is, the large diameter of the above piece should be $3\frac{1}{8}$ in.

Example 4:

Determine the diameter of a tapered roller that measures 12 in. long with a taper of $\frac{1}{4}$ in. per inch and a large diameter of 6 in.

Solution and Explanation:

Small diameter $d = D$ − (length in inches $\times$ taper per inch).

By substituting the numerical values in this formula there results:

Small diameter $d = 6 - (12 \times \frac{1}{4})$, or, $6 - 3$, which equals 3.

That is, the small diameter of the tapered roller above is 3 in.

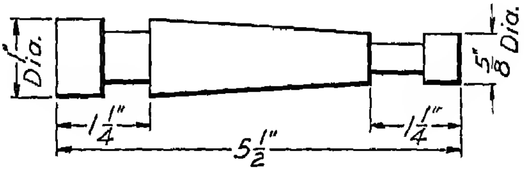
Problems Involving Calculations of Tapers

1. In turning a piece that measures 6 in. long, what will the diameter be at the small end when the large end measures $2\frac{1}{2}$ in. and the taper is 3 in. to the foot?

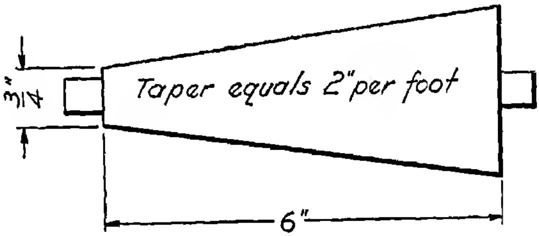
2. A work order issued to a cabinet shop calls for a piece of mahogany that measures $1\frac{1}{2}$ in. in diameter at the small end, and $3\frac{1}{2}$ in. in diameter at the large end. The taper is specified as $\frac{1}{8}$ in. to the inch. In turning this piece on the lathe it is

necessary to add 1 in. to the length of the tapered portion to allow for trimming and finishing. How long a piece should be cut for this job?

3. Determine the taper per inch, also the taper per foot, on the spindle in the sketch below.

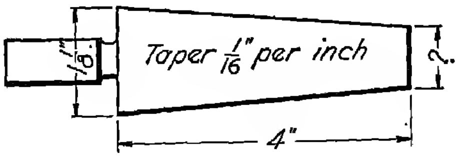


4. The following drawing for a tapered roller was sent to the machine shop. What is the smallest diameter of round stock that may be used in making this piece?



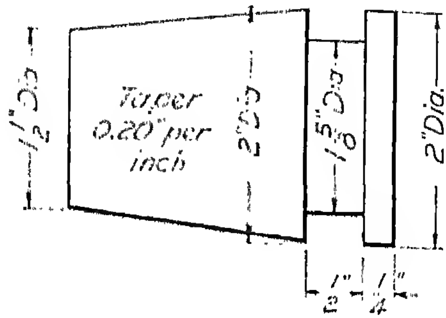
5. Determine the taper per inch, also the taper per foot, on a tapered shank that measures 6 in. long and is .750 in. in diameter at the large end and .500 in. at the other end.

6. What is the smallest diameter on the following gauge as used for tapered holes?

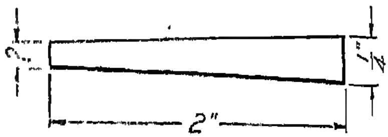


7. The measurement on the small end of a lathe center is 0.778 in. At a point 3 in. from this small end the measurement is 0.928 in. What is the taper per foot of this center?

8. Determine the length of stock in the tapered piece shown below.



9. If the taper per foot for the standard taper pins, such as are used in fastening small gears to shafts, is recommended as being ¼ in. to the foot, what should be the measurement of the small end of the tapered pin in the drawing above?



10. The tapered hole in a drill-press spindle measures 0.875 in. in diameter at a certain point. Three inches farther it measures 0.750 in. From these measurements calculate the taper per foot of this hole.*

HOW TAPERS ARE TURNED

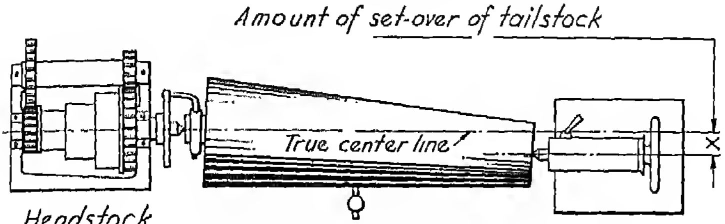
The Offset Method

In lathe work, as practiced in machine shops, tapers are turned as a rule, by "setting over" the tailstock, or "offsetting" it, a definite amount.

As illustrated in the sketch on page 405, the amount of the "set-over" is indicated by the letter "X." This equals the

*Answers to these problems will be found on page 417.

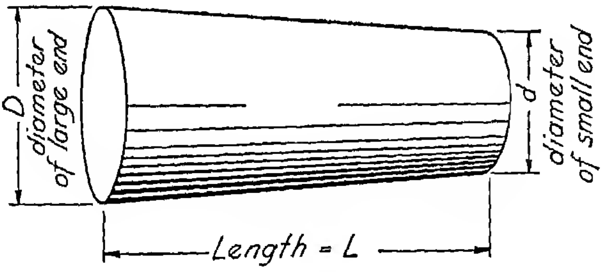
amount the tailstock is moved "out of line" from the *true line of centers.* This movement may be either forward or backward, depending upon the piece being tapered.



Taper turning in lathe
by set-over method

The formulas used in determining the amount of this offset relate to two different conditions, which are described and illustrated as follows:

1st condition: Where the piece is tapered throughout its entire length. This is illustrated in the drawing below.



In calculating the offset necessary for turning such pieces the following formula is used.

Offset $= \frac{1}{2}(D - d)$, or one half the difference in diameters.

This formula is applied in such cases as the following.

Example 1:

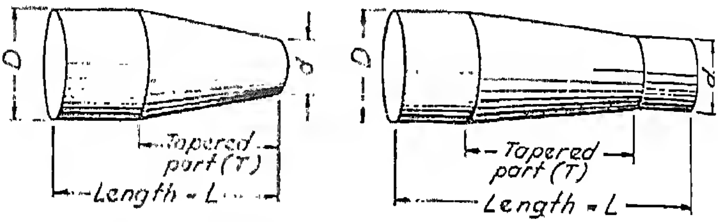
What amount must the tailstock on a lathe be set over in order to cut a taper on a piece 5 in. long that measures $1\frac{3}{4}$ in. on one end, and $1\frac{1}{4}$ in. on the other end?

Solution and Explanation:

$$\text{Offset} = \tfrac{1}{2}(D - d)$$

As applied to the above condition this equals, $\frac{1}{2}(1\frac{3}{4}$ in. $- 1\frac{1}{4}$ in.), or $\frac{1}{4}$ in. That is, the tailstock should be set over $\frac{1}{4}$ in. to cut this taper.

2nd condition: Where the tapered portion does *not* extend the full length of the piece, but only a part of it, as illustrated below.



In determining the tailstock offset for turning such tapers as those above, the total length of the piece must be used in the calculation.

The shop formula for this is:

$$\text{Offset} = \tfrac{1}{2} \times \frac{(D - d)}{\text{tapered length in inches}} \times (\text{length of piece in inches}),$$

or, substituting the above letters:

$\text{Offset} = \frac{1}{2} \times \frac{(D - d)}{T} \times (L)$; which is also, $\frac{1}{2} \times$ (taper per inch) $\times (L)$.

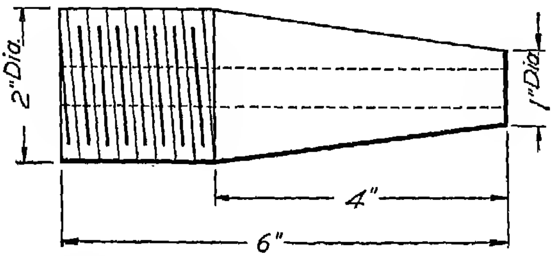
When the taper per foot is given instead of the taper per inch, this formula becomes changed to:

$$\text{Offset} = \tfrac{1}{2} \times \frac{\text{taper per foot}}{12} \times L$$

This same formula is also used in calculating tapers of pieces that are turned on *mandrels*, or *arbors*. The *length* of the arbor then becomes the length, L, in the above formula. How these formulas are used is illustrated in the following problems.

Example 2:

How much should the tailstock of a lathe be set over to turn the taper on the nozzle in the following drawing?



Solution and Explanation:

$$\text{Setover} = \tfrac{1}{2} \times \frac{(D - d)}{\text{tapered length in inches}} \times \text{length of piece}$$
in inches.

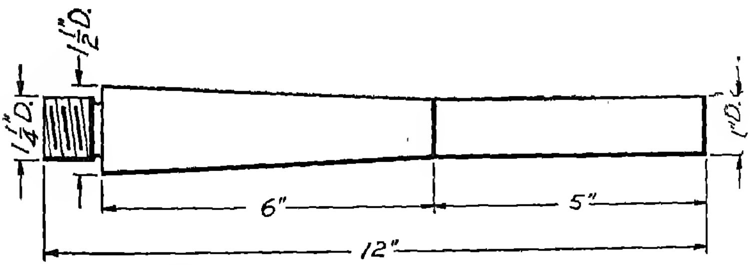
Using this in the above problem, the result works out as:

$$\tfrac{1}{2} \times \frac{(2 - 1)}{4} \times 6; \text{ or, } \tfrac{1}{2} \times \tfrac{1}{4} \times 6; \text{ which equals } \tfrac{3}{4}.$$

That is, the tailstock must be set over $\frac{3}{4}$ in. for this job.

Example 3:

Determine the amount the tailstock on a lathe must be set over in order to cut the taper on the following piece.



Solution and Explanation:

$$\text{Offset} = \tfrac{1}{2} \times \frac{(D - d)}{\text{tapered length in inches}} \times (\text{length in inches}).$$

Applying this to the above conditions the result works out as: $\frac{1}{2} \times \frac{(1\frac{1}{2} - 1)}{6} \times 12$, or $\frac{1}{2} \times \frac{1}{12} \times \frac{12}{1}$, which equals $\frac{1}{2}$ in.

This indicates that the tailstock should be set over $\frac{1}{2}$ in.

Example 4:

What should be the offset of the tailstock on a lathe in turning a spindle 2 ft. long, having a taper of $\frac{1}{4}$ in. to the foot?

Solution and Explanation:

$$\text{Offset} = \frac{1}{2} \times \frac{\text{taper per foot}}{12} \times (\text{length in inches}).$$

Using this rule in the above problem the result becomes: $\frac{1}{2} \times \frac{\frac{1}{4}}{12} \times 24$; or $\frac{1}{2} \times \frac{1}{4 \times 12} \times 24$, which reduces to $\frac{1}{4}$ in.

Accordingly, the offset for turning this taper is $\frac{1}{4}$ in.

Example 5:

A bearing 2 in. long that is to be tapered, is placed on a 6-in. arbor. The large diameter of the bearing is to be $2\frac{1}{4}$ in. while the small diameter is to be $2\frac{1}{8}$ in. How much should the offset be?

Solution and Explanation:

$$\text{Offset} = \frac{1}{2} \times \frac{(D - d)}{\text{tapered length in inches}} \times (\text{length of arbor in inches}).$$

Using the above formula, with the length of the arbor substituted for L the length of the piece, the problem is solved as follows:

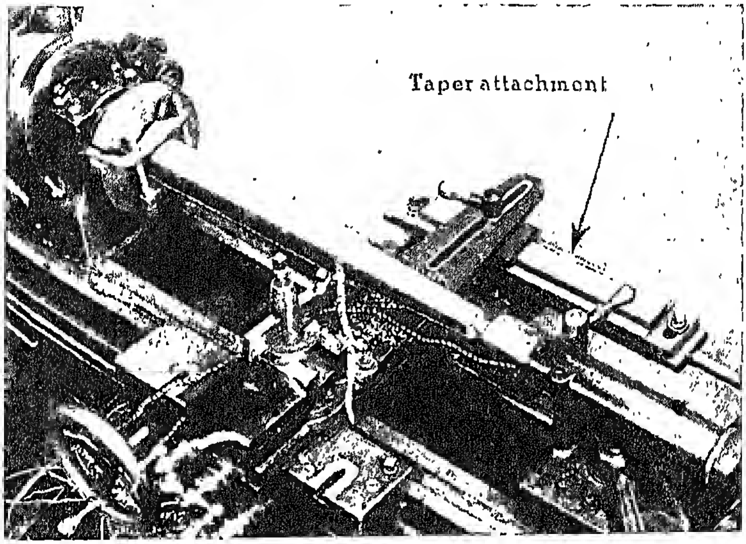
$\frac{1}{2} \times \frac{(2\frac{1}{4} - 2\frac{1}{8})}{2} \times 6$; which reduces to, $\frac{1}{2} \times \frac{1}{8} \times \frac{1}{2} \times \frac{6}{1}$, or $\frac{3}{16}$ in.

In turning the above taper the tailstock should be offset $\frac{3}{16}$ in.

Turning Tapers By Use of a Taper Attachment

While the "offset method" of turning a taper on a lathe is more generally used, some lathes are, however, equipped with a special device for this purpose, known as a "taper attach-

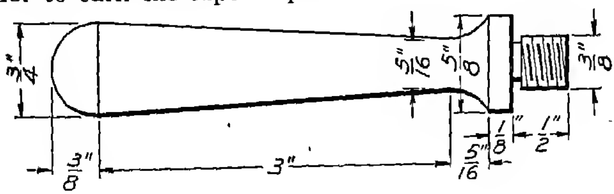
ment." The use of this makes taper turning rather simple, as it avoids having to move the tailstock off center. With this device one need only know the amount of the taper per foot or the taper per inch. The attachment is then set at that amount and it guides the cutting tool at the proper angle as the taper is being turned.



The following problem illustrates the calculations involved in the use of this attachment.

Example:

The handle in the drawing below is to be turned on a machine-shop lathe equipped with a taper attachment indicating taper per foot. How should this attachment be set in order to turn the tapered portion?



Solution and Explanation:

Since the taper attachment is to be used in the turning of the tapered portion of this handle, it is necessary only to determine the amount of the taper per foot of that portion, which in this case measures 3 in. in length.

The formula for this is: Taper per foot $= \dfrac{D - d}{L} \times 12$

Substituting the various values in this the result becomes,

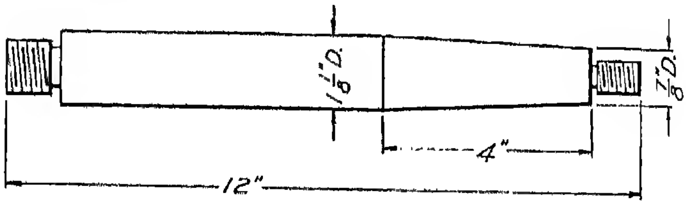
Taper per foot $= \dfrac{(\frac{5}{8} - \frac{5}{16})}{3} \times 12$; which works out as follows:

$$\frac{\frac{5}{16}}{3} \times 12; \text{ or } \frac{5}{16} \times \frac{1}{3} \times \frac{12}{1}, \text{ which reduces to } 1\tfrac{1}{4} \text{ in.}$$

That is, the taper per foot is $1\frac{1}{4}$ in. This is also the amount that the taper attachment should be set at in turning the tapered portion in the handle.

Problems Involving Calculations Relating to Taper Turning

1. It is desired to turn an arbor 12 in. long as per the dimensions in the drawing below. Determine the setover of the tailstock.

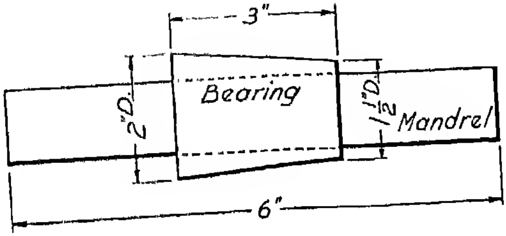


2. What is the amount of tailstock offset needed to turn a tapered piece that measures 4 in. long? The large end is to be 1 in. in diameter and the small end is to be $\frac{3}{4}$ in. in diameter.

3. Find the offset of tailstock needed to turn a piece 9 in. long having a taper of $\frac{1}{2}$ in. per foot.

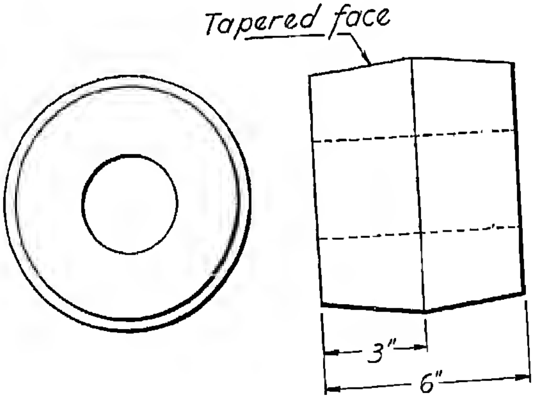
4. How much should the tailstock be moved off center to permit the machinist to turn a tapered piece 4 in. long so that the small diameter will be $\frac{5}{8}$ in. and the large diameter $1\frac{7}{8}$ in.

5. A tapered bearing is to be turned on a mandrel as illustrated below. What is the setting for this job if done on a lathe having a taper attachment?



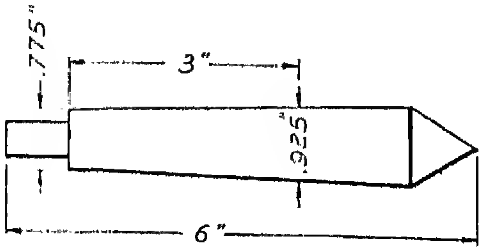
6. A special brass plug 6 in. long is to be turned with a taper of ½ in. to the foot. What would be the tailstock offset for this?

7. The pulley sketched below is to be given a tapered crown of ¾ in. to the foot. To do this job the pulley is forced on a 10-in. mandrel and is then turned on a lathe. Calculate the amount of offset necessary for this job.

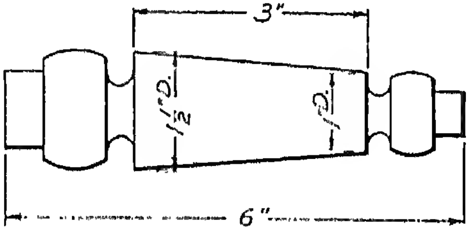


8. Calculate the setover of the tailstock on a lathe in order to turn a taper of 0.375 in. per foot on a piece that measures 8 in. long.

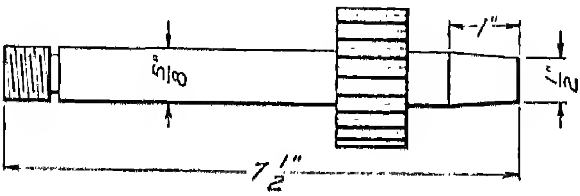
9. Determine the taper per foot on the following lathe center. In turning this taper what would be the setting of the taper attachment?



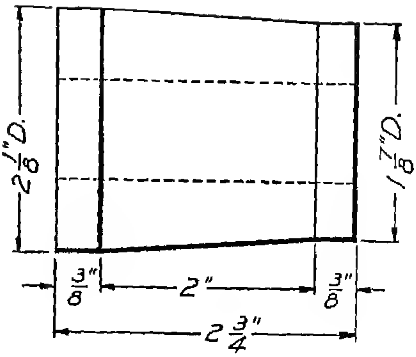
10. Determine the amount of offset necessary to turn the taper on the brass spindle as shown below. If this were done on a lathe having a taper attachment what would be the setting?



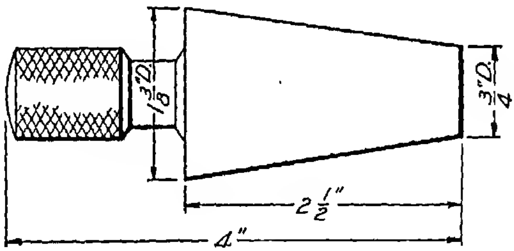
11. At what amount should the taper attachment be set in turning the taper on the spindle shown below? What is the taper per inch?



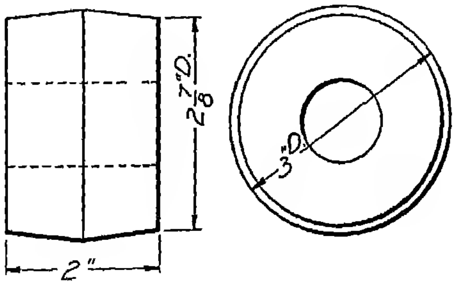
12. Calculate the taper per foot of the piece in the drawing to the right. If this is cut on a 9-in. mandrel for taper turning, what is the amount that the tailstock is set over?



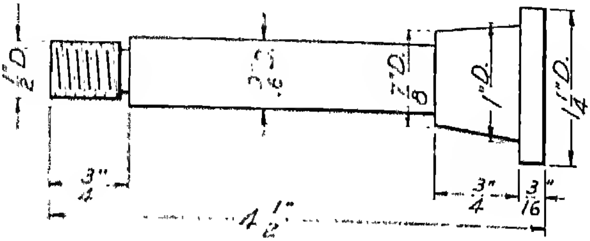
13. What is the taper per foot on the following plug gauge? How much is the tailstock set over in turning this taper?



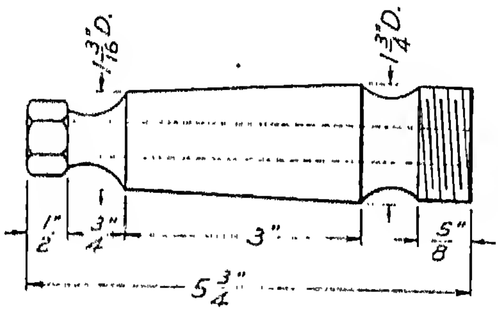
14. In order to turn the crown on the 3-in. pulley as illustrated, it is mounted on a 9-in. mandrel. Calculate the taper per foot also the amount of tailstock setover needed in turning the taper.



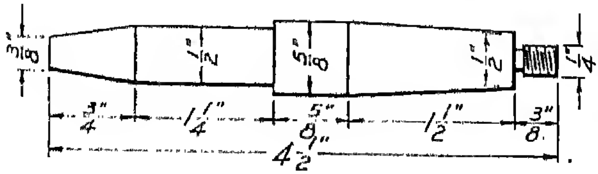
15. Calculate the taper per foot on the following piece. If turned by setting over tailstock, how much would this set-over amount to?



16. An 8-in. mandrel is used in turning the taper on the following piece. Calculate the amount of the setting on the taper attachment.



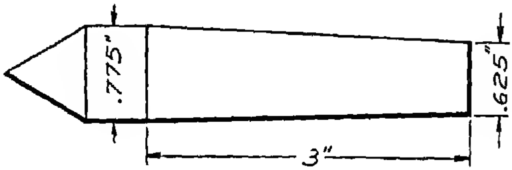
17. Determine the amount of tailstock setover needed in turning each of the following tapers.



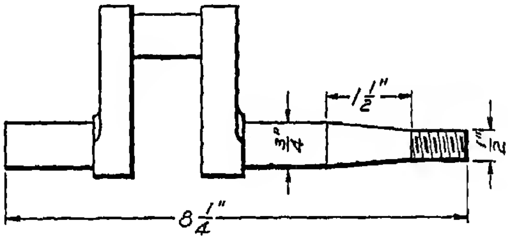
18. A special milling-machine arbor 20 in. long is tapered $5\frac{1}{4}$ in. on one end with a taper of 0.600 in. to the foot. How

much must the lathe tailstock be set over in order to turn this taper?

19. Determine the taper per foot in the following lathe center. What setover is necessary in turning this taper if made from a piece of stock 5 in. long?

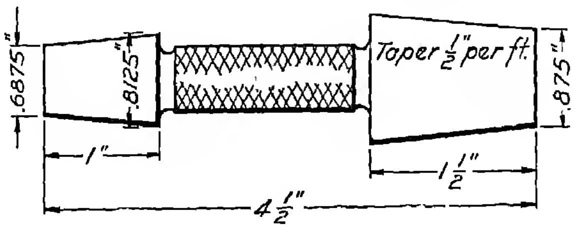


20. In order to turn the taper on the small crank illustrated below, what should be the setting of the taper attachment on the lathe used for this job?



21. With a taper per foot of .750 in., what would be the setover required to make a tapered shank on a piece $7\frac{1}{2}$ in. long?

22. A plug gauge having tapers on each end as shown below is required for a special job. What is the setting of the taper attachment required in each case? What is the smallest diameter stock that can be used in making this gauge?



23. A taper of $\frac{1}{4}$ in. to the foot is required on the end of a special shafting 18 in. long. How much should the tailstock be set over in order to cut this taper?

24. What is the taper per foot on a piece 4 in. long measuring .330 in. diameter on the small end and .580 in. diameter on the other end? If this taper were to be turned by offsetting the tailstock, what would be the amount of this offset?

25. A lathe center having a taper of .600 in. per foot is to be turned by the offset method. The length of the stock used for this job is 5 in. What is the tailstock offset needed in turning this taper?*

*Answers to these problems will be found on page 417.

ANSWERS TO PROBLEMS

Pages 402 to 404.

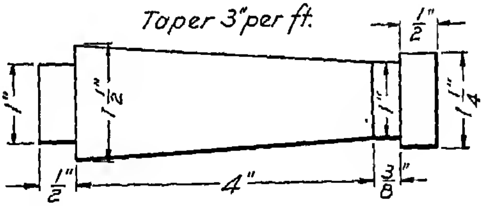
1. 1 in.
2. 17 in.
3. $\frac{1}{8}$ in.; $1\frac{1}{2}$ in.
4. $1\frac{3}{4}$ in.
5. .0417 in. per in.; $\frac{1}{2}$ in. per foot.
6. $\frac{7}{8}$ in.
7. .600 in.
8. $3\frac{1}{4}$ in.
9. .208 in. dia.
10. .500 in.

Pages 410 to 416.

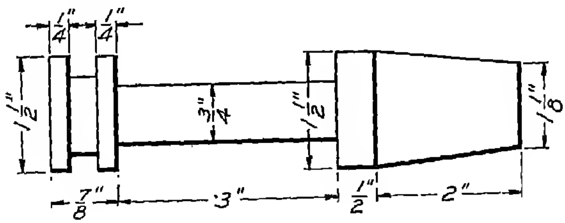
1. $\frac{3}{8}$ in.
2. $\frac{1}{8}$ in.
3. $\frac{3}{16}$ in.
4. $\frac{5}{8}$ in.
5. 2 in.
6. $\frac{1}{8}$ in.
7. $\frac{5}{16}$ in.
8. $\frac{1}{8}$ in.
9. .600 in.
10. $\frac{1}{2}$ in.; 2 in.
11. $1\frac{1}{2}$ in.; $\frac{1}{8}$ in.
12. $1\frac{1}{2}$ in; $\frac{9}{16}$ in.
13. 3 in.; $\frac{1}{2}$ in.
14. $1\frac{1}{2}$ in.; $\frac{9}{16}$ in.
15. 2 in.; $\frac{3}{8}$ in.
16. $2\frac{1}{4}$ in.
17. $\frac{3}{8}$ in.; $\frac{3}{16}$ in.
18. $\frac{1}{2}$ in.
19. .600 in.; .125 in.
20. 2 in.
21. $\frac{15}{64}$ in.
22. $1\frac{1}{2}$ in.; $\frac{1}{2}$ in.; $\frac{15}{16}$ in.
23. $\frac{3}{16}$ in.
24. $\frac{3}{4}$ in.
25. $\frac{1}{8}$ in.

Review Problems Involving Tapers

1. What is wrong with the following drawing? How would you correct it?

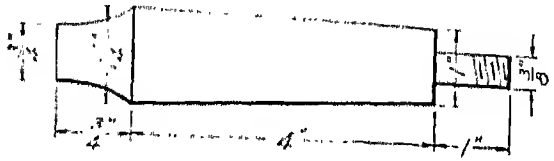


2. How would the taper attachment be set on a lathe to turn the taper as specified in the following drawing?



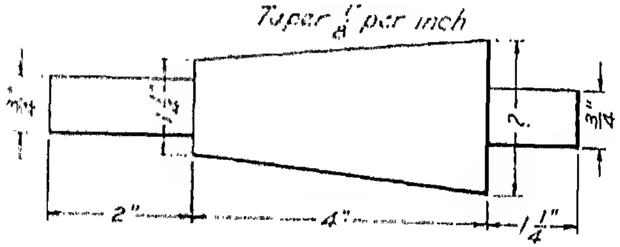
3. In order to turn a taper of $\frac{3}{16}$ in. to the inch on the end of a piece measuring 8 in. long, how much should the tailstock be set over? If this job were done on a lathe having a taper attachment what would be the setting of this attachment?

4. What is the taper per foot on the piece illustrated in the following sketch?

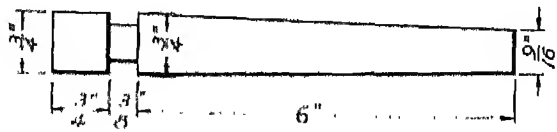


5. A lathe center which is to be made from a piece of steel 6 in. long is to have a taper of .500 in. to the foot. How much should the tailstock be offset in order to turn this taper?

6. What is the smallest diameter stock that can be used in turning the following piece?



7. How much does the pin in the following drawing taper? How would this be expressed in taper per foot?



SUMMARY

Tables

and

Formulas

Summary of Tables and Formulas

Linear Measure

12 inches (in.) (″)	= 1 foot (ft.) (′)
3 feet	= 1 yard (yd.) = 36 in.
$5\frac{1}{2}$ yards	= 1 rod (rd.) = $16\frac{1}{2}$ ft. = 198 in.
320 rods	= 1 mile (mi.) = 1,760 yd. = 5,280 ft.
1 meter (m.)	= 39.37 inches = 3.28 ft. = 1.09 yd.

Square or Surface Measure

144 square inches (sq. in.) (□ ″)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.) = 1,296 sq. in.
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.) = $272\frac{1}{4}$ sq. ft.
160 square rods	= 1 acre (A) = 4,840 sq. yd. = 43,560 sq. ft.
640 acres	= 1 square mile (sq. mi.) = 102,400 sq. rd. = 3,097,600 sq. yd. = 27,878,400 sq. ft.
1 square meter	= 1.196 sq. yd. = 10.764 sq. ft.

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
1 cubic meter	= 1.308 cu. yd. = 35.314 cu. ft.

Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight	= 1 ton (T) = 2,000 pounds
2,240 pounds	= 1 long ton

Dry Measure

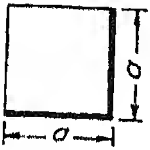
2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.) = 16 pt.
4 pecks	= 1 bushel (bu.) = 32 qt. = 64 pt.
1 bushel	= 2150.4 (cu. in.), 1.244 or $1\frac{1}{4}$ (cu. ft.)
1 bushel corn	= 56 lb.
1 bushel potatoes	= 60 lb.
1 bushel wheat	= 60 lb.
U. S. dry quart	= $67\frac{1}{5}$ cu. in.

Liquid Measure

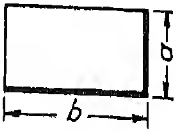
4 gills	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.) = 8 pt.
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.) = 126 qt. = 252 pt.
2 barrels	= 1 hogshead (hhd.) = 63 gal. = 252 qt. = 504 pt.
1 gallon	= 231 cu. in. or .134 cu. ft.
7.48 gallons	= 1 cu. ft. or $7\frac{1}{2}$ gal. approx.
U. S. liquid quart	= $57\frac{3}{4}$ cu. in.
1 gallon water	= 8.34 lb. or $8\frac{1}{3}$ lb. approx.
1 gallon gasoline	= 6.32 lb.
1 gallon linseed oil	= 7.84 lb.

Areas

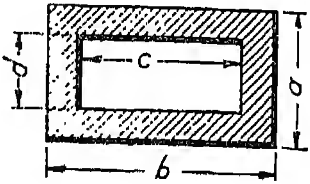
Square
Area = $a \times a$

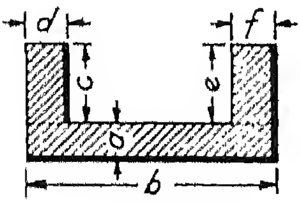


Rectangle
Area = $a \times b$



Hollow Rectangle
Area cross section
= $(a \times b) - (d \times c)$



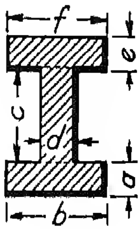


Channels

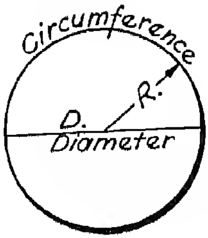
Area of channel
= $(a \times b) + (c \times d) + (e \times f)$

I sections

Area of I section
= $(a \times b) + (c \times d) + (e \times f)$



Circles



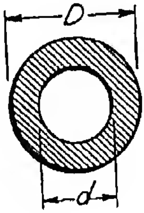
Circumference $= 3.1416 \times D$
$\pi D = \pi 2R$

Diameter $= D = \dfrac{\text{Circumference}}{3.1416}$

Area $= 3.1416 \dfrac{(D \times D)}{4}$, or
$3.1416\ R^2$
$= .7854\ (D \times D)$ or $.7854\ D^2$
$= .0796\ (\text{circum.})^2$

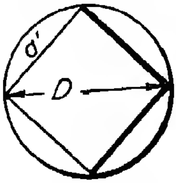
Volume of sphere
$= .5236 \times D \times D \times D$, or $.5236 D^3$
Area of surface of sphere
$=$ Dia. $\times$ circumference

Area of circular ring



$= .7854\ (D \times D) - .7854\ (d \times d)$
$= .7854\ (D^2 - d^2)$

Side of largest square inside given circle



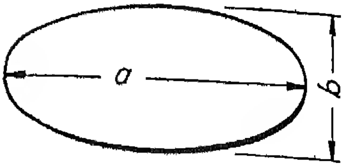
Side of square $= a$

$\dfrac{1.414}{2} \times D = .707D$

Diameter of circle that will just enclose square
$D = 1.414 \times a$

Ellipse

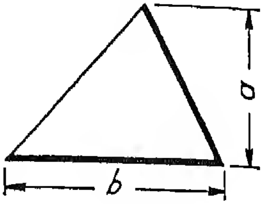
Area = $a \times b \times .7854$



Triangles

a = altitude $\qquad b$ = base

Area = $\dfrac{a \times b}{2}$

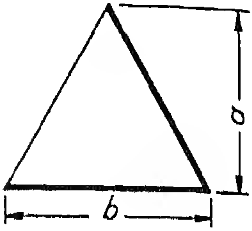


Equal-sided, or equilateral, triangle

All sides equal

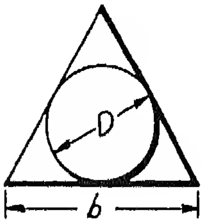
$a = .866 \times b$

Area = $\dfrac{.866 \times b \times b}{2}$



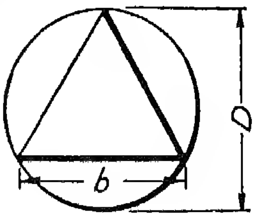
Largest circle within equilateral triangle

$D = .5774 \times b$

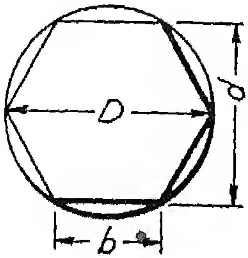


Circle that will just enclose the equilateral

$D = 1.1546 \times b$



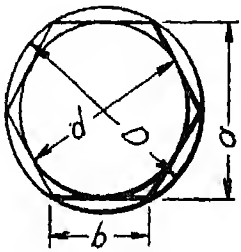
Hexagon



$$\text{Area} = \frac{6 \times .866 \times b \times b}{2}$$
$$= 2.598 \times b \times b$$
d = distance across flats
$d = 1.732 \times b$, or $.866 \times D$
$D = 2b = 1.155d$

Enclosed circle

Diameter $= d = 1.732b$



Diameter of Circle that will just enclose a hexagon,

Diameter $= D = 2b = 1.155d$

Volumes, or Cubic Measure

Volume of cylinder = Area of circular base × height
Volume of sphere = $D^3 \times 0.5236$

$$\text{Gallons in cylinder} = \frac{\text{Area of base in sq.in.} \times \text{height in inches;}}{231}$$

also, $$\frac{\text{Area of base in sq. ft.} \times \text{height in feet;}}{.134}$$

or, Area of base in sq. ft. × height in feet × 7.48

Weight of water in a tank = Volume of tank in cubic feet × 62.425

Weight of iron castings as made from pine patterns without cores equals weight of wood pattern × 16

Weight of iron castings as made from mahogany patterns without cores equals weight of pattern × 12

Weight of timbers = Volume in cubic feet × weight per cubic foot

Weight of building materials = Volume in cubic feet × weight per cu. ft.

Weights of materials (approximate)

Aluminum	0.09	lb. per cu. in.
Birch	40.	lb. per cu. ft.
Brass	0.30	lb. per cu. in.
Brick (common)	112.	lb. per cu. ft.
Brickwork (ordinary)	112.	lb. per cu. ft.
Bronze	0.32	lb. per cu. in.
Chestnut	37.	lb. per cu. ft.
Copper	0.32	lb. per cu. in.
Concrete	150.	lb. per cu. ft.
Earth (packed)	100.	lb. per cu. ft.
Granite	166.	lb. per cu. ft.
Gravel	110.	lb. per cu. ft.
Hemlock	25.	lb. per cu. ft.
Hickory	50.	lb. per cu. ft.
Ice	58.7	lb. per cu. ft.
Iron (cast)	.26	lb. per cu. in.
Iron (wrought)	.28	lb. per cu. in.
Lead	.41	lb. per cu. in.
Limestone	168.	lb. per cu. ft.
Mahogany	56.	lb. per cu. ft.
Maple	44.	lb. per cu. ft.
Oak (white)	50.	lb. per cu. ft.
Oak (red)	47.	lb. per cu. ft.
Pine (southern, yellow)	40.	lb. per cu. ft.
Pine (white)	30.	lb. per cu. ft.
Sand	100.	lb. per cu. ft.
Spruce	28.	lb. per cu. ft.
Snow	8.5	lb. per cu. ft.
Steel	.28	lb. per cu. in.
Walnut	40.	lb. per cu. ft.
Water	62.425	lb. per cu. ft.
Water (sea)	64.	lb. per cu. ft.
Zinc	.26	lb. per cu. in.

Speeds

Surface speed of pulley, in feet per minute = Circumference of pulley in feet × R.P.M.

$$\text{Circumference in feet} = \frac{\text{Surface speed in feet per minute}}{\text{R.P.M.}}$$

$$\text{R.P.M.} = \frac{\text{Surface speed in feet per minute}}{\text{Circumference in feet}}$$

Cutting speed of circular saws, band saws, swing saws = Circumference of saw in feet × R.P.M.

Cutting speed of lathe tool = Circumference in feet of piece being turned × R.P.M. of piece.

Cutting speed of grinding wheels = Circumference of wheel in feet × R.P.M.

Belt speed = Circumference of pulley in feet × R.P.M.

Length in feet of belt in coil = $.131\ (D + d) \times$ Number of laps.

Tapers

$$\text{Taper per inch} = \frac{(D - d)}{\text{length in inches}}$$

$$\text{Taper per foot} = \frac{(D - d)}{\text{length in inches}} \times 12$$

Tailstock setover for taper turning

$$= \frac{1}{2}\,\frac{(\text{length of piece} \times \text{taper per foot})}{12}$$

$$= \frac{1}{2}\,\frac{(D - d) \times \text{length of piece}}{\text{tapered length}}$$

Screw Threads

$$\text{Pitch} = \frac{1 \text{ in.}}{\text{Threads per inch}}$$

$$\text{Number of threads per inch} = \frac{1}{\text{Pitch}}$$

For "V" thread:

$$\text{Depth} = \text{Pitch} \times .866$$

$$= \frac{.866}{\text{Number threads per inch}}$$

$$\text{Root diameter} = \text{Dia. of screw} - 2 \times \frac{.866}{\text{threads per inch}}$$

$$= \text{Dia. of screw} - 2 \times \text{Pitch} \times .866$$

For American National threads:

$$\text{Depth} = \text{Pitch} \times .6495$$

$$= \frac{.6495}{\text{Number threads per inch}}$$

$$\text{Root diameter} = \text{Dia. of screw} - 2 \times \frac{.6495}{\text{threads per inch}}$$

$$= \text{Dia. of screw} - (2 \times \text{Pitch} \times .6495)$$

Tap-Drill Sizes

For "V" thread:

$$= \text{Dia. of screw} - (\text{Pitch} \times 1.3)$$

$$= \text{Dia. of screw} - \frac{1.3}{\text{threads per inch}}$$

For American National threads both coarse and fine:

$$= \text{Dia. of screw} - (\text{Pitch} \times .97)$$

$$= \text{Dia. of screw} - \frac{.97}{\text{threads per inch}}$$

Bolts and Nuts (Rough)

Hexagon head bolts

Distance across flat side = $1\frac{1}{2}$ dia. of bolt reduced to sixteenths of an inch
Distance across corners = 1.155 × distance across flats
Thickness of hex nut = $\frac{7}{8}$ dia. of bolt reduced to nearest sixty-fourth of an inch
Thickness of hex head = $\frac{3}{4}$ dia. bolt reduced to nearest sixty-fourth of an inch

Square head bolts

Distance across flat side = $1\frac{1}{2}$ dia. of bolt reduced to sixteenths of an inch
Distance across corners = 1.414 × dist. across flat sides
Thickness of square nut = $\frac{7}{8}$ dia. of bolt reduced to nearest sixty-fourth of an inch
Thickness of square head = $\frac{3}{4}$ dia. of bolt reduced to nearest sixty-fourth of an inch

AMERICAN STANDARD REGULAR BOLTS AND NUTS

Hexagon and Square (Rough)

Dia. of Bolt	*Threads per Inch*	*Across Flats*	*Thickness*	
			Head	*Nut*
$\frac{1}{4}$	20	$\frac{3}{8}$	$\frac{11}{64}$	$\frac{7}{32}$
$\frac{5}{16}$	18	$\frac{1}{2}$	$\frac{13}{64}$	$\frac{17}{64}$
$\frac{3}{8}$	16	$\frac{9}{16}$	$\frac{1}{4}$	$\frac{21}{64}$
$\frac{7}{16}$	14	$\frac{5}{8}$	$\frac{19}{64}$	$\frac{3}{8}$
$\frac{1}{2}$	13	$\frac{3}{4}$	$\frac{21}{64}$	$\frac{7}{16}$
$\frac{9}{16}$	12	$\frac{7}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
$\frac{5}{8}$	11	$\frac{15}{16}$	$\frac{27}{64}$	$\frac{35}{64}$
$\frac{3}{4}$	10	$1\frac{1}{8}$	$\frac{1}{2}$	$\frac{21}{32}$
$\frac{7}{8}$	9	$1\frac{5}{16}$	$\frac{19}{32}$	$\frac{49}{64}$
1	8	$1\frac{1}{2}$	$\frac{21}{32}$	$\frac{7}{8}$
$1\frac{1}{8}$	7	$1\frac{11}{16}$	$\frac{3}{4}$	1
$1\frac{1}{4}$	7	$1\frac{7}{8}$	$\frac{27}{32}$	$1\frac{3}{32}$
$1\frac{3}{8}$	6	$2\frac{1}{16}$	$\frac{29}{32}$	$1\frac{13}{64}$
$1\frac{1}{2}$	6	$2\frac{1}{4}$	1	$1\frac{5}{16}$
$1\frac{5}{8}$	$5\frac{1}{2}$	$2\frac{7}{16}$	$\frac{13}{32}$	$1\frac{27}{64}$
$1\frac{3}{4}$	5	$2\frac{5}{8}$	$\frac{15}{32}$	$1\frac{17}{32}$
$1\frac{7}{8}$	5	$2\frac{13}{16}$	$1\frac{1}{4}$	$1\frac{41}{64}$
2	$4\frac{1}{2}$	3	$1\frac{11}{32}$	$1\frac{3}{4}$

TAP-DRILL SIZES*

American National Coarse Threads—(N.C.)

Thread Diameter	*Threads per In.*	*Dia. of Tap Drill*	*Thread Diameter*	*Threads per In.*	*Dia. of Tap Drill*
$\frac{1}{4}$	20	$\frac{13}{64}$	1	8	$\frac{7}{8}$
$\frac{5}{16}$	18	$\frac{1}{4}$	$1\frac{1}{16}$	8	$\frac{15}{16}$
$\frac{3}{8}$	16	$\frac{5}{16}$	$1\frac{1}{8}$	7	$\frac{63}{64}$
$\frac{7}{16}$	14	$\frac{23}{64}$	$1\frac{3}{16}$	7	$1\frac{3}{64}$
$\frac{1}{2}$	13	$\frac{27}{64}$	$1\frac{1}{4}$	7	$1\frac{7}{64}$
$\frac{9}{16}$	12	$\frac{15}{32}$	$1\frac{5}{16}$	7	$1\frac{11}{64}$
$\frac{5}{8}$	11	$\frac{17}{32}$	$1\frac{3}{8}$	6	$1\frac{7}{32}$
$\frac{11}{16}$	11	$\frac{19}{32}$	$1\frac{1}{2}$	6	$1\frac{11}{32}$
$\frac{3}{4}$	10	$\frac{41}{64}$	$1\frac{5}{8}$	$5\frac{1}{2}$	$1\frac{7}{16}$
$\frac{13}{16}$	10	$\frac{45}{64}$	$1\frac{3}{4}$	5	$1\frac{35}{64}$
$\frac{7}{8}$	9	$\frac{49}{64}$	$1\frac{7}{8}$	5	$1\frac{43}{64}$
$\frac{15}{16}$	9	$\frac{53}{64}$	2	$4\frac{1}{2}$	$1\frac{25}{32}$

*By 64ths of an inch, and for 75% of a full thread.

TAP-DRILL SIZES*

American National Fine Thread (N.F.)

Dia. of Screw	*Thds. per In.*	*Dia. of Tap Drill*	*Dia. of Screw*	*Thds. per In.*	*Dia. of Tap Drill*
$\frac{1}{4}$	28	$\frac{13}{64}$	$\frac{5}{8}$	18	$\frac{9}{16}$
$\frac{5}{16}$	24	$\frac{17}{64}$	$\frac{11}{16}$	16	$\frac{39}{64}$
$\frac{3}{8}$	24	$\frac{21}{64}$	$\frac{3}{4}$	16	$\frac{43}{64}$
$\frac{7}{16}$	20	$\frac{3}{8}$	$\frac{7}{8}$	14	$\frac{51}{64}$
$\frac{1}{2}$	20	$\frac{7}{16}$	1	14	$\frac{59}{64}$
$\frac{9}{16}$	18	$\frac{1}{2}$			

TAP-DRILL SIZES FOR "V" THREADS*

Thread Diameter	*Threads per In.*	*Dia. of Tap Drill*	*Thread Diameter*	*Threads per In.*	*Dia. of Tap Drill*
$\frac{1}{4}$	20	$\frac{3}{16}$	1	8	$\frac{53}{64}$
$\frac{5}{16}$	18	$\frac{15}{64}$	$1\frac{1}{16}$	8	$\frac{57}{64}$
$\frac{3}{8}$	16	$\frac{19}{64}$	$1\frac{1}{8}$	7	$\frac{15}{16}$
$\frac{7}{16}$	14	$\frac{11}{32}$	$1\frac{3}{16}$	7	1
$\frac{1}{2}$	12	$\frac{25}{64}$	$1\frac{1}{4}$	7	$1\frac{1}{16}$
$\frac{9}{16}$	12	$\frac{29}{64}$	$1\frac{5}{16}$	7	$1\frac{1}{8}$
$\frac{5}{8}$	11	$\frac{1}{2}$	$1\frac{3}{8}$	6	$1\frac{5}{32}$
$\frac{11}{16}$	11	$\frac{9}{16}$	$1\frac{1}{2}$	6	$1\frac{9}{32}$
$\frac{3}{4}$	10	$\frac{39}{64}$	$1\frac{5}{8}$	5	$1\frac{23}{64}$
$\frac{13}{16}$	10	$\frac{43}{64}$	$1\frac{3}{4}$	5	$1\frac{31}{64}$
$\frac{7}{8}$	9	$\frac{23}{32}$	$1\frac{7}{8}$	$4\frac{1}{2}$	$1\frac{37}{64}$
$\frac{15}{16}$	9	$\frac{25}{32}$	2	$4\frac{1}{2}$	$1\frac{45}{64}$

CAP SCREWS

		Hexagonal Head		*Fillister Head*	
Diameter of Screw	*Threads per Inch*	*Distance Across Flats*	*Thickness of Head*	*Diameter of Head*	*Thickness of Head*
$\frac{1}{4}$	20	$\frac{7}{16}$	$\frac{3}{16}$	$\frac{3}{8}$	$\frac{7}{32}$
$\frac{5}{16}$	18	$\frac{1}{2}$	$\frac{15}{64}$	$\frac{7}{16}$	$\frac{1}{4}$
$\frac{3}{8}$	16	$\frac{9}{16}$	$\frac{9}{32}$	$\frac{9}{16}$	$\frac{5}{16}$
$\frac{7}{16}$	14	$\frac{5}{8}$	$\frac{21}{64}$	$\frac{5}{8}$	$\frac{3}{8}$
$\frac{1}{2}$	13	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{13}{32}$

*By 64ths of an inch, and for 75% of a full thread.

WEIGHTS OF SHEET STEEL AND IRON

Number of Gauge	Approx. Thickness in Inches	Weight per Square Foot		
		Galvanized Iron	Black Iron	Black Steel
10	.138	5.781	5.625	5.737
11	.123	5.156	5.000	5.100
12	.107	4.531	4.375	4.462
13	.092	3.906	3.750	3.825
14	.077	3.281	3.125	3.156
15	.069	2.969	2.812	2.869
16	.061	2.656	2.50	2.550
17	.055	2.406	2.25	2.295
18	.049	2.156	2.00	2.040
19	.044	1.906	1.75	1.785
20	.037	1.656	1.50	1.530
21	.034	1.531	1.375	1.402
22	.031	1.406	1.25	1.275
23	.028	1.281	1.125	1.147
24	.025	1.156	1.00	1.020
25	.022	1.031	.875	.892
26	.019	.906	.75	.765
27	.017	.844	.687	.701
28	.016	.781	.625	.637
29	.014	.719	.562	.574
30	.012	.656	.5	.510

WEIGHTS OF FLAT SIZES OF STEEL IN POUNDS PER LINEAR FOOT

Thickness	Width of Stock						
	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
$\frac{1}{8}$	.213	.320	.426	.530	.640	.745	.850
$\frac{3}{16}$	.319	.480	.639	.790	.960	1.12	1.28
$\frac{1}{4}$	.425	.640	.852	1.06	1.28	1.49	1.70
$\frac{5}{16}$	.531	.800	1.06	1.33	1.60	1.86	2.13
$\frac{3}{8}$	.638	.960	1.28	1.59	1.91	2.23	2.55
$\frac{7}{16}$	.744	1.12	1.49	1.86	2.23	2.60	2.98
$\frac{1}{2}$	....	1.28	1.70	2.13	2.55	2.98	3.40
$\frac{9}{16}$	....	1.44	1.91	2.39	2.87	3.35	3.83
$\frac{5}{8}$	....	1.60	2.12	2.66	3.19	3.72	4.26
$\frac{11}{16}$	....	1.76	2.34	2.92	3.51	4.09	4.68
$\frac{3}{4}$	....	....	2.55	3.19	3.83	4.46	5.10
$\frac{13}{16}$	....	....	2.76	3.45	4.14	4.83	5.53
$\frac{7}{8}$	....	....	2.98	3.72	4.46	5.21	5.96
$\frac{15}{16}$	....	....	3.19	3.98	4.78	5.58	6.38
1	....	....	....	4.25	5.10	5.96	6.80

WEIGHTS OF STEEL PER LINEAR FOOT

Diameter or Dist. Across Flats	*Steel—Weight per Foot*			
	Round	*Square*	*Hexagon*	*Octagon*
1/8	.042	.053	.046	.044
1/4	.167	.212	.185	.177
3/8	.375	.478	.414	.398
1/2	.667	.850	.737	.708
5/8	1.043	1.328	1.151	1.107
3/4	1.502	1.913	1.658	1.584
7/8	2.044	2.603	2.256	2.156
1	2.670	3.400	2.947	2.817
1 1/8	3.379	4.303	3.730	3.568
1 1/4	4.173	5.312	4.605	4.407
1 3/8	5.049	6.428	5.571	5.331
1 1/2	6.008	7.650	6.631	6.344
1 5/8	7.051	8.978	7.776	7.446
1 3/4	8.178	10.41	9.025	8.635
1 7/8	9.388	11.95	10.36	9.918
2	10.68	13.60	11.79	11.28
2 1/4	13.52	17.22	14.92	14.24
2 1/2	16.69	21.25	18.42	17.65
2 3/4	20.20	25.71	22.29	21.28
3	24.03	30.60	26.53	25.36

WEIGHTS OF EQUIVALENT SIZES OF OTHER STOCKS

1. To determine the weights of equivalent sizes and shapes of *Iron* rods, multiply the weights of the respective sizes in steel by: 1.006.
2. To determine the weights of equivalent sizes and shapes of *Brass* rods, multiply the weights of the respective sizes in steel by: 1.072.
3. To determine the weights of equivalent sizes and shapes of *Copper* rods, multiply the weights of the respective sizes in steel by: 1.131.
4. To determine the weights of equivalent sizes and shapes of *Aluminum* rods multiply the weights of the respective sizes in steel by: 0.328.

NUMBER OF WORDS PER SQUARE INCH OF PRINTED MATTER

Size Type	*Approximate No.*
12-point solid	14
12-point leaded	11
11-point solid	17
11-point leaded	14
10-point solid	21
10-point leaded	16
9-point solid	28
9-point leaded	21
8-point solid	32
8-point leaded	23
7-point solid	38
7-point leaded	27
6-point solid	47
6-point leaded	34

DECIMAL EQUIVALENTS OF FRACTIONAL PARTS OF ONE INCH

$\frac{1}{64}$	.015625	$\frac{33}{64}$	.515625
$\frac{1}{32}$	.031250	$\frac{17}{32}$	.531250
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.062500	$\frac{9}{16}$	.562500
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.093750	$\frac{19}{32}$	.593750
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.125000	$\frac{5}{8}$	.625000
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.156250	$\frac{21}{32}$	.656250
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.187500	$\frac{11}{16}$	.687500
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125
$\frac{7}{32}$	.218750	$\frac{23}{32}$	.718750
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.250000	$\frac{3}{4}$	.750000
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.281250	$\frac{25}{32}$	.781250
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.312500	$\frac{13}{16}$	.812500
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.343750	$\frac{27}{32}$	.843750
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.375000	$\frac{7}{8}$	.875000
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.406250	$\frac{29}{32}$	.906250
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.437500	$\frac{15}{16}$	.937500
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.468750	$\frac{31}{32}$	.968750
$\frac{31}{64}$	.484375	$\frac{63}{64}$	.984375
$\frac{1}{2}$	.500000	1	1.000000

LUMBER TABLE

Board-Foot Measure of Standard Sizes for Given Lengths

Size in Inches	*Length in Feet*			
	10	*12*	*14*	*16*
1 x 6	5	6	7	8
1 x 8	$6\frac{2}{3}$	8	$9\frac{1}{3}$	$10\frac{2}{3}$
1 x10	$8\frac{1}{3}$	10	$11\frac{2}{3}$	$13\frac{1}{3}$
1 x12	10	12	14	16
1 x14	$11\frac{2}{3}$	14	$16\frac{1}{3}$	$18\frac{2}{3}$
$1\frac{1}{4}$x 4	$4\frac{1}{6}$	5	$5\frac{5}{6}$	$6\frac{2}{3}$
$1\frac{1}{4}$x 6	$6\frac{1}{4}$	$7\frac{1}{2}$	$8\frac{3}{4}$	10
$1\frac{1}{4}$x 8	$8\frac{1}{3}$	10	$11\frac{2}{3}$	$13\frac{1}{3}$
$1\frac{1}{2}$x 4	5	6	7	8
$1\frac{1}{2}$x 6	$7\frac{1}{2}$	9	$10\frac{1}{2}$	12
$1\frac{1}{2}$x 8	10	12	14	16
2 x 4	$6\frac{2}{3}$	8	$9\frac{1}{3}$	$10\frac{2}{3}$
2 x 6	10	12	14	16
2 x 8	$13\frac{1}{3}$	16	$18\frac{2}{3}$	$21\frac{1}{3}$

AREAS AND CIRCUMFERENCES OF CIRCLES

Dia.	*Area*	*Circum.*	*Dia.*	*Area*	*Circum.*
$\frac{1}{8}$	0.01227	0.392699	1	0.7854	3.14159
$\frac{3}{16}$	0.02761	0.589049	$1\frac{1}{8}$	0.99402	3.53429
$\frac{1}{4}$	0.04909	0.785398	$1\frac{1}{4}$	1.2272	3.92699
$\frac{5}{16}$	0.07670	0.981748	$1\frac{3}{8}$	1.4849	4.31969
$\frac{3}{8}$	0.11045	1.17810	$1\frac{1}{2}$	1.7671	4.71239
$\frac{7}{16}$	0.15033	1.37445	$1\frac{5}{8}$	2.0739	5.10509
$\frac{1}{2}$	0.19635	1.57080	$1\frac{3}{4}$	2.4053	5.49780
$\frac{9}{16}$	0.24850	1.76715	$1\frac{7}{8}$	2.7612	5.89049
$\frac{5}{8}$	0.30680	1.96350	2	3.1416	6.28319
$\frac{11}{16}$	0.37122	2.15984	$2\frac{1}{4}$	3.9761	7.06858
$\frac{3}{4}$	0.44179	2.35619	$2\frac{1}{2}$	4.9087	7.85398
$\frac{13}{16}$	0.51849	2.55254	$2\frac{3}{4}$	5.9396	8.63938
$\frac{7}{8}$	0.60132	2.74889	3	7.0686	9.42478
$\frac{15}{16}$	0.69029	2.94524			

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